

Research Article

Heat Generation and Thermal Radiation Effects over a Stretching Sheet in a Micropolar Fluid

M. Ganeswara Reddy

Department of Mathematics, Acharya Nagarjuna University, Ongole Campus, Ongole 523 001, India

Correspondence should be addressed to M. Ganeswara Reddy, mgrsvu@gmail.com

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The effects of radiation and heat generation on steady thermal boundary layer flow induced by a linearly stretching sheet immersed in an incompressible micropolar fluid with constant surface temperature are investigated. Similarity transformation is employed to transform the governing partial differential equations into ordinary ones, which are then solved numerically using the Runge-Kutta fourth order along shooting method. Results for the local Nusselt number as well as the temperature profiles are presented for different values of the governing parameters. It is observed that the velocity increases with an increase in the material parameter. It is seen that the temperature profile is influenced considerably and increases when the value of heat generation parameter increases along the boundary layer. Also, the temperature distribution of the fluid increases with an increase in the radiation parameter. Comparisons with previously published work are performed and the results are found to be in very good agreement.

1. Introduction

Flow of a viscous fluid past a stretching sheet is a classical problem in fluid dynamics. The development of boundary layer flow induced solely by a stretching sheet was first studied by Crane [1] who first obtained an elegant analytical solution to the boundary layer equations for the problem of steady two dimensional flow due to a stretching surface in a quiescent incompressible fluid. Flow and heat transfer characteristics due to a stretching sheet in a stationary fluid occur in a number of industrial manufacturing processes and include both metal and polymer sheets, for example, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, paper production, metal spinning, and drawing plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface. This problem was then extended by P. S. Gupta and A. S. Gupta [2] to a permeable surface. The flow problem due to a linearly stretching sheet belongs to a class of exact solutions of the Navier-Stokes equations. Thus, the

exact solutions reported by Crane [1] and P. S. Gupta and A. S. Gupta [2] are also the exact solutions to the Navier-Stokes equations. The heat transfer aspects of similar problems were studied by Grubka and Bobba [3], Chen and Char [4], Dutta et al. [5], Ali [6, 7], Afzal and Varshney [8], Afzal [9], and many others. On the other hand, the effects of buoyancy force on the development of velocity and thermal boundary layer flows over a stretching sheet have been investigated by Chen [10], Ali and Al-Yousef [11], Daskalakis [12], Partha et al. [13], Abd El-Aziz [14], Mahapatra et al. [15], and Ishak et al. [16–20], among others.

The study of flow and heat transfer past a stretching sheet has gained tremendous interest among researchers due to its industrial and engineering applications. This includes extrusion of plastic sheets, annealing and tinning of copper wire, paper production, crystal growing, and glass blowing. The final products depend mainly on the stretching and cooling rates at the surface. Their studies are not only restricted to the Newtonian fluids but also include the non-Newtonian fluids such as micropolar fluids. Such studies have been carried out by Chiam [21], Heruska et al. [22],

Agarwal et al. [23], Hassanian and Gorla [24], Kelson and Desseaux [25], Kelson and Farrell [26], Nazar et al. [27], and very recently by Hayat et al. [28].

In the present paper we study the development of thermal boundary layer flow induced by a stretching sheet immersed in a micropolar fluid with the effect of heat generation and thermal radiation is taken into consideration. The governing partial differential equations are transformed into ordinary ones using similarity transformation, before being solved numerically by the Runge-Kutta fourth order along shooting method.

2. Mathematical Model

A steady laminar boundary layer flow over a stretching sheet immersed in a quiescent and incompressible micropolar fluid with uniform surface temperature T_w is considered. It is assumed that the sheet is stretched with a linear velocity $U_w = ax$, where a is a positive constant and x is the distance from the slit where the sheet is issued. The simplified two-dimensional equations governing the flow may be written as

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y}. \quad (2)$$

Angular Momentum equation,

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j} \left(2N + \frac{\partial u}{\partial y} \right). \quad (3)$$

Energy equation,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p} (T - T_\infty). \quad (4)$$

The spin-gradient viscosity γ (Ahmadi [29]) can be defined as

$$\gamma = \left(\mu + \frac{\kappa}{2} \right) j = \mu \left(1 + \frac{K}{2} \right) j, \quad (5)$$

where μ is the dynamic viscosity, $K = \kappa/\mu$ is the dimensionless viscosity ratio and is called the material parameter, and we take $j = \nu/a$ as a reference length. The Equation (5) is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity.

