Research Article

Geometric Location Based on TDOA for Wireless Sensor Networks

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Wireless sensor networks (WSNs) consist of a large number of low-cost miniature sensors, which can be applied to battlefield surveillance, environmental monitoring, target tracking, and other applications related to the positions of sensors. The location information of sensors is of great importance for wireless sensor networks. In this paper, we propose a new localization algorithm for the wireless sensor network based on time difference of arrival (TDOA), which is a typical algorithm in the wireless localization field. In order to improve the localization accuracy of a sensor, a new strategy is proposed for a localized sensor being upgraded to an anchor node, which is used to localize the position of the next sensor. Performance analysis and simulation results show that the revised TODA localization algorithm has the higher localization accuracy when compared with the original TDOA location method.

1. Introduction

Wireless sensor networks (WSNs) are a technique which has a wide variety of applications, such as target tracking, battlefield surveillance, and environmental monitoring [1], and the position of the nodes is a crucial issue for that all the applications are based on the location information. GPS (global positioning system) is widely used today, and it could achieve the satisfied accuracy outdoors. However, it is relatively expensive when all the sensors are equipped with GPS receivers [2], and it is not working inside buildings. Therefore, many algorithms that do not rely on GPS are proposed to estimate the positions of the sensor nodes in the network in recent years [3–8].

The trilateral positioning [2] is commonly adopted for the sensor localization. Its main idea can be illustrated by Figure 1. Let (x_a, y_a) , (x_b, y_b) , and (x_c, y_c) be coordinate vectors of the anchor nodes A, B, and C, respectively. The measurement distances from those anchors to the unknown sensor D are denoted by d_a , d_b , and d_c . Let $D(x, y)$ be the coordinate vector

Figure 1: Trilateral positioning method.

of the unknown sensor, which needs to be determined. Then, from the Euclidean distance formula, we obtain

$$\begin{aligned}\sqrt{(x - x_a)^2 + (y - y_a)^2} &= d_a, \\ \sqrt{(x - x_b)^2 + (y - y_b)^2} &= d_b, \\ \sqrt{(x - x_c)^2 + (y - y_c)^2} &= d_c.\end{aligned}\tag{1.1}$$

When those three points A, B, C are not collinear, that is, the following matrix:

$$\begin{bmatrix} x_a - x_c & y_a - y_c \\ x_b - x_c & y_b - y_c \end{bmatrix}\tag{1.2}$$

is nonsingular, then, from solving (1.1), we obtain the estimation position of the unknown sensor for $D(x, y)$:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x_a - x_c & y_a - y_c \\ x_b - x_c & y_b - y_c \end{bmatrix}^{-1} \begin{bmatrix} x_a^2 + y_a^2 - x_c^2 - y_c^2 - d_a^2 + d_c^2 \\ x_b^2 + y_b^2 - x_c^2 - y_c^2 - d_b^2 + d_c^2 \end{bmatrix}.\tag{1.3}$$

The basic assumption for the trilateral positioning method (1.1) is that the measurement distances from anchors to the unknown sensor do not have errors. However, this assumption is almost impossible in practice. Furthermore, when the matrix (1.2) is ill-conditioned, the small measurement errors result in a large amplified error of the estimated position of the unknown sensors. Therefore, in order to improve the localization precision in the case of measurement errors, we consider the sensor network localization method based on the TDOA method [7, 8], of which localization precision is not so sensitive to measurement errors.

The localization method based on TDOA estimates the unknown sensor node $D(x, y)$ by the given coordinates of anchors and time difference of arrival from those anchors to the unknown sensor D , which can be illustrated by Figure 2.

Figure 2: The localization method based on TDOA.

The difference of the distances from the anchors P_i and P_j to the unknown sensor D is denoted by Δd_{ij}

$$\Delta d_{ij} \triangleq \|P_i - D\| - \|P_j - D\|, \quad i, j = 1, 2, \dots, N, \quad (1.4)$$

where $\|\cdot\|$ denotes the Euclidean norm, that is, $\|x_i - x_j\|$ denotes the Euclidean distance between x_i and x_j .

