Research Article

The Effect of Energy Leakage Probability on Burn Propagation in an Optically Thick Fusion Plasma

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In an optically thick plasma, the mean free path of bremsstrahlung photons is smaller than the plasma radius, and radiation can be treated as a photon gas in thermal equilibrium. In these conditions, the black body radiation spectrum exceeds the number of hot photons, and reabsorption processes such as inverse bremsstrahlung radiation and inverse Compton scattering become important. It has been shown that a dense fusion plasma like the one being used in ICF method is initially optically thick. When the fuel pellet is burning, the temperature of its electrons rises (approximately greater than 90 KeV), and the pellet becomes rapidly optically thin. In this paper, we have shown that the energy leakage probability makes electron temperature remain low (approximately smaller than 55 KeV), and as a result the fuel pellet remains optically thick during burning.

1. Introduction

Research in laser driven inertial confinement fusion (ICF) began soon after the invention of the laser (see the historical notes in [1, 2]). In recent years, because of the development of high intensity lasers (especially in NIF: National Ignition Facility, USA [3], LMJ: Laser Megajoule, France [4]), the ICF method is in the forefront of energy production from fusion process. The main components of the ICF fuel pellets are the hydrogen isotopes, that is, deuterium and tritium (DT). The DT mixture is the fuel with the lowest ignition temperature and has the highest specific yield. The problem is that tritium is an unstable artificial isotope, decaying to ³He with a half-life of 12.3 years and, as a result, needs to be produced within the DT fuel cycle. Therefore, the tritium production has the most significant radiological problem in the

future of DT fusion reactors. Furthermore, another problem is energetic neutrons (14 MeV) that are yielded from DT reaction.

One of the solutions to overcome the problem of DT fuel utilization is using Proton-Boron-11 [$P + {}^{11}\text{B} \rightarrow 3\alpha(2.7\text{MeV})$] and deuterium-helium-3 [$D + {}^3\text{He} \rightarrow p(14.7\text{MeV}) + \alpha(3.6\text{MeV})$] fuel configuration. These configurations are called *advanced fusion fuels*. In classical plasmas, self-burning of advanced fuels is unlikely to occur [5], because at high temperatures the bremsstrahlung loss may exceed the fusion power produced. However, if the final temperature reached is low enough, the electrons of the plasma can be degenerated [6]. If this is the case, bremsstrahlung emission is strongly suppressed [7–12], and ignition temperature becomes lower than in the classical plasmas. In addition, several research showed that in Fermi degenerate plasmas, the properties of the aneutronic fuel burning can be very different from those of classical plasmas, due to the reduction of ion-electron (i-e) collisions, both of which allow the ion temperature to exceed the electron temperature and reduce the bremsstrahlung loss [13–17]. Nevertheless, low gain is the main failure of using advanced fusion fuels [15].

Another solution has been proposed to be adding a small amount of tritium to the deuterium fuel in the form of DT_x , where x is the ratio of tritium to deuterium in initial fuel [18, 19]. It has been shown that in $x \sim 0.01$ and with a given initial condition, namely, density $\rho = 5000 \text{ (g/cm}^3\text{)}$, areal density $\rho R = 12.5 \text{ (g/cm}^2\text{)}$, and temperature $T = 10 \text{ (KeV)}$, internal breeding of tritium takes place. Internal breeding of tritium means that the remaining amount of tritium after the burning of fuel is equal to initial content. However, the low gain of fuel pellet is still the main failure of this fuel configuration.

The other solution for the above problem is adding ${}^3\text{He}$ to DT_x fuel to form a new configuration as $\text{DT}_x{}^3\text{He}_y$, where x is the ratio of tritium to deuterium, and y is the ratio of helium-3 to deuterium in the initial pellet. It has been shown that the configuration fuel as $\text{DT}_{x=0.0112}{}^3\text{He}_{y=0.0399}$, and in the same initial conditions as those described for the case just mentioned, internal breeding of tritium and helium-3 takes place. Furthermore, the energy gain is increased and neutrons produced are decreased due to the reduction in the initial content of deuterium and tritium.

When the size of the fusion fuel pellet is comparable with the thermalization range of fusion products, a fraction of the energy will escape from the burning pellet. This fraction was calculated for $\text{DT}_{x=0.0112}$, with the same initial conditions but was not applied for burn propagation simulation [20]. In an earlier study, we inserted the energy leakage probability to the relevant calculations to obtain an ignition condition for configuration fuel $\text{DT}_x{}^3\text{He}_y$ in different ratios and densities [21]. In the present study, we applied this effect to recalculate the burn simulation in more detail.

