Research Article

A New System of Random Generalized Variational Inclusions with Random Fuzzy Mappings and Random $(H(\cdot, \cdot), \phi)$ - η -Accretive Mappings in Banach Spaces

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We introduce a new notion of random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and prove the Lipschitz continuity of the random resolvent operator associated with the random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings. We introduce and study a new system of random generalized variational inclusions with random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and random fuzzy mappings in Banach spaces. By using the random resolvent operator, an iterative algorithm for solving such system of random generalized variational inclusions is constructed in Banach spaces. Under some suitable conditions, we prove the convergence of the iterative sequences generated by the algorithm.

1. Introduction

Variational inclusions and variational inequalities have wide applications to many fields including, for example, economics, optimization and control theory, operators research, transportation network modeling, and mathematical programming. For these reasons, various variational inclusions and variational inequalities have been intensively studied in recent years. For details, we refer the reader to [1–3] and references therein.

Recently, Wang and Ding [4], and Nazemi [5] introduced the concepts of $(H(\cdot, \cdot), \eta)$ -accretive mappings, and (H, ϕ) - η -accretive mappings, respectively, which generalize the notion of H -accretive mappings, (H, η) -accretive mappings, $H(\cdot, \cdot)$ -accretive mappings, (A, η) -accretive mappings and other existing accretive mappings as special cases. They also defined the resolvent operators associated with this mappings, and shown their Lipschitz continuity.

On the other hand, it is well known that the study of the random equations involving the random operators in view of their need for dealing with probabilistic models in applied

sciences is very important. In 1989, Chang and Zhu [6] and Chang [7] first introduced the concepts of variational inequalities for fuzzy mappings. Since then several classes of variational inequalities for fuzzy mappings were considered by Chang and Huang [8], Noor [9, 10], Noor and Al-Said [11], Ding [12], Ding and Park [13], and Park and Jeong [14, 15] in the setting of Hilbert spaces. Recently, the random variational inequalities and random variational inclusion problems have been introduced and studied by Dai [16], Cho and Lan [17], Lan [18], Ahmad and Bazán [19], Chang and Huang [20, 21], Zang and Zhu [22], Huang [23, 24], Husain et al. [25], etc.

Motivated and inspired by the recent research works in this fascinating area, in this paper, we introduce a new class of random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and study a new system of random generalized variational inclusions with random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and random fuzzy mappings in Banach spaces. An iterative algorithm is defined to compute the approximate solutions of system of random generalized variational inclusions. The convergence of iterative sequences generated by the algorithm is also shown. Our results improve and generalize many known corresponding results.

2. Preliminaries

Let E be a real Banach space with dual space E^* and $\langle \cdot, \cdot \rangle$ the dual pair between E^* and E . Let $F(E)$ be a collection of all fuzzy sets over E . A mapping F from E into $F(E)$ is called a fuzzy mapping on E . If F is a fuzzy mapping on E , then $F(x)$ (denote it by F_x in the sequel) is a fuzzy set on E and $F_x(y)$ is the membership function of y in F_x . Let $N \in F(E)$, $q \in [0, 1]$, then the set

$$(N)_q = \{x \in E : N(x) \geq q\} \quad (2.1)$$

is called a q -cut set of N .

Let (Ω, Σ) be a measurable space, where Ω is a set and Σ is a σ -algebra of subsets of Ω . Let $\beta(E)$, 2^E , $\text{CB}(E)$, and $\widetilde{H}(\cdot, \cdot)$ be the class of Borel σ -fields in E , the family of all nonempty subsets of E , the family of all nonempty, closed and bounded subsets of E , and the Hausdorff metric on $\text{CB}(E)$, respectively. Recall that the generalized duality mapping $J_q : E \rightarrow 2^{E^*}$ is defined by

$$J_q(x) = \left\{ f \in E^* : \langle x, f \rangle = \|x\|^q, \|x\|^{q-1} = \|f\| \right\}, \quad \forall x \in E, \quad (2.2)$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-1} J_2(x)$ for all $x \neq 0$, and J_q is single-valued if E^* is strictly convex. In the sequel, unless otherwise specified, we always suppose that E is a real Banach space such that J_q is single-valued. If $E = \mathcal{H}$ is a Hilbert space, then J_2 becomes the identity mapping on H .

The modulus of smoothness of E is the function $\rho_E : [0, \infty) \rightarrow [0, \infty)$ defined by

$$\rho_E(t) = \sup \left\{ \frac{1}{2} (\|x + y\| + \|x - y\|) - 1 : \|x\| \leq 1, \|y\| \leq t \right\}. \quad (2.3)$$

A Banach space E is called uniformly smooth if

$$\lim_{t \rightarrow 0} \frac{\rho_E(t)}{t} = 0. \quad (2.4)$$

E is called q -uniformly smooth if there exists a constant $c > 0$ such that

$$\rho_E(t) \leq ct^q, \quad q > 1. \quad (2.5)$$

Note that J_q is single-valued if E is uniformly smooth. In the following, we give an important inequality due to Xu [26] in q -uniformly smooth Banach spaces, which will play a crucial role in constructing the iterative algorithm to approximate the solution of system of variational inclusions.

Lemma 2.1 (see [26]). *Let E be a real uniformly smooth Banach space. Then E is q -uniformly smooth if and only if there exists a constant $c_q > 0$ such that*

$$\|x + y\|^q \leq \|x\|^q + q\langle y, J_q(x) \rangle + c_q \|y\|^q. \quad (2.6)$$

Definition 2.2. A mapping $x : \Omega \rightarrow E$ is said to be measurable if for any $B \in \beta(E)$, $\{t \in \Omega : x(t) \in B\} \in \Sigma$.

Definition 2.3. A mapping $f : \Omega \times E \rightarrow E$ is called a random single-valued mapping if for any $x \in E$, $f(t, x) = y(t)$ is measurable. A random mapping f is said to be continuous if for any $t \in \Omega$, the mapping $f(t, \cdot) : E \rightarrow E$ is continuous. Similarly, we can define a random mapping $b : \Omega \times E \times E \rightarrow E$. We will write $f_t(x) = f(t, x(t))$ and $b_t(x, y) = b(t, x(t), y(t))$ for all $t \in \Omega$ and $x(t), y(t) \in E$.

It is well known that a measurable mapping is necessarily a random mapping.

Definition 2.4. A multivalued mapping $G : \Omega \rightarrow 2^E$ is said to be measurable if for any $B \in \beta(E)$, $G^{-1}(B) = \{t \in \Omega : G(t) \cap B \neq \emptyset\} \in \Sigma$.

Definition 2.5. A mapping $u : \Omega \rightarrow E$ is called a measurable section of a multivalued mapping $\Gamma : \Omega \rightarrow 2^E$ if u is measurable and for any $t \in \Omega$, $u(t) \in \Gamma(t)$.

Definition 2.6. A mapping $T : \Omega \times E \rightarrow 2^E$ is called a random multivalued mapping if for any $x \in E$, $T(\cdot, x)$ is measurable. A random multivalued mapping $T : \Omega \times E \rightarrow \text{CB}(E)$ is said to be $\widetilde{H}$ -continuous if for any $t \in \Omega$, $T(t, \cdot)$ is continuous in $\widetilde{H}(\cdot, \cdot)$, where $\widetilde{H}(\cdot, \cdot)$ is the Hausdorff metric on $\text{CB}(E)$ defined as follows: for any given $A, B \in \text{CB}(E)$,

$$\widetilde{H}(A, B) = \max \left\{ \sup_{x \in A} \inf_{y \in B} \|x - y\|, \sup_{y \in B} \inf_{x \in A} \|x - y\| \right\}. \quad (2.7)$$

Definition 2.7. A fuzzy mapping $F : \Omega \rightarrow F(E)$ is called measurable if for any $\alpha \in (0, 1]$, $(F(\cdot))_\alpha : \Omega \rightarrow 2^E$ is a measurable multivalued mapping.

Definition 2.8. A fuzzy mapping $F : \Omega \times E \rightarrow F(E)$ is called a random fuzzy mapping if for any $x \in E$, $F(\cdot, x) : \Omega \rightarrow F(E)$ is a measurable fuzzy mapping.

Lemma 2.9 (see [27]). Let $T : \Omega \times E \rightarrow CB(E)$ be a $\widetilde{H}$ -continuous random multivalued mapping. Then for any measurable mapping $w : \Omega \rightarrow E$ the multivalued mapping $T(\cdot, w(\cdot)) : \Omega \rightarrow CB(E)$ is measurable.

Lemma 2.10 (see [27]). Let $S, T : \Omega \rightarrow CB(E)$ be two measurable multivalued mappings, $\epsilon > 0$ a constant, and $v : \Omega \rightarrow E$ a measurable selection of S . Then there exists a measurable selection $w : \Omega \rightarrow E$ of T such that for all $t \in \Omega$,

$$\|v(t) - w(t)\| \leq (1 + \epsilon)\widetilde{H}(S(t), T(t)). \quad (2.8)$$

Definition 2.11. A random mapping $P : \Omega \times E \rightarrow E$ is said to be

(i) accretive if

$$\langle P_t(x_1) - P_t(x_2), J_q(x_1(t) - x_2(t)) \rangle \geq 0, \quad \forall x_1(t), x_2(t) \in E, t \in \Omega; \quad (2.9)$$

(ii) r -strongly accretive if there exists a measurable function $r : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle P_t(x_1) - P_t(x_2), J_q(x_1(t) - x_2(t)) \rangle \\ & \geq r(t)\|x_1(t) - x_2(t)\|^q, \quad \forall x_1(t), x_2(t) \in E, t \in \Omega; \end{aligned} \quad (2.10)$$

(iii) m -relaxed accretive if there exists a measurable function $m : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle P_t(x_1) - P_t(x_2), J_q(x_1(t) - x_2(t)) \rangle \\ & \geq -m(t)\|x_1(t) - x_2(t)\|^q, \quad \forall x_1(t), x_2(t) \in E, t \in \Omega; \end{aligned} \quad (2.11)$$

(iv) (α, ξ) -relaxed cocoercive if there exist measurable functions $\alpha : \Omega \rightarrow (0, \infty)$, $\xi : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle P_t(x_1) - P_t(x_2), J_q(x_1(t) - x_2(t)) \rangle \\ & \geq -\alpha(t)\|P_t(x_1) - P_t(x_2)\|^q + \xi(t)\|x_1(t) - x_2(t)\|^q, \\ & \quad \forall x_1(t), x_2(t) \in E, t \in \Omega; \end{aligned} \quad (2.12)$$

(v) λ -Lipschitz continuous if there exists a measurable function $\lambda : \Omega \rightarrow (0, \infty)$ such that

$$\|p_t(x_1) - p_t(x_2)\| \leq \lambda(t)\|x_1(t) - x_2(t)\|, \quad \forall x_1(t), x_2(t) \in E, t \in \Omega. \quad (2.13)$$

Definition 2.12. A random multivalued mapping $M : \Omega \times E \rightarrow 2^E$ is said to be

(i) accretive if

$$\begin{aligned} & \langle u(t) - v(t), J_q(x_1(t) - x_2(t)) \rangle \\ & \geq 0, \quad \forall u(t) \in M_t(x_1), v(t) \in M_t(x_2), \quad x_1(t), x_2(t) \in E, \quad t \in \Omega; \end{aligned} \quad (2.14)$$

(ii) m -accretive if M is accretive and $(I_t + \rho(t)M_t)(E) = E$ holds for all $t \in \Omega$ and $\rho(t) > 0$.

Remark 2.13. If $E = \mathcal{H}$ is a Hilbert space, then we can obtain corresponding definitions of monotonicity, strongly monotonicity, relaxed monotonicity, and maximal monotonicity from Definitions 2.11 and 2.12.