From (1.4), we obtain

$$\|P_i - D\| = \|P_j - D\| + \Delta d_{ij}, \quad (1.5)$$

which gives

$$\|P_i\|^2 - \|P_j\|^2 - \Delta d_{ij}^2 = 2(P_i - P_j)D^T + 2\Delta d_{ij}\|P_j - D\|. \quad (1.6)$$

All terms divide by Δd_{ij} in (1.6); then we obtain

$$\frac{2}{\Delta d_{ij}}(P_i - P_j)D^T + 2\|P_j - D\| = \frac{(\|P_i\|^2 - \|P_j\|^2)}{\Delta d_{ij} - \Delta d_{ij}}. \quad (1.7)$$

Let $j = 1$ and $i = 2$ in (1.7), then we get

$$\frac{2}{\Delta d_{21}}(P_2 - P_1)D^T + 2\|P_1 - D\| = \frac{\|P_2\|^2 - \|P_1\|^2}{\Delta d_{21}} - \Delta d_{21}. \quad (1.8)$$

Let $j = 1$ and $i = 3, \dots, N$ in (1.8) and two sides of (1.7) subtract two sides of (1.8), then we obtain

$$\left(\frac{2(P_i - P_1)}{\Delta d_{i1}} - \frac{2(P_2 - P_1)}{\Delta d_{21}} \right) D^T = \frac{\|P_i\|^2 - \|P_1\|^2}{\Delta d_{i1}} - \frac{\|P_2\|^2 - \|P_1\|^2}{\Delta d_{21}} - (\Delta d_{i1} - \Delta d_{21}). \quad (1.9)$$

Therefore, via solving the linear system (1.9), we obtain the coordinate vector $D(x, y)$ of the current undetermined sensor. When the number of anchors N is greater than 4, the system (1.9) is overdetermined. In this case, from the linear least-squares method, we obtain the approximation solution D by

$$D^T = (A^T A)^{-1} A^T b, \quad (1.10)$$

where

$$A = \begin{bmatrix} \frac{2(P_3 - P_1)}{\Delta d_{31}} - \frac{2(P_2 - P_1)}{\Delta d_{21}} \\ \vdots \\ \frac{2(P_N - P_1)}{\Delta d_{N1}} - \frac{2(P_2 - P_1)}{\Delta d_{21}} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{\|P_3\|^2 - \|P_1\|^2}{\Delta d_{31}} - \frac{\|P_2\|^2 - \|P_1\|^2}{\Delta d_{21}} - (\Delta d_{31} - \Delta d_{21}) \\ \vdots \\ \frac{\|P_N\|^2 - \|P_1\|^2}{\Delta d_{N1}} - \frac{\|P_2\|^2 - \|P_1\|^2}{\Delta d_{21}} - (\Delta d_{N1} - \Delta d_{21}) \end{bmatrix}. \quad (1.11)$$

This paper is organized as follows. In the next section, we give a revised geometric localization algorithm based on TDOA for the sensor network. In Section 3, for typical sensor network localization model, we compare our revised localization algorithm with the original TDOA localization method (1.10). In Section 4, some discussions are also given.

2. Revised Localization Algorithm Based on TDOA

It needs at least four anchors ($N \geq 4$) to determine uniquely the position of a sensor by TDOA method. When there are not enough anchors for the unknown sensor, Savarese et al. [5] and Savvides et al. [6] proposed an iterative multilateral positioning method to handle this case. Its key point is that the localized sensor can be regarded as a new anchor to locate the next sensor. Evidently, this strategy could help locate more sensors when the anchors are sparse. And in this paper, we propose a revised strategy when a determined sensor is upgraded to an anchor node, that is, only the localized sensors with high precision can be upgraded to be anchor nodes, for it is obvious that the error propagates to the later sensor when the lower precision sensor is used to localize the later sensor node.