The boundary conditions for the velocity, angular velocity, and fields are

$$\begin{aligned} u &= U_w, & v &= 0, & N &= -m \frac{\partial u}{\partial y}, \\ T &= T_w & \text{at } y &= 0 \\ u &\longrightarrow 0, & N &\longrightarrow 0, & T &\longrightarrow T_\infty \quad \text{as } y \longrightarrow \infty, \end{aligned} \quad (6)$$

where T_∞ is the ambient fluid temperature and m is the boundary parameter with $0 \leq m \leq 1$.

By using the Rosseland approximation, Brewster [30], the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y}, \quad (7)$$

where σ_s is the Stefan-Boltzmann constant and k_e is the mean absorption coefficient. It should be noted that by using the Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature differences within the flow are sufficiently small, then (7) can be linearized by expanding T^4 into the Taylor series about T_∞ , which after neglecting higher order terms takes the form

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \quad (8)$$

In view of (7) and (8), (4) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha(1+R) \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty), \quad (9)$$

where $\alpha = k/\rho c_p$ is the thermal diffusivity and $R = 16\sigma^* T_\infty^3 / 3kk^*$ is the radiation parameter.

We introduce now the following similarity transformation:

$$\begin{aligned} \eta &= \left(\frac{U_w}{\nu x} \right)^{1/2} y, & \psi &= (\nu x U_w)^{1/2} f(\eta), \\ N &= U_w \left(\frac{U_w}{\nu x} \right)^{1/2} h(\eta), & \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \\ Q &= \frac{Q_0 \nu}{\rho c_p U_w^2}, \end{aligned} \quad (10)$$

where η is the similarity variable and ψ is the stream function defined as $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ which identically satisfies the mass conservation equation (1). Substituting (10) into (2), (3), and (9) we obtain the following ordinary differential equations:

$$(1+K)f'''' + f f'' - f'^2 + K h' = 0, \quad (11)$$

$$\left(1 + \frac{K}{2} \right) h'' + f h' - f' h - K(2h + f'') = 0, \quad (12)$$

$$(1+R)\theta'' + \text{Pr} f \theta' + \text{Pr} Q \theta = 0, \quad (13)$$

where primes denote differentiation with respect to η and $\text{Pr} = \mu/\alpha$ is the Prandtl number.

The corresponding dimensionless boundary conditions are

$$\begin{aligned} f &= 0, & f' &= 1, & h &= -m f''(0), & \theta &= 1 \quad \text{at } \eta = 0 \\ f' &\longrightarrow 0, & h &\longrightarrow 0, & \theta &\longrightarrow 0 \quad \text{as } \eta \longrightarrow \infty. \end{aligned} \quad (14)$$

It is note that $K = 0$ corresponds to viscous fluid.

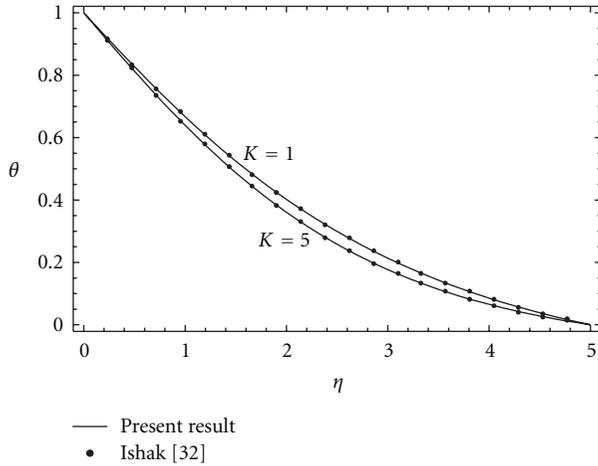


FIGURE 1: Comparison of temperature profiles.