The organization of this paper is as follows. The one-dimensional power balance equations are introduced in Section 2. Energy deposition of charged particles and energy leakage probability are given in Section 3, optical depth are discussed in Section 4, and finally, conclusion is placed in Section 5.

2. One-Dimensional Power Balance Equations

Here we consider thermonuclear reaction starts in a hot spot with radius $2.5 \times 10^{-3} \text{ cm}$, density of 5000 g/cm^3 , and temperature of 10 keV which has been produced inside the surrounding shell after compression phase. The equation of energy balance for ions, electrons, and radiation are given by

$$\begin{aligned}
\frac{dE_i}{dt} &= \frac{3}{2} \frac{d}{dt} (n_i T_i) = \sum_j \sum_k (1 - \eta_k^j) f_k^j P_{fk}^j - P_{ie} - P_{mi}, \\
\frac{dE_e}{dt} &= \frac{3}{2} \frac{d}{dt} (n_e T_e) = \sum_j \sum_k (1 - \eta_k^j) (1 - f_k^j) P_{fk}^j + P_{ie} - P_B - P_C - P_e - P_{me}, \\
\frac{dE_r}{dt} &= \frac{1}{V} \frac{dT_r}{dt} = P_B + P_C - P_r,
\end{aligned} \tag{2.1}$$

where T_i (T_e) is the ion (electron) temperature, n_i (n_e) is the ion (electron) number density, η_k^j is the energy leakage probability of the product k created in the reaction j , and f_k^j is the fraction of the energy of the product k created in the reaction j that is deposited into the plasma ions. P_{fk}^j is the fusion power of product k created in the reaction j , P_{mi} , and P_{me} are the mechanical expansion loss power of ion and electron, respectively, and P_e is the thermal conduction of electrons [1]. T_r is the radiation temperature, and P_r is the photon energy loss power [19].

For the optically thick plasma the mean free path of bremsstrahlung photons is smaller than the plasma radius and radiation can be treated as a photon gas in thermal equilibrium. Thus, a black body radiation spectrum is obtained, and the radiation temperature can be defined. In this case, the reabsorption process, such as inverse bremsstrahlung and inverse Compton effect are more important. Then, we can write the bremsstrahlung loss, P_B , as in the following form:

$$P_B \left(\frac{\text{KeV}}{\text{cm}^3 \text{s}} \right) = 2.94 \times 10^{-15} n_e \sum_k n_k Z_k^2 T_e^{0.5} \times G(\gamma) \left(\frac{T_e - T_r}{T_e} \right) \times \left(1 + \frac{2T_e}{511.1} \right), \tag{2.2}$$

with

$$G(\gamma) = \frac{\pi^2}{4} \frac{(T_r - 1.03T_e)}{(T_r - 2.54T_e)}, \tag{2.3}$$

where $G(\gamma)(T_e - T_r/T_e)$ is the inverse bremsstrahlung term [19], and $(1 + 2T_e/511.1)$ is a relativistic correction term [22].

From the Planck distribution for the photons, we can easily estimate the inverse Compton scattering contribution. Provided the photons are of low energy, $h\nu/m_e c^2 \ll 1$, and the electron temperature moderate, $kT_e/m_e c^2 \ll 1$, the energy lost by the electrons in inverse Compton scattering is given by [23] as follows:

$$P_C \left(\frac{\text{KeV}}{\text{cm}^3 \text{s}} \right) = 4E_r n_e c \left(\frac{8}{3} \pi r_e^2 \right) \left(\frac{T_e (\text{keV}) - T_r (\text{keV})}{511.1} \right), \tag{2.4}$$

where $(8/3)\pi r_e^2$ is the Thomson cross-section, and r_e is the classical electron radius.