Definition 2.14. A random mapping $\eta : \Omega \times E \times E \rightarrow E$ is said to be τ -Lipschitz continuous if there exists a measurable function $\tau : \Omega \rightarrow (0, \infty)$ such that

$$\|\eta_t(x_1, x_2)\| \leq \tau(t)\|x_1(t) - x_2(t)\|, \quad \forall x_1(t), x_2(t) \in E, t \in \Omega. \quad (2.15)$$

Definition 2.15. Let $H, A : \Omega \times E \rightarrow E$ and $\eta : \Omega \times E \times E \rightarrow E$ be random single-valued mappings. Then a random multivalued mapping $T : \Omega \times E \rightarrow 2^E$ is said to be

(i) η -accretive if

$$\begin{aligned} & \langle u(t) - v(t), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq 0, \quad \forall u(t) \in T_t(x_1), v(t) \in T_t(x_2), \quad x_1(t), x_2(t) \in E, \quad t \in \Omega; \end{aligned} \quad (2.16)$$

(ii) strictly η -accretive if

$$\begin{aligned} & \langle u(t) - v(t), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq 0, \quad \forall u(t) \in T_t(x_1), v(t) \in T_t(x_2), \quad x_1(t), x_2(t) \in E, \quad t \in \Omega, \end{aligned} \quad (2.17)$$

and equality holds if and only if $x_1(t) = x_2(t)$;

(iii) γ -strongly η -accretive if there exists a measurable function $\gamma : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle u(t) - v(t), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq \gamma(t)\|x_1(t) - x_2(t)\|^q, \quad \forall u(t) \in T_t(x_1), v(t) \in T_t(x_2), \quad x_1(t), x_2(t) \in E, \quad t \in \Omega; \end{aligned} \quad (2.18)$$

(iv) m -relaxed η -accretive if there exists a measurable function $m : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle u(t) - v(t), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq -m(t)\|x_1(t) - x_2(t)\|^q, \quad \forall u(t) \in T_t(x_1), v(t) \in T_t(x_2), \quad x_1(t), x_2(t) \in E, \quad t \in \Omega; \end{aligned} \quad (2.19)$$

- (v) η - m -accretive if T is η -accretive and $(I_t + \rho(t)T_t)(E) = E$ holds for all $t \in \Omega$ and $\rho(t) > 0$;
- (vi) H -accretive if T is accretive and $(H_t + \rho(t)T_t)(E) = E$ holds for all $t \in \Omega$ and $\rho(t) > 0$;
- (vii) (H, η) -accretive if T is η -accretive and $(H_t + \rho(t)T_t)(E) = E$ holds for all $t \in \Omega$ and $\rho(t) > 0$;
- (viii) (A, η) -accretive if T is m -relaxed η -accretive and $(A_t + \rho(t)T_t)(E) = E$ holds for all $t \in \Omega$ and $\rho(t) > 0$.

Definition 2.16. Let $H : E \times E \rightarrow E$ be a single-valued mapping and $\eta : \Omega \times E \times E \rightarrow E$ and $A, B : \Omega \times E \rightarrow E$ random single-valued mappings. Then

- (i) $H(A, \cdot)$ is said to be α -strongly η -accretive with respect to A if there exists a measurable function $\alpha : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle H(A_t(x_1), u(t)) - H(A_t(x_2), u(t)), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq \alpha(t) \|x_1(t) - x_2(t)\|^q, \quad \forall x_1(t), x_2(t), u(t) \in E, t \in \Omega; \end{aligned} \quad (2.20)$$

- (ii) $H(\cdot, B)$ is said to be β -relaxed η -accretive with respect to B if there exists a measurable function $\beta : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \langle H(u(t), B_t(x_1)) - H(u(t), B_t(x_2)), J_q(\eta_t(x_1, x_2)) \rangle \\ & \geq -\beta(t) \|x_1(t) - x_2(t)\|^q, \quad \forall x_1(t), x_2(t), u(t) \in E, t \in \Omega; \end{aligned} \quad (2.21)$$

- (iii) H is said to be λ -Lipschitz continuous with respect to A in the first argument if there exists a measurable function $\lambda : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \|H(A_t(x_1), u(t)) - H(A_t(x_2), u(t))\| \\ & \leq \lambda(t) \|x_1(t) - x_2(t)\|, \quad \forall x_1(t), x_2(t), u(t) \in E, t \in \Omega. \end{aligned} \quad (2.22)$$

Example 2.17. Let $\mathcal{H}$ be a Hilbert space and $S : \Omega \times \mathcal{H} \rightarrow \mathcal{H}$ a s -Lipschitz continuous and accretive mapping, $\eta_t(x, y) = x(t) - y(t)$, $H(x(t), y(t)) = x(t) + y(t)$, $A_t(x) = (1 + c_k/16)x(t)$, and $B_t(x) = -c_k S_t(x) - (c_k/8)x(t)$, for all $(x(t), y(t)) \in \mathcal{H} \times \mathcal{H}$, $t \in \Omega$ and $c_k > 0$. Then $H(A, \cdot)$ is $(1 + c_k/16)$ -strongly η -accretive with respect to A and $H(\cdot, B)$ is $c_k(s + 1/8)$ -relaxed η -accretive with respect to B .

Definition 2.18. Let $E_1, E_2, \dots, E_k$ be real Banach spaces and for each $i = 1, 2, \dots, k$, let $N_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ be a random single-valued mapping. Then N_i is said to be ξ_{ij} -Lipschitz continuous in the $(j + 1)$ th argument if there exists a measurable function $\xi_{ij} : \Omega \rightarrow (0, \infty)$ such that

$$\begin{aligned} & \|N_i(x_1, \dots, x_{j-1}, y_{j1}, x_{j+1}, \dots, x_k) - N_i(x_1, \dots, x_{j-1}, y_{j2}, x_{j+1}, \dots, x_k)\| \\ & \leq \xi_{ij}(t) \|y_{j1}(t) - y_{j2}(t)\|, \quad \forall y_{j1}(t), y_{j2}(t) \in E_j, x_i(t) \in E_i, \\ & t \in \Omega \ (i, j \in \{1, 2, \dots, k\}, i \neq j). \end{aligned} \quad (2.23)$$

Definition 2.19. Let $\mathcal{H}$ be a Hilbert space, $H : \mathcal{H} \times \mathcal{H} \rightarrow \mathcal{H}$ a single-valued mapping, and $A, B : \Omega \times \mathcal{H} \rightarrow \mathcal{H}$ random single-valued mappings. H is said to be coercive with respect to A and B if

$$\lim_{\|x(t)\| \rightarrow \infty} \frac{\langle H(A_t(x), B_t(x)), x(t) \rangle}{\|x(t)\|} = +\infty. \quad (2.24)$$

Definition 2.20. Let $\mathcal{H}$ be a Hilbert space, $H : \mathcal{H} \times \mathcal{H} \rightarrow \mathcal{H}$ a single-valued mapping and $A, B : \Omega \times \mathcal{H} \rightarrow \mathcal{H}$ random single-valued mappings. H is said to be bounded with respect to A and B if $H(A_t(P), B_t(P))$ is bounded for every bounded subset P of $\mathcal{H}$ and $t \in \Omega$. H is said to be hemicontinuous with respect to A and B if for any fixed $x(t), y(t), z(t) \in \mathcal{H}$, the function $k \mapsto \langle H(A_t(x + ky), B_t(x + ky)), z(t) \rangle$ is continuous at 0^+ .

3. Random $(H(\cdot, \cdot), \phi)$ - η -Accretive Mappings

In this section, we will introduce a new class of random accretive mappings- $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and discuss some properties of random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings.

Definition 3.1. Let $H : E \times E \rightarrow E$ be a single-valued mapping and $\eta : \Omega \times E \times E \rightarrow E$ and $\phi, A, B : \Omega \times E \rightarrow E$ random single-valued mappings. Then the random multivalued mapping $M : \Omega \times E \rightarrow 2^E$ is said to be $(H(\cdot, \cdot), \phi)$ - η -accretive with respect to A and B if $\phi \circ M$ is m -relaxed η -accretive and

$$(H(A_t, B_t) + \phi_t \circ M_t)(E) = E, \quad \forall t \in \Omega. \quad (3.1)$$

Remark 3.2. (i) If $H(A_t, B_t) = H_t$ and $\phi \circ M$ is η -accretive, then the random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping reduces to the random (H, ϕ) - η -accretive mapping.

(ii) If $\phi_t(x) = \lambda(t)x(t)$, for all $x(t) \in E$, $\lambda(t) > 0$, then the random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping reduces to the random $(H(\cdot, \cdot), \eta)$ -accretive mapping.

Remark 3.3. (i) If M is η -accretive, $H(A_t, B_t) = H_t$, and $\phi_t(x) = \lambda(t)x(t)$, for all $x(t) \in E$, $\lambda(t) > 0$, then the random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping reduces to the random (H, η) -accretive mapping studied by Uea and Kumam [28].

(ii) If M is m -relaxed η -accretive, $H(A_t, B_t) = H_t$, and $\phi_t(x) = \lambda(t)x(t)$, for all $x(t) \in E$, $\lambda(t) > 0$, then the random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping reduces to the random (A, η) -accretive mapping studied by Cho and Lan [17].

(iii) If M is η -accretive, $\eta_t(x, y) = x(t) - y(t)$, $\phi_t(x) = \lambda(t)x(t)$, for all $x(t), y(t) \in E$, $\lambda(t) > 0$, then the random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping reduces to the random $H(\cdot, \cdot)$ -accretive mapping studied by Zhang [29].

Example 3.4. Let $\mathcal{H}$ be a Hilbert space, $\lambda : \Omega \rightarrow (0, \infty)$ a measurable function; for every $(x(t), y(t)) \in \mathcal{H} \times \mathcal{H}$, $\eta_t(x, y) = x(t) - y(t)$, $\phi_t(x) = \lambda(t)x(t)$; $M : \Omega \times \mathcal{H} \rightarrow 2^{\mathcal{H}}$ a random maximal monotone mapping, $H : \mathcal{H} \times \mathcal{H} \rightarrow \mathcal{H}$ a bounded, coercive, and hemicontinuous mapping, with respect to A and B , and α -strongly η -accretive with respect to A in the first argument and β -relaxed η -accretive with respect to B in second argument. Then M is a random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping with respect to A and B .

Proof. For every $t \in \Omega$, $\lambda(t)M$ is maximal monotone since M is maximal monotone. Since H is bounded, coercive, and hemicontinuous with respect to A and B , α -strongly η -accretive with respect to A in the first argument and β -relaxed η -accretive with respect to B in the second argument, it follows from Corollary 32.26 of [30] that $(H(A_t, B_t) + \phi_t \circ M_t)(\mathcal{L}) = \mathcal{L}$ holds for every $t \in \Omega$. Thus M is a random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping with respect to A and B . $\square$

Example 3.5. Let $\mathcal{L} = \mathcal{R} = (-\infty, +\infty)$, $M_t(x) = 1/(1 + x(t)^2)$, $A_t(x) = 2x(t)$, $B_t(y) = -y(t)$, $H(A_t(x), B_t(y)) = A_t(x) + B_t(y)$, $\eta_t(x, y) = \arctan(x(t)) - \arctan(y(t))$, and $\phi_t \circ M_t(x) = \int_{-\infty}^{x(t)} M_t(s)ds$, for all $x(t), y(t) \in \mathcal{R}$. Then

$$\begin{aligned} & \langle \phi_t \circ M_t(x) - \phi_t \circ M_t(y), \eta_t(x, y) \rangle \\ &= \left\langle \left(\arctan(x(t)) + \frac{\pi}{2} \right) - \left(\arctan(y(t)) + \frac{\pi}{2} \right), \arctan(x(t)) - \arctan(y(t)) \right\rangle \\ &= (\arctan(x(t)) - \arctan(y(t)))^2 \geq 0, \\ & (H(A_t, B_t) + \phi_t \circ M_t)(x) = x(t) + \arctan(x(t)) + \frac{\pi}{2}. \end{aligned} \quad (3.2)$$

This implies that $(H(A_t, B_t) + \phi_t \circ M_t)$ is surjective. Thus M is a random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping with respect to A and B .

Definition 3.6. Let $\eta : \Omega \times E \times E \rightarrow E$, $\phi : \Omega \times E \rightarrow E$ be random single-valued mappings, $H(A, B)$ be α -strongly η -accretive with respect to A and β -relaxed η -accretive with respect to B and $M : \Omega \times E \rightarrow 2^E$ be a random $(H(\cdot, \cdot), \phi)$ - η -accretive mapping with respect to A and B . Then the general resolvent operator $R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t} : \Omega \times E \rightarrow 2^E$ is defined by

$$R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) = (H(A_t, B_t) + \phi_t \circ M_t)^{-1}(u), \quad \forall u(t) \in E, \quad t \in \Omega. \quad (3.3)$$

Remark 3.7. (i) If M is η -accretive, $H(A_t, B_t) = H_t$, and $\phi_t(x) = \lambda(t)x(t)$, for all $x(t) \in E$, $\lambda(t) > 0$, then the resolvent operator $R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}$ reduces to the resolvent operator $J_{H_t}^{\lambda(t), M_t}$ introduced by Uea and Kumam [28].

(ii) If M is m -relaxed η -accretive, $H(A_t, B_t) = H_t$ and $\phi_t(x) = \lambda(t)x(t)$, for all $x(t) \in E$, $\lambda(t) > 0$, then the resolvent operator $R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}$ reduces to the resolvent operator $R_{\eta_t, M_t}^{\lambda(t), A_t}$ introduced by Cho and Lan [17].

(iii) If M is η -accretive, $\eta_t(x, y) = x(t) - y(t)$, $\phi_t(x) = \lambda(t)x(t)$, for all $x(t), y(t) \in E$, $\lambda(t) > 0$, then the resolvent operator $R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}$ reduces to the resolvent operator $R_{M_t, \lambda}^{H(\cdot, \cdot)}$ introduced by Zhang [29].