3. Results and Discussion

The set of nonlinear ordinary differential equations (11)–(13) with boundary conditions (14) have been solved by using the Runge-Kutta fourth order along with Shooting method. First of all, higher order nonlinear differential Equations (11)–(13) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain et al. [31]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.01$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. To analyze the results, numerical computation has been carried out using the method described in the previous section for variations in the governing parameters, namely, material parameter K , radiation parameter R , heat generation parameter Q , the Prandtl number Pr , and boundary parameter m . In the present study following default parameter values are adopted for computations: $K = 1.0$, $R = 1.0$, $Q = 1.0$, $Pr = 0.71$, and $m = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

In order to assess the accuracy of our computed results, the present result has been compared with Ishak [32] for different values of K as shown in Figure 1 with $Q = 0.0$. It is observed that the agreements with the solution of velocity profiles are excellent.

Figure 2 presents the velocity profiles for various values of K when $m = 0.5$. We note that the parameters R and Pr have no influence on the flow field, which is clear from (11)–(13). It is evident from this figure that the boundary layer thickness increases with K . The velocity gradient at the surface $f''(0)$ decreases (in absolute sense) as K increases. Thus, micropolar fluids show drag reduction compared to viscous fluids. The negative velocity gradient at the surface $f''(0) < 0$ as shown in Figure 2 means the stretching sheet exerts a drag force on the fluid. This is not surprising since the development of the boundary layer is solely induced by

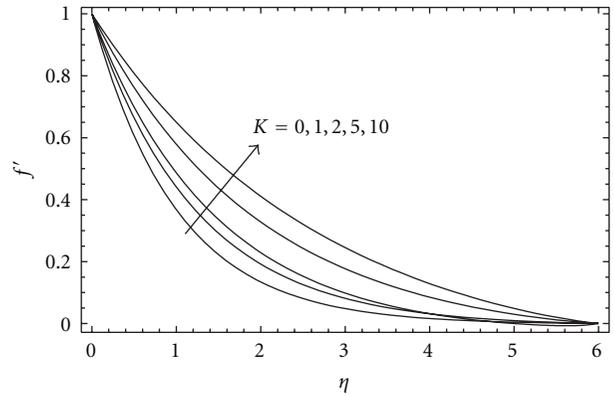


FIGURE 2: Velocity profiles for different values of K .