In a burning plasma, the fusion energy will be deposited mainly into the ions [2, 20]. Therefore, the ion temperature will be higher than the electron temperature, and an energy flow from the ions to the electrons by collisions is expected. In a dense plasma scattering is predominantly quantum mechanical and entails not only the first Fermi-Dirac correction,

but also the first classical correction when the electron density starts to increase. P_{ie} is the ion-electron energy exchange term that is achieved by [24] as follows:

$$P_{ie} \left(\frac{\text{KeV}}{\text{cm}^3 \text{s}} \right) = -C_{ie}(T_e - T_i), \quad (2.5)$$

where,

$$C_{ie} = \frac{\omega_I^2}{2\pi} \sqrt{\frac{m_e}{2\pi T_e}} \left(\frac{4\pi e^2 n_e}{T_e} \right) \ln \Lambda_{ie}, \quad (2.6)$$

and $\ln \Lambda_{ie}$ is named Coulomb Logarithm, including quantum and degeneracy effects and first classical correction is given by [25] as follows:

$$\begin{aligned} \ln \Lambda_{ie} &= \ln \Lambda_{\text{BPS}}^{\text{QM}} + \ln \Lambda_{\text{BPS}}^{\text{FD}} + \ln \Lambda_{\text{BPS}}^{\Delta \text{C}} \\ &= \frac{1}{2} \left[\ln \left\{ \frac{8T_e^2}{\hbar^2 \omega_e^2} \right\} - \gamma - 1 \right] \\ &\quad + \frac{n_e \lambda_e^2}{2} \left[- \left(1 - \frac{1}{2^{3/2}} \right) \frac{1}{2} \left[\ln \left\{ \frac{8T_e^2}{\hbar^2 \omega_e^2} \right\} - \gamma - 1 \right] + \left(\frac{1}{2} \ln 2 - \frac{1}{2^{5/2}} \right) \right] \\ &\quad - \frac{\epsilon_H}{T_e} \sum_i \frac{Z_i^2 \omega_i^2}{\omega_I^2} \left[\zeta(3) \left(\ln \left\{ \frac{T_e}{Z_i^2 \epsilon_H} \right\} - \gamma \right) - 2\zeta'(3) \right], \end{aligned} \quad (2.7)$$

where $\ln \Lambda_{\text{BPS}}^{\text{QM}}$ is the leading term in the quantum limit together with the Δ correction that exhibited by, $\ln \Lambda_{\text{BPS}}^{\Delta \text{C}}$ is the first classical correction, and $\ln \Lambda_{\text{BPS}}^{\text{FD}}$ is the first correction when Fermi-Dirac statistics start to become important. The numerical values of the zeta-function and its derivative are

$$\begin{aligned} \zeta(3) &= \sum_{k=1}^{\infty} \frac{1}{k^3} \simeq 1.20205, \\ \zeta'(3) &= - \sum_{k=1}^{\infty} \frac{1}{k^3} \ln k \simeq -0.19812, \end{aligned} \quad (2.8)$$

in the last line of (2.7), the ratio ϵ_H/T_e describes the relative size of the correction, where,

$$\epsilon_H = \left(\frac{e^2}{4\pi} \right) \frac{m_e}{2\hbar^2} \simeq 13.606 \text{ eV} \quad (2.9)$$

is the binding energy of the hydrogen atom. λ_e is the thermal wavelength for electron as

$$\lambda_e = \hbar \sqrt{\frac{2\pi}{m_e T_e}}, \quad (2.10)$$

and ω_b is the plasma frequency for species b ,

$$\omega_b^2 = \frac{4\pi e^2 n_b}{M_b}, \quad (2.11)$$

where $\omega_I^2 = \sum_i \omega_i^2$.

The energy gain of burning region is defined as the ratio between the total energy deposited by fusion products and the energy contained in the originally heated plasma,

$$\text{GAIN} = \frac{E_{\text{fus}}^{\text{tot}}}{E_{\text{input}}} = \frac{\int_0^\infty dt \sum_j \sum_k (1 - \eta_k^j) P_{fk}^j}{(3/2)(n_e(0)T_e(0) + n_i(0)T_i(0))}. \quad (2.12)$$

The catalytic regime of tritium is obtained when the final amount of tritium is slightly higher than the initial amount. In this case, external tritium breeding is avoided and is replaced by internal tritium breeding. The internal tritium breeding (ITB) is defined as

$$\text{ITB} = \frac{N_T(t = \infty)}{N_T(t = 0)}. \quad (2.13)$$

3. Energy Deposition of Charged Particles and Energy Leakage Probability

When two ions fuse together, the energy E_j is yielded from fusion reaction j . The reaction product k carries with itself a fraction of w_k^j of energy E_j . Next, the reaction product, k , with an energy equal to $w_k^j E_j$ slows down and deposits a fraction of f_k^j of its energy to the ions. This fraction is calculated through the following:

$$f_k^j = \frac{1}{w_k^j E_j - (3/2)T_i} \int_{(3/2)T_i}^{w_k^j E_j} \frac{(dE/dx)_i}{(dE/dx)_e + (dE/dx)_i} dE, \quad (3.1)$$

where dE/dx is the stopping power of charged particle with an initial energy of $w_k^j E_j$ which starts to slow down to the final energy of $3/2T_i$, [26]. Subscripts i and e represent ions and electrons, respectively.