Theorem 3.8. Let $\phi : \Omega \times E \rightarrow E$ be a random single-valued mapping, $\eta : \Omega \times E \times E \rightarrow E$ be τ -Lipschitz continuous and $H(A, B)$ be α -strongly η -accretive with respect to A and β -relaxed η -accretive with respect to B and $\alpha(t) \geq \beta(t)$, $\forall t \in \Omega$. Let $M : \Omega \times E \rightarrow 2^E$ be a random

$(H(\cdot, \cdot), \phi)$ - η -accretive mapping with respect to A and B . Then the resolvent operator $R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}$ is $\tau(t)^{q-1} / (r(t) - m(t))$ -Lipschitz continuous for $r(t) > m(t)$, where $r(t) = \alpha(t) - \beta(t)$, $\forall t \in \Omega$, that is,

$$\|R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)\| \leq \frac{\tau(t)^{q-1}}{r(t) - m(t)} \|u(t) - v(t)\|, \quad \forall u(t), v(t) \in E, \quad t \in \Omega. \quad (3.4)$$

Proof. Let $u(t), v(t) \in E$ be any given points; it follows from Definition 3.6 that

$$\begin{aligned} R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) &= (H(A_t, B_t) + \phi_t \circ M_t)^{-1}(u), \\ R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) &= (H(A_t, B_t) + \phi_t \circ M_t)^{-1}(v). \end{aligned} \quad (3.5)$$

This implies that

$$\begin{aligned} (u(t) - H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u)) &\in \phi_t \circ M_t(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u)), \\ (v(t) - H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)) &\in \phi_t \circ M_t(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)). \end{aligned} \quad (3.6)$$

Since $\phi \circ M$ is m -relaxed η -accretive, we get

$$\begin{aligned} &-m(t) \left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\|^q \\ &\leq \left\langle u(t) - H(A_t, B_t) + \phi_t \circ M_t, R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\rangle \\ &= \left\langle u(t) - v(t) - (H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u)) \right. \\ &\quad \left. - (H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)) \right\rangle \\ &= \left\langle u(t) - v(t) - (H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u)) \right. \\ &\quad \left. - (H(A_t, B_t) + \phi_t \circ M_t)(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)) \right\rangle. \end{aligned} \quad (3.7)$$

From (3.7) and Lipschitz continuity of η , we have

$$\begin{aligned} &\tau(t)^{q-1} \|u(t) - v(t)\| \left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\|^{q-1} \\ &\geq \|u(t) - v(t)\| \left\| \eta_t(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u), R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v)) \right\|^{q-1} \\ &\geq \left\langle u(t) - v(t), J_q(\eta_t(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u), R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v))) \right\rangle \\ &\geq \left\langle H(A_t, B_t) + \phi_t \circ M_t, R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\rangle, \end{aligned}$$

$$\begin{aligned}
& J_q \left(\eta_t \left(R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u), R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right) \right) - m(t) \left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\|^q \\
& \geq (\alpha(t) - \beta(t) - m(t)) \left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\|^q \\
& = (r(t) - m(t)) \left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\|^q.
\end{aligned} \tag{3.8}$$

Hence

$$\left\| R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(u) - R_{M_t, \phi_t}^{H(\cdot, \cdot), \eta_t}(v) \right\| \leq \frac{\tau(t)^{q-1}}{r(t) - m(t)} \|u(t) - v(t)\|, \quad \forall u(t), v(t) \in E, t \in \Omega. \tag{3.9}$$

This completes the proof. $\square$

4. A New System of Random Generalized Variational Inclusions

In this section, we will introduce a new system of random generalized variational inclusions with random $(H(\cdot, \cdot), \phi)$ - η -accretive mappings and construct an iterative algorithm for solving this system of random generalized variational inclusions.

Let for $i, j = 1, 2, \dots, k$, $T_{ji} : \Omega \times E_j \rightarrow F(E_i)$, and $S_i : \Omega \times E_i \rightarrow F(E_i)$ be random fuzzy mappings satisfying the following condition (C).

(C) For $i, j = 1, 2, \dots, k$, there exist mappings $a_j : E_j \rightarrow [0, 1]$, $b_i : E_i \rightarrow [0, 1]$, such that

$$\begin{aligned}
& \left(T_{ji, y_j} \right)_{a_j(y_j)} \in \text{CB}(E_i), \quad \forall (t, y_j) \in \Omega \times E_j \quad (i, j = 1, 2, \dots, k), \\
& \left(S_{i, z_i} \right)_{b_i(z_i)} \in \text{CB}(E_i), \quad \forall (t, z_i) \in \Omega \times E_i \quad (i = 1, 2, \dots, k).
\end{aligned} \tag{4.1}$$

By using the random fuzzy mappings T_{ji} , S_i ($i, j = 1, 2, \dots, k$), we can define random multivalued mappings $\tilde{T}_{ji}$, $\tilde{S}_i$ as follows.

$$\begin{aligned}
& \tilde{T}_{ji} : \Omega \times E_j \longrightarrow \text{CB}(E_i), \quad y_j \longmapsto \left(T_{ji, y_j} \right)_{a_j(y_j)} \quad (i, j = 1, 2, \dots, k), \\
& \tilde{S}_i : \Omega \times E_i \longrightarrow \text{CB}(E_i), \quad z_i \longmapsto \left(S_{i, z_i} \right)_{b_i(z_i)} \quad (i = 1, 2, \dots, k),
\end{aligned} \tag{4.2}$$

where $(S_{i, z_i})_{b_i(z_i)} = S_i(t, z_i(t))$.

In the sequel, for $i, j = 1, 2, \dots, k$, $\tilde{T}_{ji}$ and $\tilde{S}_i$ are called the random multivalued mappings induced by the random fuzzy mappings T_{ji} and S_i , respectively.

For $i = 1, 2, \dots, k$, given mappings $a_i : E_i \rightarrow [0, 1]$, $b_i : E_i \rightarrow [0, 1]$, let $T_{1i} : \Omega \times E_1 \rightarrow F(E_i)$, $T_{2i} : \Omega \times E_2 \rightarrow F(E_i)$, $\dots$, $T_{ki} : \Omega \times E_k \rightarrow F(E_i)$, $S_i : \Omega \times E_i \rightarrow F(E_i)$ be random fuzzy mappings, $H_i : E_i \times E_i \rightarrow E_i$ be a single-valued mapping and $\phi_i, A_i, B_i, f_i, p_i : \Omega \times E_i \rightarrow E_i$, $\eta_i : \Omega \times E_i \times E_i \rightarrow E_i$, $F_i, N_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ be random single-valued mappings and

$M_i : \Omega \times E_i \times E_i \rightarrow 2^{E_i}$ be a random $(H_i(\cdot, \cdot), \phi_i)$ - η_i -accretive mapping with respect to A_i and B_i in the second argument with $\text{Im } f_i \cap \text{dom } M_i(\cdot, w_i) \neq \emptyset$, for each $w_i(t) \in E_i$ and $t \in \Omega$. Now we consider the following system of random generalized variational inclusions:

For $i = 1, 2, \dots, n$, find measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki}, v_i : \Omega \rightarrow E_i$ such that for all $t \in \Omega$ and for each $i = 1, 2, \dots, k, x_i(t) \in E_i, T_{1i, x_1(t)}(u_{1i}(t)) \geq a_1(x_1(t)), T_{2i, x_2(t)}(u_{2i}(t)) \geq a_2(x_2(t)), \dots, T_{ki, x_k(t)}(u_{ki}(t)) \geq a_k(x_k(t)), S_{i, x_i(t)}(v_i(t)) \geq b_i(x_i(t))$ such that

$$0 \in F_i(x_1 - p_{1i}(x_1), x_2 - p_{2i}(x_2), \dots, x_k - p_{ki}(x_k)) + N_i(u_{i1}, u_{i2}, \dots, u_{ik}) + M_i(f_i(x_i), v_i). \quad (4.3)$$

If for $i = 1, 2, \dots, k, E_i = \mathcal{H}_i$ is a Hilbert space and $M_i(\cdot, v_i) = \partial\varphi_i(\cdot)$, for all $t \in \Omega, x_i(t) \in \mathcal{H}_i$ and $v_i(t) \in \tilde{S}_i(x_i)$, where $\partial\varphi_i$ denote the subdifferential of a proper, convex, and lower semicontinuous function $\varphi_i : \Omega \times \mathcal{H}_i \rightarrow R \cup \{+\infty\}$, then system of random generalized variational inclusions (4.3) reduces to the following system of random generalized mixed variational inequalities.

For $i = 1, 2, \dots, k$, find measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki} : \Omega \rightarrow \mathcal{H}_i$ such that for all $t \in \Omega$ and for each $i = 1, 2, \dots, k, x_i(t) \in \mathcal{H}_i, T_{1i, x_1(t)}(u_{1i}(t)) \geq a_1(x_1(t)), T_{2i, x_2(t)}(u_{2i}(t)) \geq a_2(x_2(t)), \dots, T_{ki, x_k(t)}(u_{ki}(t)) \geq a_k(x_k(t))$ such that

$$\begin{aligned} & \langle F_i(x_1 - p_{1i}(x_1), x_2 - p_{2i}(x_2), \dots, x_k - p_{ki}(x_k)) + N_i(u_{i1}, u_{i2}, \dots, u_{ik}), y_i(t) - f_i(x_i) \rangle \\ & \geq \varphi_i(f_i(x_i)) - \varphi_i(y_i), \quad \forall t \in \Omega, y_i(t) \in \mathcal{H}_i. \end{aligned} \quad (4.4)$$

We remark that system of random generalized variational inclusions (4.3) and system of random generalized mixed variational inequalities (4.4) include as special cases, many kinds of random variational inequalities and random variational inclusions of [17, 19–21], etc.

Lemma 4.1. For $i = 1, 2, \dots, k$, let $H_i : E_i \times E_i \rightarrow E_i$ be a single-valued mapping and $\eta_i : \Omega \times E_i \times E_i \rightarrow E_i, \phi_i, A_i, B_i : \Omega \times E_i \rightarrow E_i$ random single-valued mappings such that $H_i(A_i, B_i)$ be α_i -strongly η_i -accretive with respect to A_i and β_i -relaxed η_i -accretive with respect to B_i . Then the set of measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki} : \Omega \rightarrow E_i$ ($i = 1, 2, \dots, k$) is a random solution of problem (4.3) if and only if for all $t \in \Omega, x_i(t) \in E_i, u_{1i}(t) \in \tilde{T}_{1i}(x_1), u_{2i}(t) \in \tilde{T}_{2i}(x_2), \dots, u_{ki}(t) \in \tilde{T}_{ki}(x_k), v_i(t) \in \tilde{S}_i(x_i)$, and

$$\begin{aligned} f_i(x_i) &= R_{M_i(\cdot, v_i), \phi_i}^{H_i(\cdot, \cdot), \eta_i} [H_i(A_i(f_i(x_i)), B_i(f_i(x_i))) \\ &\quad - \phi_i \circ F_i(x_1 - p_{1i}(x_1), x_2 - p_{2i}(x_2), \dots, x_k - p_{ki}(x_k)) \\ &\quad - \phi_i \circ N_i(u_{i1}, u_{i2}, \dots, u_{ik})], \end{aligned} \quad (4.5)$$

where $R_{M_i(\cdot, v_i), \phi_i}^{H_i(\cdot, \cdot), \eta_i} = (H_i(A_i, B_i) + \phi_i \circ M_i(\cdot, v_i))^{-1}$.