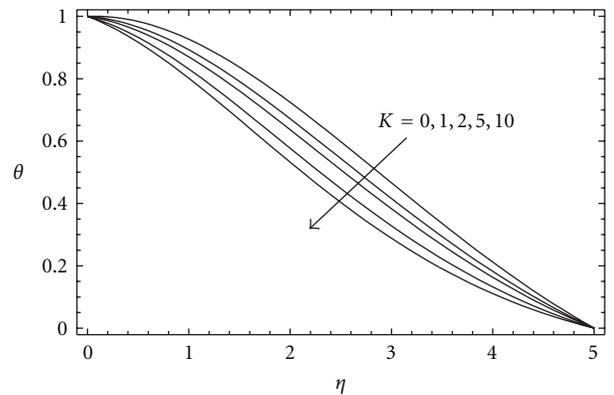
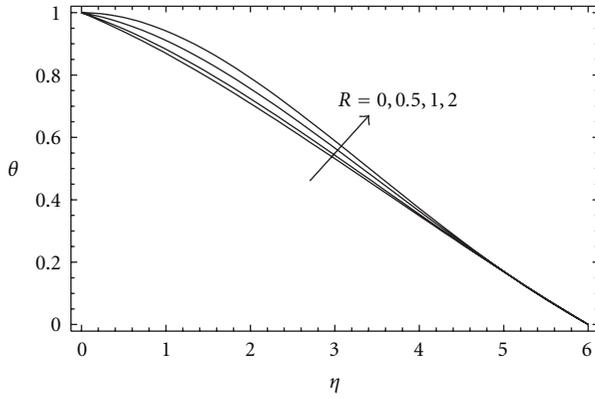
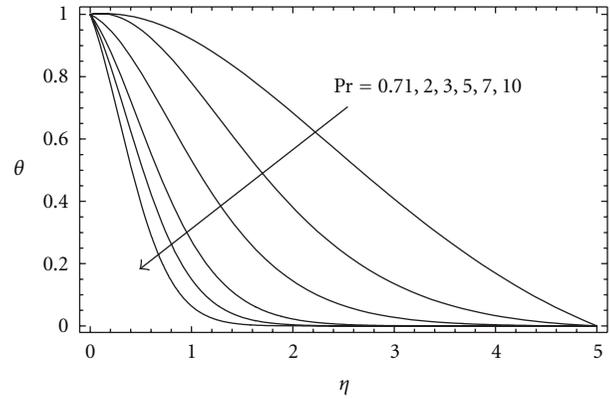
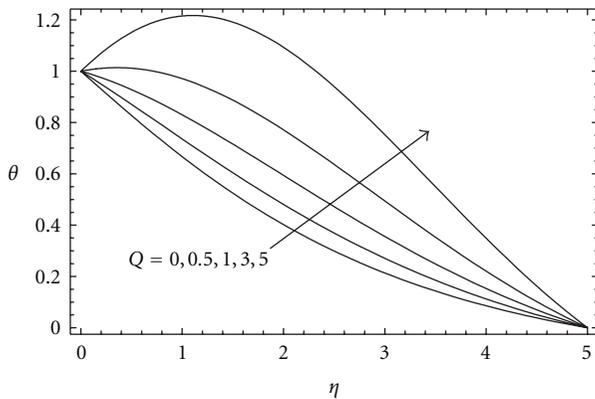
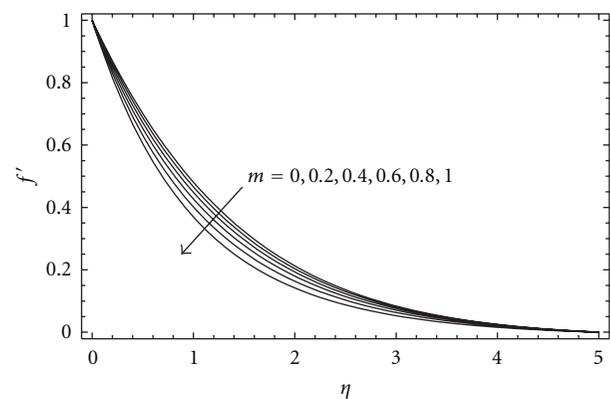


FIGURE 3: Temperature profiles for different values of K .

it. The effect of material parameter K on the temperature is shown in Figure 3. The temperature decreases with an increase in the material parameter.

Figure 4 shows the temperature profiles for different values of R . The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in increasing temperature within the boundary layer. The effect of heat generation parameter Q on the temperature is shown in Figure 5. From this figure, we observe that when the value of heat generation parameter increases, the temperature distribution also increases along the boundary layer.

The influence of the Prandtl number Pr on temperature field is shown in Figure 6. The numerical results show that the effect of increasing values of the Prandtl number results in a decreasing temperature. It is observed that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr . Hence in the case of the smaller Prandtl numbers

FIGURE 4: Temperature profiles for different values of R .FIGURE 6: Temperature profiles for different values of Pr .FIGURE 5: Temperature profiles for different values of Q .FIGURE 7: Velocity profiles for different values of m .

as the boundary layer is thicker and the rate of heat transfer is reduced.