Since there are errors in the measurement distances and the propagating errors in the process of numerical computation, recalculating the positions of the determined sensors help to reduce the accumulated errors, which are used by generic localized algorithm [9], n -hop multilateral primitive algorithm, and the two-phase positioning algorithm [10]. Here, we also borrow this refinement idea to improve the computing accuracy of the sensor position. In order to overcome the overhead and reduce the cost of refinement computation, we set the threshold of the number of the iteration refinement (the typical value is 3). Specifically, our revised geometric localization algorithm based on TDOA for sensor networks can be described as follows.

Algorithm 2.1 (geometric localization method based on TDOA). Firstly, search for at least four anchors around the current undetermined sensor node and use their distances with this sensor node and the TDOA method (1.10) to locate this unknown sensor node.

Secondly, use the geometric distance measurement error (GDME) to measure the localization accuracy. The GDME is defined as follows:

$$\text{GDME} = \max \left\{ \frac{|d_{ij} - \hat{d}_{ij}|}{d_{ij}}, j = 1 : N \right\}, \quad (2.1)$$

where d_{ij} is the measurement distance between the i th unknown sensor and the j th anchor around this sensor and $\hat{d}_{ij}$ is their distance computed according to the estimated coordinate of this sensor and the coordinate of the j th anchor by Euclidean distance formula. When the GDME is smaller than the prescribed threshold, this current determined sensor node is upgraded to the anchor node, that is, the lower precision localization sensor is not involved in the later sensor localization. Update the anchor set and the unknown sensor set, and repeat this step, until every unknown sensor is scanned for localization one time.

Thirdly, for those lower precision sensors which are not upgraded to the anchor nodes, in this refinement stage, we use the coordinate information of determined sensors and anchors and their distances recalculate the coordinates of those lower precision sensors.

3. Simulation Results

The simulation model is generated as follows. There are 60 points randomly generated in a one by one square area. Then, we choose the first 10% points as the anchor nodes and compute their all Euclidean distances d_{ij} . Only those distances which are less than the given radio range RD are viewed as the given distances and the rest distances are looked as unknown. Add a multiplicative random noise to every given distance as the measurement distance:

$$\hat{d}_{ij} = d_{ij}(1 + nf \cdot \text{randn}(1)), \quad (3.1)$$

where nf is a given noise factor and $\text{randn}(1)$ is a standard Gaussian random variable function.

We use the positioning accuracy and the successful localization probability to evaluate the performance of an algorithm. The positioning accuracy is defined by GDME (2.1) or defined by root mean square deviation (RMSD) as follows:

$$\text{RMSD} = \frac{1}{\sqrt{2}} \|x_i - \bar{x}_i\|, \quad (3.2)$$

where x_i is the real position of the i th sensor and $\bar{x}_i$ is the estimated position by the localization algorithm. The successful localization probability is defined by

$$\text{LP} = \frac{m}{n} \times 100\%, \quad (3.3)$$

where n is the number of sensors and m is the number of located sensors in the network.

Figure 3: Localization results of Algorithm 2.1 and the original TDOA method.

In order to verify the performance of the revised geometric localization method based on (Algorithm 2.1), we compare it with the original TDOA method. Figure 3 is their localization results for the test model with the small measurement noise case. The blue diamond node refers to the position of an anchor, and the blue cross node refers to the original location of an unknown sensor. The red star node refers to the located sensor whose localization accuracy meets the localization requirement. The green circle node refers to the located sensor whose localization accuracy does not meet the localization requirement, and the black circle node refers to the sensor which cannot be determined by the localization algorithm. The discrepancy between the original position and the localized position of a sensor node is indicated by the solid line.

From Figure 3, we find that all unknown sensors are located by Algorithm 2.1 (the revised geometric localization method based on TDOA), and some sensors cannot be located and some located sensors have high errors for the original TDOA method.