Algorithm 4.2. Suppose that for $i, j = 1, 2, \dots, k, T_{ji} : \Omega \times E_j \rightarrow F(E_i), S_i : \Omega \times E_i \rightarrow F(E_i)$ be random fuzzy mappings satisfying the condition (C). Let for $i = 1, 2, \dots, k, \tilde{T}_{1i} : \Omega \times E_1 \rightarrow \text{CB}(E_i), \tilde{T}_{2i} : \Omega \times E_2 \rightarrow \text{CB}(E_i), \dots, \tilde{T}_{ki} : \Omega \times E_k \rightarrow \text{CB}(E_i), \tilde{S}_i : \Omega \times E_i \rightarrow \text{CB}(E_i)$ be

$\widetilde{H}$ -continuous random multivalued mappings induced by $T_{1i}, T_{2i}, \dots, T_{ki}$ and S_i , respectively. Let $H_i : E_i \times E_i \rightarrow E_i$ be a single-valued mapping; $\eta_i : \Omega \times E_i \times E_i \rightarrow E_i$, $\phi_i, A_i, B_i : \Omega \times E_i \rightarrow E_i$, $F_i, N_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ random single-valued mappings; the random multivalued mapping $M_i : \Omega \times E_i \times E_i \rightarrow 2^{E_i}$ be $(H_i(\cdot, \cdot), \phi_i)$ - η_i -accretive with respect to A_i and B_i in the second argument. For any given measurable mapping $x_i : \Omega \rightarrow E_i$ ($i = 1, 2, \dots, k$), the multivalued mappings $\tilde{T}_{1i}(\cdot, x_1^0(\cdot)), \tilde{T}_{2i}(\cdot, x_2^0(\cdot)), \dots, \tilde{T}_{ki}(\cdot, x_k^0(\cdot)), \tilde{S}_i(\cdot, x_i^0(\cdot))$ ($i = 1, 2, \dots, k$) are measurable by Lemma 2.9. Hence for $i = 1, 2, \dots, k$, there exist measurable selections $u_{1i}^0 : \Omega \rightarrow E_i$ of $\tilde{T}_{1i}(\cdot, x_1^0(\cdot))$, $u_{2i}^0 : \Omega \rightarrow E_i$ of $\tilde{T}_{2i}(\cdot, x_2^0(\cdot))$, $\dots$, $u_{ki}^0 : \Omega \rightarrow E_i$ of $\tilde{T}_{ki}(\cdot, x_k^0(\cdot))$ and $v_i^0 : \Omega \rightarrow E_i$ of $\tilde{S}_i(\cdot, x_i^0(\cdot))$, by Himmelberg [31]. Let for $i = 1, 2, \dots, k$,

$$\begin{aligned} & x_i^1(t) \\ &= x_i^0(t) - f_{i_t}(x_i^0) + R_{M_{i_t}(\cdot, v_i^0), \phi_{i_t}}^{H_i(\cdot, \cdot), \eta_{i_t}} \left[H_i \left(A_{i_t} \left(f_{i_t}(x_i^0) \right), B_{i_t} \left(f_{i_t}(x_i^0) \right) \right) \right. \\ & \quad \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^0 - p_{1_t}(x_1^0), x_2^0 - p_{2_t}(x_2^0), \dots, x_k^0 - p_{k_t}(x_k^0) \right) \right. \\ & \quad \left. - \phi_{i_t} \circ N_{i_t} \left(u_{i1}^0, u_{i2}^0, \dots, u_{ik}^0 \right) \right]. \end{aligned} \quad (4.6)$$

It is easy to see that for $i = 1, 2, \dots, k$, $x_i^1 : \Omega \rightarrow E_i$ is measurable. By Lemma 2.10, there exist measurable selections $u_{1i}^1 : \Omega \rightarrow E_i$ of $\tilde{T}_{1i}(\cdot, x_1^1(\cdot))$, $u_{2i}^1 : \Omega \rightarrow E_i$ of $\tilde{T}_{2i}(\cdot, x_2^1(\cdot))$, $\dots$, $u_{ki}^1 : \Omega \rightarrow E_i$ of $\tilde{T}_{ki}(\cdot, x_k^1(\cdot))$ and $v_i^1 : \Omega \rightarrow E_i$ of $\tilde{S}_i(\cdot, x_i^1(\cdot))$ ($i = 1, 2, \dots, k$) such that for all $t \in \Omega$ and $i = 1, 2, \dots, k$,

$$\begin{aligned} & \|u_{1i}^0(t) - u_{1i}^1(t)\| \leq (1 + 1)\widetilde{H}(\tilde{T}_{1i}(x_1^0), \tilde{T}_{1i}(x_1^1)), \\ & \|u_{2i}^0(t) - u_{2i}^1(t)\| \leq (1 + 1)\widetilde{H}(\tilde{T}_{2i}(x_2^0), \tilde{T}_{2i}(x_2^1)), \\ & \quad \vdots \\ & \|u_{ki}^0(t) - u_{ki}^1(t)\| \leq (1 + 1)\widetilde{H}(\tilde{T}_{ki}(x_k^0), \tilde{T}_{ki}(x_k^1)), \\ & \|v_i^0(t) - v_i^1(t)\| \leq (1 + 1)\widetilde{H}(\tilde{S}_i(x_i^0), \tilde{S}_i(x_i^1)). \end{aligned} \quad (4.7)$$

Let for $i = 1, 2, \dots, k$,

$$\begin{aligned} & x_i^2(t) = x_i^1(t) - f_{i_t}(x_i^1) + R_{M_{i_t}(\cdot, v_i^1), \phi_{i_t}}^{H_i(\cdot, \cdot), \eta_{i_t}} \left[H_i \left(A_{i_t} \left(f_{i_t}(x_i^1) \right), B_{i_t} \left(f_{i_t}(x_i^1) \right) \right) \right. \\ & \quad \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^1 - p_{1_t}(x_1^1), x_2^1 - p_{2_t}(x_2^1), \dots, x_k^1 - p_{k_t}(x_k^1) \right) \right. \\ & \quad \left. - \phi_{i_t} \circ N_{i_t} \left(u_{i1}^1, u_{i2}^1, \dots, u_{ik}^1 \right) \right], \end{aligned} \quad (4.8)$$

then $x_i^2 : \Omega \rightarrow E_i$ is measurable. Continuing the above process inductively, we can define the following random iterative sequences $\{x_i^n(t)\}$, $\{u_{1i}^n(t)\}$, $\{u_{2i}^n(t)\}$, $\dots$, $\{u_{ki}^n(t)\}$ and $\{v_i^n(t)\}$ ($i = 1, 2, \dots, k$) for solving problem (4.3) as follows:

$$\begin{aligned}
 & x_i^{n+1}(t) \\
 &= x_i^n(t) - f_i(x_i^n) + R_{M_i(\cdot, v_i^n), \phi_i}^{H_i(\cdot, \cdot), \eta_i} [H_i(A_i(f_i(x_i^n)), B_i(f_i(x_i^n))) \\
 &\quad - \phi_i \circ F_i(x_1^n - p_{1i}(x_1^n), x_2^n - p_{2i}(x_2^n), \dots, x_k^n - p_{ki}(x_k^n)) \\
 &\quad - \phi_i \circ N_i(u_{1i}^n, u_{2i}^n, \dots, u_{ki}^n)], \\
 & u_{1i}^n(t) \in \tilde{T}_{1i}(x_1^n), \quad u_{2i}^n(t) \in \tilde{T}_{2i}(x_2^n), \dots, u_{ki}^n(t) \in \tilde{T}_{ki}(x_k^n), \quad v_i^n(t) \in \tilde{S}_i(x_i^n), \\
 & \|u_{1i}^n(t) - u_{1i}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{1i}(x_1^n), \tilde{T}_{1i}(x_1^{n+1})), \\
 & \|u_{2i}^n(t) - u_{2i}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{2i}(x_2^n), \tilde{T}_{2i}(x_2^{n+1})), \\
 & \quad \vdots \\
 & \|u_{ki}^n(t) - u_{ki}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{ki}(x_k^n), \tilde{T}_{ki}(x_k^{n+1})), \\
 & \|v_i^n(t) - v_i^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{S}_i(x_i^n), \tilde{S}_i(x_i^{n+1})),
 \end{aligned} \tag{4.9}$$

for any $t \in \Omega$, $i = 1, 2, \dots, k$ and $n = 0, 1, 2, \dots$.

Theorem 4.3. For $i = 1, 2, \dots, k$, let E_i be a q -uniformly smooth Banach space and $\phi_i : \Omega \times E_i \rightarrow E_i$ be a q_i -Lipschitz continuous mapping satisfying $\phi_i(x + y) = \phi_i(x) + \phi_i(y)$ and $\ker \phi_i = \{0\}$, for all $t \in \Omega$. Let $\eta_i : \Omega \times E_i \times E_i \rightarrow E_i$ be τ_i -Lipschitz continuous, $H_i : E_i \times E_i \rightarrow E_i$ be a single-valued mapping, and $A_i, B_i : \Omega \times E_i \rightarrow E_i$ random single-valued mappings such that $H_i(A_i, B_i)$ be α_i -strongly η_i -accretive with respect to A_i and β_i -relaxed η_i -accretive with respect to B_i and $H_i(A_i, B_i)$ be δ_{i1} -Lipschitz continuous with respect to A_i and δ_{i2} -Lipschitz continuous with respect to B_i . Let $f_i : \Omega \times E_i \rightarrow E_i$ be (ϵ_i, μ_i) -relaxed cocoercive and ξ_i -Lipschitz continuous, and $p_i : \Omega \times E_i \rightarrow E_i$ be γ_{pi} -strongly accretive and λ_{pi} -Lipschitz continuous. Suppose that $F_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ be ζ_{ij} -Lipschitz continuous in the $(j+1)$ th argument for $j = 1, 2, \dots, k$, $N_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ be ν_{ij} -Lipschitz continuous in the $(j+1)$ th argument for $j = 1, 2, \dots, k$. Let $M_i : \Omega \times E_i \times E_i \rightarrow 2^{E_i}$ be a random multivalued mapping such that M_i is a $(H_i(\cdot, \cdot), \phi_i)$ - η_i -accretive mapping with respect to A_i and B_i in the second argument. Let $T_{1i} : \Omega \times E_1 \rightarrow F(E_i)$, $T_{2i} : \Omega \times E_2 \rightarrow F(E_i)$, $\dots$, $T_{ki} : \Omega \times E_k \rightarrow F(E_i)$, $S_i : \Omega \times E_i \rightarrow F(E_i)$ be random fuzzy mappings satisfying the condition (C) and $\tilde{T}_{1i} : \Omega \times E_1 \rightarrow CB(E_i)$, $\tilde{T}_{2i} : \Omega \times E_2 \rightarrow CB(E_i)$, $\dots$, $\tilde{T}_{ki} : \Omega \times E_k \rightarrow CB(E_i)$, and $\tilde{S}_i : \Omega \times E_i \rightarrow CB(E_i)$ random multivalued mappings induced by $T_{1i}, T_{2i}, \dots, T_{ki}$ and S_i , respectively.

Suppose that $\tilde{T}_{1i}, \tilde{T}_{2i}, \dots, \tilde{T}_{ki}, \tilde{S}_i$ are $\tilde{H}$ -continuous with constants $t_{1i}, t_{2i}, \dots, t_{ki}$, and λ_{s_i} , respectively. In addition, if

$$\begin{aligned} \|R_{M_{i_t}(\cdot, y_{i1}), \phi_{i_t}}^{H_i(\cdot, \cdot), n_{i_t}}(z_i) - R_{M_{i_t}(\cdot, y_{i2}), \phi_{i_t}}^{H_i(\cdot, \cdot), n_{i_t}}(z_i)\| &\leq \lambda_i(t) \|y_{i1}(t) - y_{i2}(t)\|, \\ \forall t \in \Omega, y_{i1}(t), y_{i2}(t), z_i(t) &\in E_i \quad (i = 1, 2, \dots, k), \end{aligned} \quad (4.10)$$

$$\begin{aligned} 0 &< (1 + q\epsilon_1(t)\xi_1(t)^q - q\mu_1(t) + c_q\xi_1(t)^q)^{1/q} + \lambda_1(t)\lambda_{s_1}(t) + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} \\ &\times \left(((\delta_{11}(t) + \delta_{12}(t))^q \xi_1(t)^q \right. \\ &\quad + qQ_1(t)(1 - q\gamma_{p_1}(t) + c_q\lambda_{p_1}(t)^q)^{1/q} \xi_{11}(t)(\delta_{11}(t) + \delta_{12}(t))^{q-1} \xi_1(t)^{q-1} \\ &\quad \left. + c_qQ_1(t)^q (1 - q\gamma_{p_1}(t) + c_q\lambda_{p_1}(t)^q) \xi_{11}(t)^q)^{1/q} + \sum_{j=1}^k Q_1(t)v_{1j}(t)t_{1j}(t) \right) \\ &+ \sum_{j=2}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t)\xi_{j1}(t)(1 - q\gamma_{p_1}(t) + c_q\lambda_{p_1}(t)^q)^{1/q} < 1, \\ 0 &< (1 + q\epsilon_2(t)\xi_2(t)^q - q\mu_2(t) + c_q\xi_2(t)^q)^{1/q} + \lambda_2(t)\lambda_{s_2}(t) + \frac{\tau_2(t)^{q-1}}{r_2(t) - m_2(t)} \\ &\times \left(((\delta_{21}(t) + \delta_{22}(t))^q \xi_2(t)^q \right. \\ &\quad + qQ_2(t)(1 - q\gamma_{p_2}(t) + c_q\lambda_{p_2}(t)^q)^{1/q} \xi_{22}(t)(\delta_{21}(t) + \delta_{22}(t))^{q-1} \xi_2(t)^{q-1} \\ &\quad \left. + c_qQ_2(t)^q (1 - q\gamma_{p_2}(t) + c_q\lambda_{p_2}(t)^q) \xi_{22}(t)^q)^{1/q} + \sum_{j=1}^k Q_2(t)v_{2j}(t)t_{2j}(t) \right) \\ &+ \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} Q_1(t)\xi_{12}(t)(1 - q\gamma_{p_2}(t) + c_q\lambda_{p_2}(t)^q)^{1/q} \\ &+ \sum_{j=3}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t)\xi_{j2}(t)(1 - q\gamma_{p_2}(t) + c_q\lambda_{p_2}(t)^q)^{1/q} < 1, \dots, \\ 0 &< (1 + q\epsilon_k(t)\xi_k(t)^q - q\mu_k(t) + c_q\xi_k(t)^q)^{1/q} + \lambda_k(t)\lambda_{s_k}(t) + \frac{\tau_k(t)^{q-1}}{r_k(t) - m_k(t)} \end{aligned}$$

$$\begin{aligned}
& \times \left(((\delta_{k1}(t) + \delta_{k2}(t))^q \xi_k(t)^q \right. \\
& \quad + q\varrho_k(t)(1 - q\gamma_{p_k}(t) + c_q\lambda_{p_k}(t)^q)^{1/q} \xi_{kk}(t)(\delta_{k1}(t) + \delta_{k2}(t))^{q-1} \xi_k(t)^{q-1} \\
& \quad \left. + c_q\varrho_k(t)^q(1 - q\gamma_{p_k}(t) + c_q\lambda_{p_k}(t)^q) \xi_{kk}(t)^q)^{1/q} + \sum_{j=1}^k \varrho_k(t) v_{kj}(t) t_{kj}(t) \right) \\
& + \sum_{j=1}^{k-1} \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} \varrho_j(t) \xi_{jk}(t) (1 - q\gamma_{p_k}(t) + c_q\lambda_{p_k}(t)^q)^{1/q} < 1,
\end{aligned} \tag{4.11}$$

then there exist measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki}, v_i : \Omega \rightarrow E_i$ such that (4.3) holds. Moreover for all $t \in \Omega$, $x_i^n(t) \rightarrow x_i(t)$, $u_{1i}^n(t) \rightarrow u_{1i}(t)$, $u_{2i}^n(t) \rightarrow u_{2i}(t), \dots, u_{ki}^n(t) \rightarrow u_{ki}(t)$, $v_i^n(t) \rightarrow v_i(t)$, where $\{x_i^n(t)\}$, and $\{u_{1i}^n(t)\}, \dots, \{u_{ki}^n(t)\}, \{v_i^n(t)\}$ are random sequences obtained by Algorithm 4.2.