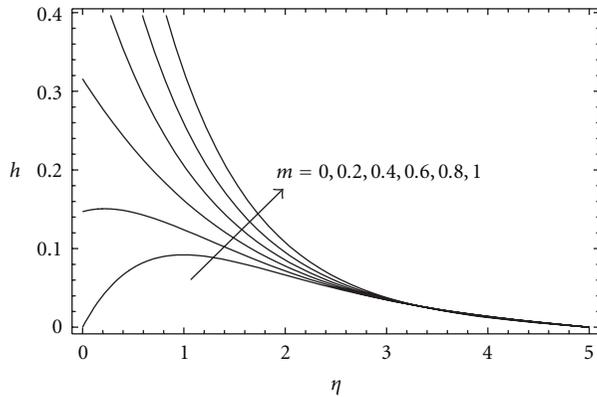
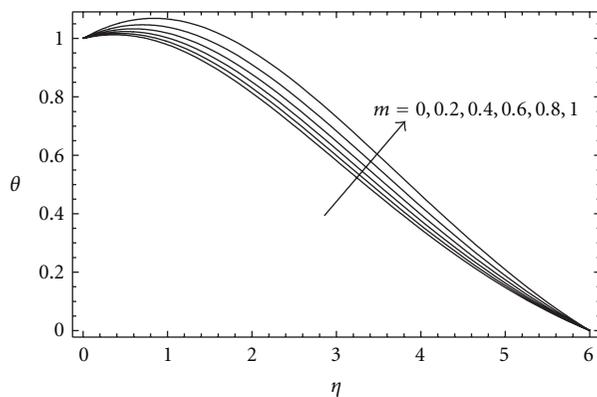
The effects of m on velocity, angular velocity, and temperature profiles are depicted in Figures 7, 8, and 9, respectively. Figure 7 shows that the velocity gradient at the surface is larger for larger values of m . Different behaviors are observed for the effect of m on the heat transfer rate at the surface as presented in Figure 9. As expected, the couple stress $h(0)$ is more dominant for larger values of m , as shown in Figure 8.

4. Conclusions

A steady two-dimensional laminar and heat transfer due to a stretching sheet immersed in an incompressible micropolar fluid has been investigated. The effect of thermal radiation and heat generation on the development of the thermal boundary layer flow has been taken into consideration. The effects of the governing parameters K, R, Q, Pr , and m on the fluid flow and heat transfer characteristics are discussed. It is found that the temperature distribution increases as the radiation parameter R increases. It is observed that the velocity gradient at the surface is larger for larger values of m .

Nomenclature

- a, b : Constants
- c_p : Specific heat at constant pressure
- f' : Dimensionless stream function
- h : Dimensionless microrotation
- j : Microinertia density
- k : Thermal conductivity
- k^* : Mean absorption coefficient
- K : Material parameter
- m : Boundary parameter
- N : Microrotation or angular velocity
- Pr : Prandtl number
- q_r : Radiative heat flux
- Q_0 : Heat generation constant
- Q : Heat generation parameter
- R : Radiation parameter
- T : Fluid temperature
- T_w : Surface temperature
- T_∞ : Ambient temperature
- u, v : Velocity components in the x - and y -directions, respectively
- U_w : Velocity of the stretching sheet
- x, y : Cartesian coordinates along the sheet and normal to it, respectively.

FIGURE 8: Angular velocity profiles for different values of m .FIGURE 9: Temperature profiles for different values of m .

Greek Letters

- α : Thermal diffusivity
- β : Thermal expansion coefficient
- γ : Spin gradient viscosity
- η : Similarity variable
- θ : Dimensionless temperature
- κ : Vortex viscosity
- ν : Kinematic viscosity
- μ : Dynamic viscosity
- ρ : Fluid density
- σ^* : Stefan-Boltzmann constant
- ψ : Stream function.

Subscripts

- w : Condition at the solid surface
- ∞ : Ambient condition.

Superscript

- $()'$: Differentiation with respect to η .

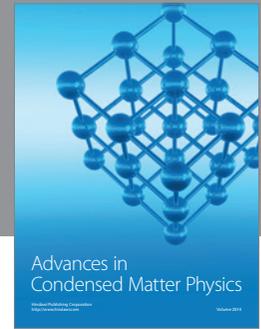
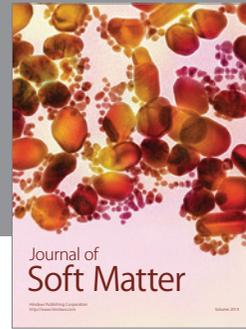
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