The distance traversed by a charged particle, $x(E_0, E)$, which loses energy from energy E_0 to E is obtained as

$$x(E_0, E) = \int_{E_0}^E \left(\frac{dE}{dx} \right)^{-1} dE. \quad (3.2)$$

We can now calculate the fraction of the energy the charged particle releases from the pellet of radius R . The parameter x can be expressed in terms of the radial distance from

Figure 1: The relation between distance, x , with radial distance from the center, r , the pellet radius, R , and the angle between the path and the line passing through the center $\mu = \cos \theta$.

the center, r , the pellet radius, R , and the angle between the path and the line passing through the center, $\mu = \cos \theta$, as shown in Figure 1:

$$x = r\mu + \sqrt{R^2 - r^2(1 - \mu^2)}. \quad (3.3)$$

If the charged particle is produced with uniform probability in the sphere, R , the probability of escaping can be calculated as

$$\eta = \frac{2\pi}{(4/3)\pi R^3} \int_{-1}^1 \int_0^R \frac{E(r, \mu)}{E_0} r^2 dr d\mu, \quad (3.4)$$

where E_0 is the initial energy of the charged particle, and $E(r, \mu)$ is the energy of the same particle after traversing a distance x from the point of birth to the spherical surface. In Figure 2, we illustrate energy leakage probability of charged particles for various reactions during the burning of fuel pellet.

4. Optical Depth

Optical depth is a parameter that determines whether the plasma is optically thick or thin. Since hot photons with $h\nu > T_e$ are important for the reabsorption process, this parameter is defined by [22] as follows:

$$\tau_B = \frac{\int_{h\nu > T_e}^{\infty} E^{\text{BB}}(\nu) d\nu}{\int_{h\nu > T_e}^{\infty} E^{\text{Br}}(\nu) d\nu}, \quad (4.1)$$

where $E^{\text{BB}}(\nu)$ is the radiation energy density of a black body, and $E^{\text{Br}}(\nu)$ is the bremsstrahlung radiation energy density [27]. When this parameter is greater than unity, the black

Figure 2: Energy leakage probability of charged particles for various reactions during the burning of fuel pellet.

Figure 3: Ion, electron, and radiation temperatures as a function of burning time.

Figure 4: The ratio between the number of hot photons with $h\nu > T_e$ obtained from the black body radiation spectrum $E^{\text{BB}}(\nu)$, and from bremsstrahlung radiation spectrum, $E^{\text{Br}}(\nu)$.

Figure 5: Ratio of the number of tritium particles at time t to the initial number of tritium particles as a function of burning time.

body radiation spectrum exceeds the number of hot photons and, as a result, plasma becomes optically thick. Earlier studies have shown that the plasma is initially optically thick but rapidly becomes optically thin (transparent) as the electron temperature rises (greater than ~ 90 KeV) [28, 29]. As shown in Figure 3, the electron temperature cannot be greater than

55 KeV if energy leakage probability is taken into consideration. As a result, burning fusion pellet will not be optically thin during burning time (Figure 4).

5. Conclusion

In the present study, the effects of burn propagation of a pellet with the configuration $DT_{x=0.0112}^3\text{He}_{y=0.0399}$ with the initial conditions, namely, density $\rho = 5000$ (g/cm³), areal density $\rho R = 12.5$ (g/cm²), and temperature $T = 10$ (KeV)—taking into consideration energy leakage probability—has been reinvestigated. The results indicate that the electrons' temperature does not exceed 55 KeV, and the fuel pellet remains optically thick during the burning process. Also, the effects of energy leakage probability resulted in the reduction of energy gain can be reduced to a gain of 70. Figure 5 illustrates internal tritium breeding during the burning process. As Figure 5 shows, energy leakage probability effect brings about a reduction in ITB, and consequently, the final ratio of tritium is less than the primary ratio.

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