Proof. For $i = 1, 2, \dots, k$, from (4.10), Lemma 4.1, and Algorithm 4.2, we have

$$\begin{aligned}
& \|x_i^{n+1}(t) - x_i^n(t)\| \\
& = \left\| x_i^n(t) - f_{i_t}(x_i^n) + R_{M_{i_t}(\cdot, v_i^n), \phi_{i_t}}^{H_{i_t}(\cdot, \eta_{i_t})} [H_{i_t}(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) \right. \\
& \quad \left. - \phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), x_2^n - p_{2_t}(x_2^n), \dots, x_k^n - p_{k_t}(x_k^n)) \right. \\
& \quad \left. - \phi_{i_t} \circ N_{i_t}(u_{i1}^n, u_{i2}^n, \dots, u_{ik}^n) \right] \\
& \quad - (x_i^{n-1}(t) - f_{i_t}(x_i^{n-1})) \\
& \quad + R_{M_{i_t}(\cdot, v_i^{n-1}), \phi_{i_t}}^{H_{i_t}(\cdot, \eta_{i_t})} [H_{i_t}(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \\
& \quad - \phi_{i_t} \circ F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), \dots, x_k^{n-1} - p_{k_t}(x_k^{n-1})) \\
& \quad - \phi_{i_t} \circ N_{i_t}(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1})] \Big\| \\
& \leq \left\| x_i^n(t) - x_i^{n-1}(t) - (f_{i_t}(x_i^n) - f_{i_t}(x_i^{n-1})) \right\| \\
& \quad + \left\| R_{M_{i_t}(\cdot, v_i^n), \phi_{i_t}}^{H_{i_t}(\cdot, \eta_{i_t})} [H_{i_t}(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) \right. \\
& \quad \left. - \phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), x_2^n - p_{2_t}(x_2^n), \dots, x_k^n - p_{k_t}(x_k^n)) \right. \\
& \quad \left. - \phi_{i_t} \circ N_{i_t}(u_{i1}^n, u_{i2}^n, \dots, u_{ik}^n) \right] \\
& \quad - R_{M_{i_t}(\cdot, v_i^{n-1}), \phi_{i_t}}^{H_{i_t}(\cdot, \eta_{i_t})} [H_{i_t}(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \\
& \quad - \phi_{i_t} \circ F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), \dots, x_k^{n-1} - p_{k_t}(x_k^{n-1})) \\
& \quad \left. - \phi_{i_t} \circ N_{i_t}(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1})] \right\|
\end{aligned}$$

$$\begin{aligned}
& + \left\| R_{M_{i_t}(\cdot, v_i^n), \phi_{i_t}}^{H_{i_t}(\cdot, \cdot), \eta_{i_t}} \left[H_{i_t} \left(A_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right), B_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right) \right) \right. \right. \\
& \quad \left. \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^{n-1} - p_{1_t} \left(x_1^{n-1} \right), x_2^{n-1} - p_{2_t} \left(x_2^{n-1} \right), \dots, x_k^{n-1} - p_{k_t} \left(x_k^{n-1} \right) \right) \right. \right. \\
& \quad \left. \left. - \phi_{i_t} \circ N_{i_t} \left(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1} \right) \right] \right. \\
& \quad \left. - R_{M_{i_t}(\cdot, v_i^{n-1}), \phi_{i_t}}^{H_{i_t}(\cdot, \cdot), \eta_{i_t}} \left[H_{i_t} \left(A_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right), B_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right) \right) \right. \right. \\
& \quad \left. \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^{n-1} - p_{1_t} \left(x_1^{n-1} \right), x_2^{n-1} - p_{2_t} \left(x_2^{n-1} \right), \dots, x_k^{n-1} - p_{k_t} \left(x_k^{n-1} \right) \right) \right. \right. \\
& \quad \left. \left. - \phi_{i_t} \circ N_{i_t} \left(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1} \right) \right] \right\| \\
& \leq \left\| x_i^n(t) - x_i^{n-1}(t) - \left(f_{i_t} \left(x_i^n \right) - f_{i_t} \left(x_i^{n-1} \right) \right) \right\| \\
& \quad + \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \left\| H_{i_t} \left(A_{i_t} \left(f_{i_t} \left(x_i^n \right) \right), B_{i_t} \left(f_{i_t} \left(x_i^n \right) \right) \right) \right. \\
& \quad \left. - H_{i_t} \left(A_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right), B_{i_t} \left(f_{i_t} \left(x_i^{n-1} \right) \right) \right) \right. \\
& \quad \left. - \left(\phi_{i_t} \circ F_{i_t} \left(x_1^n - p_{1_t} \left(x_1^n \right), x_2^n - p_{2_t} \left(x_2^n \right), \dots, x_k^n - p_{k_t} \left(x_k^n \right) \right) \right. \right. \\
& \quad \left. \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^{n-1} - p_{1_t} \left(x_1^{n-1} \right), x_2^{n-1} - p_{2_t} \left(x_2^{n-1} \right), \dots, x_k^{n-1} - p_{k_t} \left(x_k^{n-1} \right) \right) \right) \right\| \\
& \quad + \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \varrho_i(t) \left\| N_{i_t} \left(u_{i1}^n, u_{i2}^n, \dots, u_{ik}^n \right) - N_{i_t} \left(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1} \right) \right\| \\
& \quad + \lambda_i(t) \left\| v_i^n(t) - v_i^{n-1}(t) \right\|.
\end{aligned} \tag{4.12}$$

Since $f_i : \Omega \times E_i \rightarrow E_i$ be (ϵ_i, μ_i) -relaxed cocoercive and ξ_i -Lipschitz continuous, we have

$$\begin{aligned}
& \left\| x_i^n(t) - x_i^{n-1}(t) - \left(f_{i_t} \left(x_i^n \right) - f_{i_t} \left(x_i^{n-1} \right) \right) \right\|^q \\
& \leq \left\| x_i^n(t) - x_i^{n-1}(t) \right\|^q - q \left\langle f_{i_t} \left(x_i^n \right) - f_{i_t} \left(x_i^{n-1} \right), J_q \left(x_i^n(t) - x_i^{n-1}(t) \right) \right\rangle \\
& \quad + c_q \left\| f_{i_t} \left(x_i^n \right) - f_{i_t} \left(x_i^{n-1} \right) \right\|^q \\
& \leq (1 + q\epsilon_i(t)\xi_i(t)^q - q\mu_i(t) + c_q\xi_i(t)^q) \left\| x_i^n(t) - x_i^{n-1}(t) \right\|^q.
\end{aligned} \tag{4.13}$$

Using γ_{p_i} -strongly accretivity and λ_{p_i} -Lipschitz continuity of p_i , ϱ_i -Lipschitz continuity of φ_i we have

$$\begin{aligned}
& \left\| x_i^n(t) - x_i^{n-1}(t) - \left(p_{i_t} \left(x_i^n \right) - p_{i_t} \left(x_i^{n-1} \right) \right) \right\|^q \\
& \leq \left\| x_i^n(t) - x_i^{n-1}(t) \right\|^q - q \left\langle p_{i_t} \left(x_i^n \right) - p_{i_t} \left(x_i^{n-1} \right), J_q \left(x_i^n(t) - x_i^{n-1}(t) \right) \right\rangle \\
& \quad + c_q \left\| p_{i_t} \left(x_i^n \right) - p_{i_t} \left(x_i^{n-1} \right) \right\|^q
\end{aligned}$$

$$\begin{aligned}
&\leq (1 - qY_{p_i}(t) + c_q \lambda_{p_i}(t)^q) \left\| x_i^n(t) - x_i^{n-1}(t) \right\|^q, \\
&\left\| H_i(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right. \\
&\quad - (\phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), x_2^n - p_{2_t}(x_2^n), \dots, x_k^n - p_{k_t}(x_k^n)) \\
&\quad \left. - \phi_{i_t} \circ F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), \dots, x_k^{n-1} - p_{k_t}(x_k^{n-1}))) \right\| \\
&\leq \left\| H_i(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right. \\
&\quad - (\phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), \dots, x_{i-1}^n - p_{i-1_t}(x_{i-1}^n), x_i^n - p_{i_t}(x_i^n), \\
&\quad \quad x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n)) \\
&\quad \left. - \phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), \dots, x_{i-1}^n - p_{i-1_t}(x_{i-1}^n), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \\
&\quad \quad x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n)) \left. \right\| \\
&\quad + Q_i(t) \left\| F_{i_t}(x_1^n - p_{1_t}(x_1^n), \dots, x_{i-1}^n - p_{i-1_t}(x_{i-1}^n), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \\
&\quad \quad x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n)) \\
&\quad \left. - F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), \dots, x_{i-1}^{n-1} - p_{i-1_t}(x_{i-1}^{n-1}), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \\
&\quad \quad x_{i+1}^{n-1} - p_{i+1_t}(x_{i+1}^{n-1}), \dots, x_k^{n-1} - p_{k_t}(x_k^{n-1})) \left. \right\|.
\end{aligned} \tag{4.14}$$

Since $H_i(A_i, B_i)$ is δ_{i1} -Lipschitz continuous with respect to A_i and δ_{i2} -Lipschitz continuous with respect to B_i , we have

$$\begin{aligned}
&\left\| H_i(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right\| \\
&\leq \left\| H_i(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^n))) \right\| \\
&\quad + \left\| H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right\| \tag{4.15} \\
&\leq (\delta_{i1}(t) + \delta_{i2}(t)) \left\| f_{i_t}(x_i^n) - f_{i_t}(x_i^{n-1}) \right\| \\
&\leq (\delta_{i1}(t) + \delta_{i2}(t)) \xi_i(t) \left\| x_i^n(t) - x_i^{n-1}(t) \right\|.
\end{aligned}$$

Since $F_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ is ζ_{ii} -Lipschitz continuous in the $(i+1)$ th argument, by (4.15), we have

$$\begin{aligned}
&\left\| H_i(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_i(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right. \\
&\quad - (\phi_{i_t} \circ F_{i_t}(x_1^n - p_{1_t}(x_1^n), \dots, x_{i-1}^n - p_{i-1_t}(x_{i-1}^n), x_i^n - p_{i_t}(x_i^n), \\
&\quad \quad x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n))
\end{aligned}$$

$$\begin{aligned}
& -\phi_{i_t} \circ F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^{n-1} p_{i_t}(x_i^{n-1}), \right. \\
& \quad \left. x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \|^q \\
& \leq \left\| H_{i_t}(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_{i_t}(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right\|^q \\
& \quad - q \left\langle \phi_{i_t} \circ F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^n - p_{i_t}(x_i^n), \right. \right. \\
& \quad \left. \left. x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right. \\
& \quad \left. - \phi_{i_t} \circ F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \right. \\
& \quad \left. \left. x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right\rangle \\
& \quad J_q \left(H_{i_t}(A_{i_t}(f_{i_t}(x_i^n)), B_{i_t}(f_{i_t}(x_i^n))) - H_{i_t}(A_{i_t}(f_{i_t}(x_i^{n-1})), B_{i_t}(f_{i_t}(x_i^{n-1}))) \right) \rangle \\
& + c_q Q_i(t)^q \left\| F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^n - p_{i_t}(x_i^n), \right. \right. \\
& \quad \left. \left. x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right. \\
& \quad \left. - F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \right. \\
& \quad \left. \left. x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right\|^q \\
& \leq \left((\delta_{i1}(t) + \delta_{i2}(t))^q \xi_i(t)^q + q Q_i(t) (1 - q \gamma_{p_i}(t) + c_q \lambda_{p_i}(t)^q)^{1/q} \xi_{ii}(t) (\delta_{i1}(t) + \delta_{i2}(t))^{q-1} \xi_i(t)^{q-1} \right. \\
& \quad \left. + c_q Q_i(t)^q (1 - q \gamma_{p_i}(t) + c_q \lambda_{p_i}(t)^q) \xi_{ii}(t)^q \right) \left\| x_i^n(t) - x_i^{n-1}(t) \right\|^q, \quad i = 1, 2, \dots, k.
\end{aligned} \tag{4.16}$$