In order to evaluate the performance of Algorithm 2.1, we test this algorithm for simulation model with large measurement distance noises, compared with the original TDOA. Figures 4 and 5 are the simulation results of Algorithm 2.1, and the original TDOA method with the RMSD and GDME criteria when the measurement distance error is 2%, respectively. Figures 6 and 7 are the relations of localization accuracy and the successful localization probability with the RMSD and GDME criteria when the measurement distance error is 2%, respectively.

From Figure 4 to Figure 7, we find that the successful localization probability of the algorithm is superior to which of the original TDOA localization method under the same localization accuracy.

For a real wireless sensor network, the measurement error is not avoided and the localization accuracy depends on the measurement error heavily. Therefore, we evaluate the performances of Algorithm 2.1 and the original TDOA method when the relative measurement errors vary from 2% to 20%. Their successful localization probabilities and the localization accuracy are put in Tables 1 and 2, respectively. The sensor is regarded as the successful localization if the localization error is less than the measurement error.

(a)

(b)

Figure 4: $nf = 2\%$, localization errors with RMSD.

(a)

(b)

Figure 5: $nf = 2\%$, localization errors with GDME.

From Tables 1 and 2, we find that the algorithm is effective even if the relative measurement error attains to 20%, and it is superior to the original TDOA localization method according to the evaluation criteria of the successful localization probability and the localization accuracy.

Figure 6: $nf = 2\%$, localization probability with RMSD.

Figure 7: $nf = 2\%$, localization probability with GDME.

4. Conclusions

Our idea is to use the revised geometric localization method based on TDOA for the wireless sensor network with sparse anchors. For improving the localization accuracy, in the original TDOA localization method, we use the threshold to judge whether the determined sensor is upgraded an anchor node and the iterative refinement idea. Furthermore, for the typical

Table 1: The successful localization probability with different relative measurement errors.

Noise factor	Improved TDOA	Original TDOA
Null (Exact Distance)	100.00%	92.78%
2%	100.00%	90.56%
4%	100.00%	91.23%
6%	100.00%	90.56%
8%	100.00%	89.38%
10%	100.00%	88.15%
12%	100.00%	89.38%
14%	100.00%	88.27%
16%	100.00%	87.22%
18%	100.00%	87.72%
20%	100.00%	88.15%

Table 2: The localization accuracy with different relative measurement error.

Noise factor	Improved TDOA		Original TDOA	
	RMSD	GDME	RMSD	GDME
Null (Exact Distance)	$1.3464E - 15$	$1.5413E - 14$	$4.1999E - 01$	$4.4121E + 00$
2%	$5.6392E - 04$	$2.7764E - 03$	$7.1842E - 01$	$6.5894E + 00$
4%	$4.5478E - 04$	$6.5198E - 03$	$2.7080E - 01$	$2.6745E + 00$
6%	$7.7822E - 04$	$8.3309E - 03$	$4.6763E - 01$	$4.4270E + 00$
8%	$9.4769E - 04$	$9.0917E - 03$	$2.7973E - 01$	$2.2608E + 00$
10%	$7.4960E - 04$	$7.8121E - 03$	$5.9484E - 01$	$4.0471E + 00$
12%	$1.1071E - 03$	$1.8286E - 02$	$4.5207E - 01$	$5.4583E + 00$
14%	$2.2427E - 03$	$1.7131E - 02$	$1.7496E + 00$	$3.3375E + 01$
16%	$9.8465E - 04$	$1.2029E - 02$	$9.3667E - 01$	$7.5572E + 00$
18%	$6.7687E - 04$	$7.3791E - 03$	$4.9262E - 01$	$6.3574E + 00$
20%	$1.0773E - 03$	$1.0983E - 02$	$4.6257E - 01$	$4.4013E + 00$

simulation model of wireless sensor network localization problem, this revised strategy is effective, compared with the original TDOA localization method.

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