Since $F_i : \Omega \times \prod_{j=1}^k E_j \rightarrow E_i$ is ξ_{ij} -Lipschitz continuous in the $(j+1)$ th argument, for $j = 1, \dots, i-1, i+1, \dots, k$, we have

$$\begin{aligned}
& \left\| F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, \right. \right. \\
& \quad \left. \left. x_k^n - p_{k_{i_t}}(x_k^n) \right) - F_{i_t} \left(x_1^{n-1} - p_{1_{i_t}}(x_1^{n-1}), \dots, x_{i-1}^{n-1} - p_{i-1_{i_t}}(x_{i-1}^{n-1}), x_i^{n-1} - p_{i_t}(x_i^{n-1}), \right. \right. \\
& \quad \left. \left. x_{i+1}^{n-1} - p_{i+1_{i_t}}(x_{i+1}^{n-1}), \dots, x_k^{n-1} - p_{k_{i_t}}(x_k^{n-1}) \right) \right\| \\
& \leq \left\| F_{i_t} \left(x_1^n - p_{1_{i_t}}(x_1^n), x_2^n - p_{2_{i_t}}(x_2^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), \right. \right. \\
& \quad \left. \left. x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right. \\
& \quad \left. - F_{i_t} \left(x_1^{n-1} - p_{1_{i_t}}(x_1^{n-1}), x_2^n - p_{2_{i_t}}(x_2^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), \right. \right. \\
& \quad \left. \left. x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^n - p_{i+1_{i_t}}(x_{i+1}^n), \dots, x_k^n - p_{k_{i_t}}(x_k^n) \right) \right\| \\
& + \left\| F_{i_t} \left(x_1^{n-1} - p_{1_{i_t}}(x_1^{n-1}), x_2^n - p_{2_{i_t}}(x_2^n), x_3^n - p_{3_{i_t}}(x_3^n), \dots, x_{i-1}^n - p_{i-1_{i_t}}(x_{i-1}^n), \right. \right.
\end{aligned}$$

$$\begin{aligned}
& x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n) \\
& - F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), x_3^n - p_{3_t}(x_3^n), \dots, x_{i-1}^n - p_{i-1_t}(x_{i-1}^n), \\
& x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^n - p_{i+1_t}(x_{i+1}^n), \dots, x_k^n - p_{k_t}(x_k^n)) \Big\| \\
& + \dots + \Big\| F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), x_3^{n-1} - p_{3_t}(x_3^{n-1}), \dots, \\
& x_{i-1}^{n-1} - p_{i-1_t}(x_{i-1}^{n-1}), x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^{n-1} - p_{i+1_t}(x_{i+1}^{n-1}), \dots, \\
& x_{k-1}^{n-1} - p_{k-1_t}(x_{k-1}^{n-1}), x_k^n - p_{k_t}(x_k^n) \\
& - F_{i_t}(x_1^{n-1} - p_{1_t}(x_1^{n-1}), x_2^{n-1} - p_{2_t}(x_2^{n-1}), x_3^{n-1} - p_{3_t}(x_3^{n-1}), \dots, \\
& x_{i-1}^{n-1} - p_{i-1_t}(x_{i-1}^{n-1}), x_i^{n-1} - p_{i_t}(x_i^{n-1}), x_{i+1}^{n-1} - p_{i+1_t}(x_{i+1}^{n-1}), \dots, \\
& x_{k-1}^{n-1} - p_{k-1_t}(x_{k-1}^{n-1}), x_k^{n-1} - p_{k_t}(x_k^{n-1})) \Big\| \\
& \leq \zeta_{i1}(t)(1 - q\gamma_{p_1}(t) + c_q\lambda_{p_1}(t)^q)^{1/q} \Big\| x_1^n(t) - x_1^{n-1}(t) \Big\| \\
& + \zeta_{i2}(t)(1 - q\gamma_{p_2}(t) + c_q\lambda_{p_2}(t)^q)^{1/q} \Big\| x_2^n(t) - x_2^{n-1}(t) \Big\| \\
& + \dots + \zeta_{i,i-1}(t)(1 - q\gamma_{p_{i-1}}(t) + c_q\lambda_{p_{i-1}}(t)^q)^{1/q} \Big\| x_{i-1}^n(t) - x_{i-1}^{n-1}(t) \Big\| \\
& + \zeta_{i,i+1}(t)(1 - q\gamma_{p_{i+1}}(t) + c_q\lambda_{p_{i+1}}(t)^q)^{1/q} \Big\| x_{i+1}^n(t) - x_{i+1}^{n-1}(t) \Big\| \\
& + \dots + \zeta_{ik}(t)(1 - q\gamma_{p_k}(t) + c_q\lambda_{p_k}(t)^q)^{1/q} \Big\| x_k^n(t) - x_k^{n-1}(t) \Big\| \\
& = \sum_{j=1}^{i-1} \zeta_{ij}(t)(1 - q\gamma_{p_j}(t) + c_q\lambda_{p_j}(t)^q)^{1/q} \Big\| x_j^n(t) - x_j^{n-1}(t) \Big\| \\
& + \sum_{j=i+1}^k \zeta_{ij}(t)(1 - q\gamma_{p_j}(t) + c_q\lambda_{p_j}(t)^q)^{1/q} \Big\| x_j^n(t) - x_j^{n-1}(t) \Big\|, \quad i = 1, 2, \dots, k.
\end{aligned} \tag{4.17}$$

It follows from the Lipschitz continuity of N_i , the $\widetilde{H}$ -continuity of $\widetilde{T}_{1i}, \widetilde{T}_{2i}, \dots, \widetilde{T}_{ki}$ ($i = 1, 2, \dots, k$) that

$$\begin{aligned}
& \Big\| N_{i_t}(u_{i1}^n, u_{i2}^n, \dots, u_{ik}^n) - N_{i_t}(u_{i1}^{n-1}, u_{i2}^{n-1}, \dots, u_{ik}^{n-1}) \Big\| \\
& \leq \Big\| N_{i_t}(u_{i1}^n, u_{i2}^n, \dots, u_{ik}^n) - N_{i_t}(u_{i1}^{n-1}, u_{i2}^n, \dots, u_{ik}^n) \Big\| \\
& + \Big\| N_{i_t}(u_{i1}^{n-1}, u_{i2}^n, u_{i3}^n, \dots, u_{ik}^n) - N_{i_t}(u_{i1}^{n-1}, u_{i2}^{n-1}, u_{i3}^n, \dots, u_{ik}^n) \Big\|
\end{aligned}$$

$$\begin{aligned}
& + \cdots + \left\| N_{i_1} \left(u_{i_1}^{n-1}, u_{i_2}^{n-1}, u_{i_3}^{n-1}, \dots, u_{i_{k-1}}^{n-1}, u_{i_k}^{n-1} \right) - N_{i_1} \left(u_{i_1}^{n-1}, u_{i_2}^{n-1}, u_{i_3}^{n-1}, \dots, u_{i_{k-1}}^{n-1}, u_{i_k}^{n-1} \right) \right\| \\
& \leq \sum_{j=1}^k v_{ij}(t) \left\| u_{ij}^n(t) - u_{ij}^{n-1}(t) \right\| \leq \sum_{j=1}^k v_{ij}(t) \left(1 + \frac{1}{n} \right) \widetilde{H} \left(\widetilde{T}_{ij_i}(x_i^n), \widetilde{T}_{ij_i}(x_i^{n-1}) \right) \\
& \leq \sum_{j=1}^k v_{ij}(t) \left(1 + \frac{1}{n} \right) t_{ij}(t) \left\| x_i^n(t) - x_i^{n-1}(t) \right\|, \quad i = 1, 2, \dots, k.
\end{aligned} \tag{4.18}$$

It follows from (4.12)–(4.18) that for each $i = 1, 2, \dots, k$,

$$\begin{aligned}
& \left\| x_i^{n+1}(t) - x_i^n(t) \right\| \\
& \leq \left[\left(1 + q\epsilon_i(t)\xi_i(t)^q - q\mu_i(t) + c_q\xi_i(t)^q \right)^{1/q} + \lambda_i(t)\lambda_{s_i}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \right. \\
& \quad \times \left(\left((\delta_{i1}(t) + \delta_{i2}(t))^q \xi_i(t)^q \right. \right. \\
& \quad \quad + qQ_i(t)(1 - q\gamma_{p_i}(t) + c_q\lambda_{p_i}(t)^q)^{1/q} \xi_{ii}(t)(\delta_{i1}(t) + \delta_{i2}(t))^{q-1} \xi_i(t)^{q-1} \\
& \quad \quad + c_qQ_i(t)^q (1 - q\gamma_{p_i}(t) + c_q\lambda_{p_i}(t)^q) \xi_{ii}(t)^q \Big)^{1/q} \\
& \quad \left. + \sum_{j=1}^k Q_i(t)v_{ij}(t) \left(1 + \frac{1}{n} \right) t_{ij}(t) \right) \left\| x_i^n(t) - x_i^{n-1}(t) \right\| \\
& + \sum_{j=1}^{i-1} \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} Q_i(t)\xi_{ij}(t) \left(1 - q\gamma_{p_j}(t) + c_q\lambda_{p_j}(t)^q \right)^{1/q} \left\| x_j^n(t) - x_j^{n-1}(t) \right\| \\
& + \sum_{j=i+1}^k \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} Q_i(t)\xi_{ij}(t) \left(1 - q\gamma_{p_j}(t) + c_q\lambda_{p_j}(t)^q \right)^{1/q} \left\| x_j^n(t) - x_j^{n-1}(t) \right\|.
\end{aligned} \tag{4.19}$$

Therefore,

$$\begin{aligned}
& \sum_{i=1}^k \left\| x_i^{n+1}(t) - x_i^n(t) \right\| \\
& \leq \sum_{i=1}^k \left\{ \left[\left(1 + q\epsilon_i(t)\xi_i(t)^q - q\mu_i(t) + c_q\xi_i(t)^q \right)^{1/q} + \lambda_i(t)\lambda_{s_i}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \right. \right. \\
& \quad \times \left(\left((\delta_{i1}(t) + \delta_{i2}(t))^q \xi_i(t)^q + qQ_i(t)(1 - q\gamma_{p_i}(t) + c_q\lambda_{p_i}(t)^q)^{1/q} \right. \right. \\
& \quad \quad \times \xi_{ii}(t)(\delta_{i1}(t) + \delta_{i2}(t))^{q-1} \xi_i(t)^{q-1} + c_qQ_i(t)^q (1 - q\gamma_{p_i}(t) + c_q\lambda_{p_i}(t)^q) \xi_{ii}(t)^q \Big)^{1/q}
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=1}^k \varrho_i(t) v_{ij}(t) \left(1 + \frac{1}{n} \right) t_{ij}(t) \Bigg] \left\| x_i^n(t) - x_i^{n-1}(t) \right\| \\
& + \sum_{j=1}^{i-1} \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \varrho_i(t) \zeta_{ij}(t) \left(1 - q\gamma_{p_j}(t) + c_q \lambda_{p_j}(t)^q \right)^{1/q} \left\| x_j^n(t) - x_j^{n-1}(t) \right\| \\
& + \sum_{j=i+1}^k \frac{\tau_i(t)^{q-1}}{r_i(t) - m_i(t)} \varrho_i(t) \zeta_{ij}(t) \left(1 - q\gamma_{p_j}(t) + c_q \lambda_{p_j}(t)^q \right)^{1/q} \left\| x_j^n(t) - x_j^{n-1}(t) \right\| \Bigg\} \\
= & \left[\left(1 + q\epsilon_1(t) \xi_1(t)^q - q\mu_1(t) + c_q \xi_1(t)^q \right)^{1/q} + \lambda_1(t) \lambda_{s_1}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} \right. \\
& \times \left(\left((\delta_{11}(t) + \delta_{12}(t))^q \xi_1(t)^q \right. \right. \\
& \quad + q\varrho_1(t) \left(1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q \right)^{1/q} \xi_{11}(t) (\delta_{11}(t) + \delta_{12}(t))^{q-1} \xi_1(t)^{q-1} \\
& \quad + c_q \varrho_1(t)^q \left(1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q \right) \xi_{11}(t)^q \Big)^{1/q} \\
& \quad \left. + \sum_{j=1}^k \varrho_1(t) v_{1j}(t) \left(1 + \frac{1}{n} \right) t_{1j}(t) \right) \\
& + \sum_{j=2}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} \varrho_j(t) \zeta_{j1}(t) \left(1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q \right)^{1/q} \Bigg] \left\| x_1^n(t) - x_1^{n-1}(t) \right\| \\
& + \left[\left(1 + q\epsilon_2(t) \xi_2(t)^q - q\mu_2(t) + c_q \xi_2(t)^q \right)^{1/q} + \lambda_2(t) \lambda_{s_2}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_2(t)^{q-1}}{r_2(t) - m_2(t)} \right. \\
& \times \left(\left((\delta_{21}(t) + \delta_{22}(t))^q \xi_2(t)^q \right. \right. \\
& \quad + q\varrho_2(t) \left(1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q \right)^{1/q} \xi_{22}(t) (\delta_{21}(t) + \delta_{22}(t))^{q-1} \xi_2(t)^{q-1} \\
& \quad + c_q \varrho_2(t)^q \left(1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q \right) \xi_{22}(t)^q \Big)^{1/q} + \sum_{j=1}^k \varrho_2(t) v_{2j}(t) \left(1 + \frac{1}{n} \right) t_{2j}(t) \Bigg) \\
& + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} \varrho_1(t) \zeta_{12}(t) \left(1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q \right)^{1/q} \\
& + \sum_{j=3}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} \varrho_j(t) \zeta_{j2}(t) \left(1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q \right)^{1/q} \Bigg] \left\| x_2^n(t) - x_2^{n-1}(t) \right\| \\
& + \cdots + \left[\left(1 + q\epsilon_k(t) \xi_k(t)^q - q\mu_k(t) + c_q \xi_k(t)^q \right)^{1/q} + \lambda_k(t) \lambda_{s_k}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_k(t)^{q-1}}{r_k(t) - m_k(t)} \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left(((\delta_{k1}(t) + \delta_{k2}(t))^q \xi_k(t)^q \right. \\
& \quad + qQ_k(t)(1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q)^{1/q} \zeta_{kk}(t)(\delta_{k1}(t) + \delta_{k2}(t))^{q-1} \xi_k(t)^{q-1} \\
& \quad + c_q Q_k(t)^q (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q) \zeta_{kk}(t)^q)^{1/q} \\
& \quad + \sum_{j=1}^k Q_k(t) v_{kj}(t) \left(1 + \frac{1}{n} \right) t_{kj}(t) \Bigg) \\
& \quad + \sum_{j=1}^{k-1} \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \zeta_{jk}(t) (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q)^{1/q} \Bigg] \|x_k^n(t) - x_k^{n-1}(t)\| \\
& \leq \theta_n(t) \left(\sum_{i=1}^k \|x_i^n(t) - x_i^{n-1}(t)\| \right),
\end{aligned} \tag{4.20}$$

where

$$\begin{aligned}
& \theta_n(t) \\
& = \max \left\{ \left(1 + qe_1(t) \xi_1(t)^q - q\mu_1(t) + c_q \xi_1(t)^q \right)^{1/q} + \lambda_1(t) \lambda_{s_1}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} \right. \\
& \quad \times \left(((\delta_{11}(t) + \delta_{12}(t))^q \xi_1(t)^q \right. \\
& \quad + qQ_1(t)(1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q)^{1/q} \zeta_{11}(t)(\delta_{11}(t) + \delta_{12}(t))^{q-1} \xi_1(t)^{q-1} \\
& \quad + c_q Q_1(t)^q (1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q) \zeta_{11}(t)^q)^{1/q} + \sum_{j=1}^k Q_1(t) v_{1j}(t) \left(1 + \frac{1}{n} \right) t_{1j}(t) \Bigg) \\
& \quad + \sum_{j=2}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \zeta_{j1}(t) (1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q)^{1/q}, \\
& \quad \left(1 + qe_2(t) \xi_2(t)^q - q\mu_2(t) + c_q \xi_2(t)^q \right)^{1/q} + \lambda_2(t) \lambda_{s_2}(t) \left(1 + \frac{1}{n} \right) + \frac{\tau_2(t)^{q-1}}{r_2(t) - m_2(t)} \\
& \quad \times \left(((\delta_{21}(t) + \delta_{22}(t))^q \xi_2(t)^q \right. \\
& \quad + qQ_2(t)(1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q} \zeta_{22}(t)(\delta_{21}(t) + \delta_{22}(t))^{q-1} \xi_2(t)^{q-1} \\
& \quad + c_q Q_2(t)^q (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q) \zeta_{22}(t)^q)^{1/q} + \sum_{j=1}^k Q_2(t) v_{2j}(t) \left(1 + \frac{1}{n} \right) t_{2j}(t) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} Q_1(t) \xi_{12}(t) (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q} \\
& + \sum_{j=3}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \xi_{j2}(t) (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q}, \dots, \\
& (1 + q\epsilon_k(t) \xi_k(t)^q - q\mu_k(t) + c_q \xi_k(t)^q)^{1/q} + \lambda_k(t) \lambda_{s_k}(t) \left(1 + \frac{1}{n}\right) + \frac{\tau_k(t)^{q-1}}{r_k(t) - m_k(t)} \\
& \times \left(((\delta_{k1}(t) + \delta_{k2}(t))^q \xi_k(t)^q \right. \\
& \quad + qQ_k(t) (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q)^{1/q} \xi_{kk}(t) (\delta_{k1}(t) + \delta_{k2}(t))^{q-1} \xi_k(t)^{q-1} \\
& \quad \left. + c_q Q_k(t)^q (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q) \xi_{kk}(t)^q)^{1/q} + \sum_{j=1}^k Q_k(t) v_{kj}(t) \left(1 + \frac{1}{n}\right) t_{kj}(t) \right) \\
& + \sum_{j=1}^{k-1} \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \xi_{jk}(t) (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q)^{1/q} \Bigg\}.
\end{aligned} \tag{4.21}$$

Let

$$\begin{aligned}
& \theta(t) \\
& = \max \left\{ (1 + q\epsilon_1(t) \xi_1(t)^q - q\mu_1(t) + c_q \xi_1(t)^q)^{1/q} + \lambda_1(t) \lambda_{s_1}(t) + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} \right. \\
& \quad \times \left(((\delta_{11}(t) + \delta_{12}(t))^q \xi_1(t)^q \right. \\
& \quad \quad + qQ_1(t) (1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q)^{1/q} \xi_{11}(t) (\delta_{11}(t) + \delta_{12}(t))^{q-1} \xi_1(t)^{q-1} \\
& \quad \quad \left. + c_q Q_1(t)^q (1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q) \xi_{11}(t)^q)^{1/q} + \sum_{j=1}^k Q_1(t) v_{1j}(t) t_{1j}(t) \right) \\
& \quad + \sum_{j=2}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \xi_{j1}(t) (1 - q\gamma_{p_1}(t) + c_q \lambda_{p_1}(t)^q)^{1/q}, \\
& \quad (1 + q\epsilon_2(t) \xi_2(t)^q - q\mu_2(t) + c_q \xi_2(t)^q)^{1/q} + \lambda_2(t) \lambda_{s_2}(t) + \frac{\tau_2(t)^{q-1}}{r_2(t) - m_2(t)} \\
& \quad \times \left(((\delta_{21}(t) + \delta_{22}(t))^q \xi_2(t)^q \right. \\
& \quad \quad + qQ_2(t) (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q} \xi_{22}(t) (\delta_{21}(t) + \delta_{22}(t))^{q-1} \xi_2(t)^{q-1}
\end{aligned}$$

$$\begin{aligned}
& + c_q Q_2(t)^q (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q) \zeta_{22}(t)^q)^{1/q} + \sum_{j=1}^k Q_2(t) v_{2j}(t) t_{2j}(t) \Bigg) \\
& + \frac{\tau_1(t)^{q-1}}{r_1(t) - m_1(t)} Q_1(t) \zeta_{12}(t) (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q} \\
& + \sum_{j=3}^k \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \zeta_{j2}(t) (1 - q\gamma_{p_2}(t) + c_q \lambda_{p_2}(t)^q)^{1/q}, \dots, \\
& (1 + qe_k(t) \zeta_k(t)^q - q\mu_k(t) + c_q \zeta_k(t)^q)^{1/q} + \lambda_k(t) \lambda_{s_k}(t) + \frac{\tau_k(t)^{q-1}}{r_k(t) - m_k(t)} \\
& \times \left(((\delta_{k1}(t) + \delta_{k2}(t))^q \zeta_k(t)^q \right. \\
& \quad + qQ_k(t) ((1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q))^{1/q} \zeta_{kk}(t) (\delta_{k1}(t) + \delta_{k2}(t))^{q-1} \zeta_k(t)^{q-1} \\
& \quad \left. + c_q Q_k(t)^q (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q) \zeta_{kk}(t)^q)^{1/q} + \sum_{j=1}^k Q_k(t) v_{kj}(t) t_{kj}(t) \right) \\
& \left. + \sum_{j=1}^{k-1} \frac{\tau_j(t)^{q-1}}{r_j(t) - m_j(t)} Q_j(t) \zeta_{jk}(t) (1 - q\gamma_{p_k}(t) + c_q \lambda_{p_k}(t)^q)^{1/q} \right\}. \tag{4.22}
\end{aligned}$$

Then $\theta_n(t) \rightarrow \theta(t)$, for all $t \in \Omega$.

Define $\|\cdot\|_*$ on $E = \prod_{i=1}^k E_i$ by

$$\begin{aligned}
\|(z_1(t), z_2(t), \dots, z_k(t))\|_* &= \|z_1(t)\| + \|z_2(t)\| + \dots + \|z_k(t)\|, \\
\forall (z_1(t), z_2(t), \dots, z_k(t)) &\in E. \tag{4.23}
\end{aligned}$$

It is easy to see that $(E, \|\cdot\|_*)$ is a Banach space.

Define $z^{n+1}(t) = (x_1^{n+1}(t), x_2^{n+1}(t), \dots, x_k^{n+1}(t))$. Then, we have

$$\|z^{n+1}(t) - z^n(t)\|_* = \|x_1^{n+1}(t) - x_1^n(t)\| + \|x_2^{n+1}(t) - x_2^n(t)\| + \dots + \|x_k^{n+1}(t) - x_k^n(t)\|. \tag{4.24}$$

From the condition (4.11), we know that for all $t \in \Omega$, $0 < \theta(t) < 1$, and hence there exists an $n_0 > 0$ and $\theta_0(t) \in (0, 1)$ such that $\theta_n(t) \leq \theta_0(t)$, for all $n \geq n_0$. Therefore, by (4.20) and (4.24), we have

$$\|z^{n+1}(t) - z^n(t)\|_* \leq \theta_0(t) \|z^n(t) - z^{n-1}(t)\|_*, \quad \forall n \geq n_0. \tag{4.25}$$

It follows from (4.25) that

$$\|z^{n+1}(t) - z^n(t)\|_* \leq \theta_0(t)^{n-n_0} \|z^{n_0+1}(t) - z^{n_0}(t)\|_*. \tag{4.26}$$

Hence, for any $m \geq n \geq n_0$, it follows that

$$\begin{aligned} \|x_1^m(t) - x_1^n(t)\| &\leq \|z^m(t) - z^n(t)\|_* \\ &\leq \sum_{i=n}^{m-1} \|z^{i+1}(t) - z^i(t)\|_* \\ &\leq \sum_{i=n}^{m-1} \theta_0(t)^{i-n_0} \|z^{n_0+1}(t) - z^{n_0}(t)\|_*. \end{aligned} \quad (4.27)$$

Since $0 < \theta_0(t) < 1$, for all $t \in \Omega$, it follows from (4.27) that $\|x_1^m(t) - x_1^n(t)\| \rightarrow 0$ as $n \rightarrow \infty$, and hence $\{x_1^n(t)\}$ is a Cauchy sequence in E_1 . By the same argument, we also have that $\{x_i^n(t)\}$ is a Cauchy sequence in E_i ($i = 2, 3, \dots, k$). Thus there exist measurable mappings $x_1 : \Omega \rightarrow E_1, x_2 : \Omega \rightarrow E_2, \dots, x_k : \Omega \rightarrow E_k$ such that for all $t \in \Omega$, $x_1^n(t) \rightarrow x_1(t), x_2^n(t) \rightarrow x_2(t), \dots, x_k^n(t) \rightarrow x_k(t)$ as $n \rightarrow \infty$.

Now we prove that $u_{1i}^n(t) \rightarrow u_{1i}(t) \in \tilde{T}_{1i}(x_1), u_{2i}^n(t) \rightarrow u_{2i}(t) \in \tilde{T}_{2i}(x_2), \dots, u_{ki}^n(t) \rightarrow u_{ki}(t) \in \tilde{T}_{ki}(x_k), v_i^n(t) \rightarrow v_i(t) \in \tilde{S}_i(x_i)$ ($i = 1, 2, \dots, k$). In fact, it follows from the Lipschitz continuity of $\tilde{T}_{1i}, \tilde{T}_{2i}, \dots, \tilde{T}_{ki}, \tilde{S}_i$ and Algorithm 4.2 that for $i = 1, 2, \dots, k$,

$$\begin{aligned} \|u_{1i}^n(t) - u_{1i}^{n-1}(t)\| &\leq \left(1 + \frac{1}{n}\right) t_{1i}(t) \|x_1^n(t) - x_1^{n-1}(t)\|, \\ \|u_{2i}^n(t) - u_{2i}^{n-1}(t)\| &\leq \left(1 + \frac{1}{n}\right) t_{2i}(t) \|x_2^n(t) - x_2^{n-1}(t)\|, \\ &\vdots \\ \|u_{ki}^n(t) - u_{ki}^{n-1}(t)\| &\leq \left(1 + \frac{1}{n}\right) t_{ki}(t) \|x_k^n(t) - x_k^{n-1}(t)\|, \\ \|v_i^n(t) - v_i^{n-1}(t)\| &\leq \left(1 + \frac{1}{n}\right) \lambda_{s_i}(t) \|x_i^n(t) - x_i^{n-1}(t)\|. \end{aligned} \quad (4.28)$$

From (4.28), we know that $\{u_{1i}^n(t)\}, \{u_{2i}^n(t)\}, \dots, \{u_{ki}^n(t)\}, \{v_i^n(t)\}$ ($i = 1, 2, \dots, k$) are also Cauchy sequences. Therefore, there exist $u_{1i}(t) \in E_i, u_{2i}(t) \in E_i, \dots, u_{ki}(t) \in E_i, v_i(t) \in E_i$ ($i = 1, 2, \dots, k$) such that $u_{1i}^n(t) \rightarrow u_{1i}(t), u_{2i}^n(t) \rightarrow u_{2i}(t), \dots, u_{ki}^n(t) \rightarrow u_{ki}(t)$ as $n \rightarrow \infty$. Further, for $i = 1, 2, \dots, k$,

$$\begin{aligned} d(u_{1i}(t), \tilde{T}_{1i}(x_1)) &\leq \|u_{1i}(t) - u_{1i}^n(t)\| + d(u_{1i}^n(t), \tilde{T}_{1i}(x_1)) \\ &\leq \|u_{1i}(t) - u_{1i}^n(t)\| + \tilde{H}(\tilde{T}_{1i}(x_1^n), \tilde{T}_{1i}(x_1)) \\ &\leq \|u_{1i}(t) - u_{1i}^n(t)\| + t_{1i}(t) \|x_1^n(t) - x_1(t)\| \rightarrow 0, \quad \forall t \in \Omega. \end{aligned} \quad (4.29)$$

Since $\tilde{T}_{1i}(x_1)$ is closed, we have $u_{1i}(t) \in \tilde{T}_{1i}(x_1)$ ($i = 1, 2, \dots, k$). Similarly, $u_{2i}(t) \in \tilde{T}_{2i}(x_2), \dots, u_{ki}(t) \in \tilde{T}_{ki}(x_k), v_i(t) \in \tilde{S}_i(x_i)$ ($i = 1, 2, \dots, k$). This completes the proof. $\square$

Now we show the existence of solutions for problem (4.4).

Lemma 4.4. *Problem (4.4) is equivalent to find measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki} : \Omega \rightarrow \mathcal{H}_i$ such that for all $t \in \Omega$ and $i = 1, 2, \dots, k$, $x_i(t) \in \mathcal{H}_i$, $T_{1i} (u_{1i}(t)) \geq a_1(x_1(t))$, $T_{2i, x_2(t)} (u_{2i}(t)) \geq a_2(x_2(t)), \dots, T_{ki, x_k(t)} (u_{ki}(t)) \geq a_k(x_k(t))$ such that*

$$\begin{aligned} & f_i(x_i) \\ &= J_{\rho_i(t)}^{\varphi_i} [f_i(x_i) - \rho_i(t)(F_i(x_1 - p_{1i}(x_1), x_2 - p_{2i}(x_2), \dots, x_k - p_{ki}(x_k)) + N_i(u_{1i}, u_{2i}, \dots, u_{ki}))], \end{aligned} \quad (4.30)$$

where $\rho_i : \Omega \rightarrow (0, \infty)$ is a measurable function and $J_{\rho_i(t)}^{\varphi_i} = (I_t + \rho_i(t)\partial\varphi_i)^{-1}$.

If for $i = 1, 2, \dots, k$, $E_i = \mathcal{H}_i$ is a Hilbert space and $M_i(\cdot, v_i) = \partial\varphi_i(\cdot)$, for all $t \in \Omega$, $x_i(t) \in \mathcal{H}_i$ and $v_i(t) \in \tilde{S}_i(x_i)$, then Algorithm 4.2 reduces to Algorithm 4.5.

Algorithm 4.5. For any given measurable mapping $x_i^0 : \Omega \rightarrow \mathcal{H}_i$ ($i = 1, 2, \dots, k$), compute the following random iterative sequences $\{x_i^n(t)\}$, $\{u_{1i}^n(t)\}$, $\{u_{2i}^n(t)\}, \dots, \{u_{ki}^n(t)\}$ ($i = 1, 2, \dots, k$) for solving problem (4.4) as follows:

$$\begin{aligned} & x_i^{n+1}(t) \\ &= x_i^n(t) - f_i(x_i^n) + J_{\rho_i(t)}^{\varphi_i} [f_i(x_i^n) - \rho_i(t)(F_i(x_1^n - p_{1i}(x_1^n), x_2^n - p_{2i}(x_2^n), \dots, x_k^n - p_{ki}(x_k^n)) + N_i(u_{1i}^n, u_{2i}^n, \dots, u_{ki}^n))], \\ & u_{1i}^n(t) \in \tilde{T}_{1i}(x_1^n), u_{2i}^n(t) \in \tilde{T}_{2i}(x_2^n), \dots, u_{ki}^n(t) \in \tilde{T}_{ki}(x_k^n), \\ & \|u_{1i}^n(t) - u_{1i}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{1i}(x_1^n), \tilde{T}_{1i}(x_1^{n+1})), \\ & \|u_{2i}^n(t) - u_{2i}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{2i}(x_2^n), \tilde{T}_{2i}(x_2^{n+1})), \\ & \vdots \\ & \|u_{ki}^n(t) - u_{ki}^{n+1}(t)\| \leq \left(1 + \frac{1}{n+1}\right) \widetilde{H}(\tilde{T}_{ki}(x_k^n), \tilde{T}_{ki}(x_k^{n+1})), \end{aligned} \quad (4.31)$$

for any $t \in \Omega$, $i = 1, 2, \dots, k$, $n = 0, 1, 2, \dots$.

From Theorem 4.3, we have the following Theorem.

Theorem 4.6. For $i = 1, 2, \dots, k$, let $\mathcal{H}_i$ be a Hilbert space, $f_i : \Omega \times \mathcal{H}_i \rightarrow \mathcal{H}_i$ be (ϵ_i, μ_i) -relaxed cocoercive and ξ_i -Lipschitz continuous, and $p_i : \Omega \times \mathcal{H}_i \rightarrow \mathcal{H}_i$ be γ_{p_i} -strongly accretive and λ_{p_i} -Lipschitz continuous. Let $F_i : \Omega \times \prod_{j=1}^k \mathcal{H}_j \rightarrow \mathcal{H}_i$ be ζ_{ij} -Lipschitz continuous in the $(j+1)$ th argument for $j = 1, 2, \dots, k$, and $N_i : \Omega \times \prod_{j=1}^k \mathcal{H}_j \rightarrow \mathcal{H}_i$ be v_{ij} -Lipschitz continuous in the $(j+1)$ th argument for $j = 1, 2, \dots, k$. Let $T_{1i} : \Omega \times \mathcal{H}_1 \rightarrow F(\mathcal{H}_i)$, $T_{2i} : \Omega \times \mathcal{H}_2 \rightarrow F(\mathcal{H}_i), \dots, T_{ki} :$

$\Omega \times \mathcal{H}_k \rightarrow F(\mathcal{H}_i)$ be random fuzzy mappings satisfying the condition (C); $\tilde{T}_{1i} : \Omega \times \mathcal{H}_1 \rightarrow CB(\mathcal{H}_i)$, $\tilde{T}_{2i} : \Omega \times \mathcal{H}_2 \rightarrow CB(\mathcal{H}_i), \dots, \tilde{T}_{ki} : \Omega \times \mathcal{H}_k \rightarrow CB(\mathcal{H}_i)$ be random multivalued mappings induced by $T_{1i}, T_{2i}, \dots, T_{ki}$, respectively. Suppose that $\tilde{T}_{1i}, \tilde{T}_{2i}, \dots, \tilde{T}_{ki}$ are $\widetilde{H}$ -continuous with constants $t_{1i}, t_{2i}, \dots, t_{ki}$, respectively. If there exists a constant $\rho_i(t) > 0$ ($i = 1, 2, \dots, k$) such that

$$\begin{aligned}
0 &< \left(1 + 2\epsilon_1(t)\xi_1(t)^2 - 2\mu_1(t) + \xi_1(t)^2\right)^{1/2} \\
&+ \left(\xi_1(t)^2 + 2\rho_1(t)\left(1 - 2\gamma_{p_1}(t) + \lambda_{p_1}(t)^2\right)^{1/2}\xi_{11}(t)\xi_1(t)\right. \\
&\quad \left.+ \rho_1(t)^2\left(1 - 2\gamma_{p_1}(t) + \lambda_{p_1}(t)^2\right)\xi_{11}(t)^2\right)^{1/2} \\
&+ \sum_{j=1}^k \rho_1(t)\nu_{1j}(t)t_{1j}(t) + \sum_{j=2}^k \rho_j(t)\xi_{j1}(t)\left(1 - 2\gamma_{p_1}(t) + \lambda_{p_1}(t)^2\right)^{1/2} < 1, \\
0 &< \left(1 + 2\epsilon_2(t)\xi_2(t)^2 - 2\mu_2(t) + \xi_2(t)^2\right)^{1/2} \\
&+ \left(\xi_2(t)^2 + 2\rho_2(t)\left(1 - 2\gamma_{p_2}(t) + \lambda_{p_2}(t)^2\right)^{1/2}\xi_{22}(t)\xi_2(t)\right. \\
&\quad \left.+ \rho_2(t)^2\left(1 - 2\gamma_{p_2}(t) + \lambda_{p_2}(t)^2\right)\xi_{22}(t)^2\right)^{1/2} \\
&+ \sum_{j=1}^k \rho_2(t)\nu_{2j}(t)t_{2j}(t) + \rho_1(t)\xi_{12}(t)\left(1 - 2\gamma_{p_2}(t) + \lambda_{p_2}(t)^2\right)^{1/2} \\
&+ \sum_{j=3}^k \rho_j(t)\xi_{j2}(t)\left(1 - 2\gamma_{p_2}(t) + \lambda_{p_2}(t)^2\right)^{1/2} < 1, \dots, \\
0 &< \left(1 + 2\epsilon_k(t)\xi_k(t)^2 - 2\mu_k(t) + \xi_k(t)^2\right)^{1/2} \\
&+ \left(\xi_k(t)^2 + 2\rho_k(t)\left(1 - 2\gamma_{p_k}(t) + \lambda_{p_k}(t)^2\right)^{1/2}\xi_{kk}(t)\xi_k(t)\right. \\
&\quad \left.+ \rho_k(t)^2\left(1 - 2\gamma_{p_k}(t) + \lambda_{p_k}(t)^2\right)\xi_{kk}(t)^2\right)^{1/2} + \sum_{j=1}^k \rho_k(t)\nu_{kj}(t)t_{kj}(t) \\
&+ \sum_{j=1}^{k-1} \rho_j(t)\xi_{jk}(t)\left(1 - 2\gamma_{p_k}(t) + \lambda_{p_k}(t)^2\right)^{1/2} < 1.
\end{aligned} \tag{4.32}$$

Then there exist measurable mappings $x_i, u_{1i}, u_{2i}, \dots, u_{ki} : \Omega \rightarrow \mathcal{H}_i$ such that (4.4) holds. Moreover, for all $t \in \Omega$, $x_i^n(t) \rightarrow x_i(t)$, $u_{1i}^n(t) \rightarrow u_{1i}(t)$, $u_{2i}^n(t) \rightarrow u_{2i}(t), \dots, u_{ki}^n(t) \rightarrow u_{ki}(t)$, where $\{x_i^n(t)\}, \{u_{1i}^n(t)\}, \dots, \{u_{ki}^n(t)\}$ are random sequences obtained by Algorithm 4.5.

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