Research Article

Obtaining an Initial Solution for Facility Layout Problem

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The facility layout approaches can generally be classified into two groups, constructive approaches and improvement approaches. All improvement procedures require an initial solution which has a significant impact on final solution. In this paper, we introduce a new technique for accruing an initial placement of facilities on extended plane. It is obtained by graph theoretic facility layout approaches and graph drawing algorithms. To evaluate the performance, this initial solution is applied to rectangular facility layout problem. The solution is improved using an analytical method. The approach is then tested on five instances from the literature. Test problems include three large size problems of 50, 100, and 125 facilities. The results demonstrate effectiveness of the technique especially for large size problems.

1. Introduction

The facility layout problem seeks the best positions of facilities to optimize some objective. The common objective is to reduce material handling costs between the facilities. The problem has been modeled by a variety of approaches. A detailed review of the different problem formulations can be found in Singh and Sharma [1]. The facility layout problem is an optimization problem which arises in a variety of problems such as placing machines on a factory floor, VLSI design, and layout design of hospitals, schools.

The facility layout approaches can generally be classified into two groups, constructive methods and improvement methods. In this paper, we consider the placement of facilities on an extended plane. Many improvement approaches have been proposed for this problem. All improvement procedures require an initial solution. Some approaches start from a good but infeasible solution [2–4]. These models contain a penalty component in their objective function. Hence, these approaches minimize objective function value for feasible solutions. But some approaches require a feasible initial solution. These approaches use a randomly generated initial solution [5, 6]. Mir and Imam [7] have proposed simulated annealing for a better initial solution. They have shown

that a good initial solution has a significant impact on final solution.

In this paper, we introduce a new technique for accruing an initial placement of facilities on an extended plane. The technique consists of two stages. In the first stage, a maximal planar graph (MPG) is obtained. In the second stage, the vertices of MPG are drawn on the plane by graph drawing algorithms. Then, vertices are replaced by facilities. Hence, an initial solution is obtained.

In an MPG, the facilities with larger flows are adjacent together. Hence, drawing the MPG on the plane can be a good idea for obtaining an initial solution. To evaluate the performance of the idea, this initial solution is applied in rectangular facility layout problem. The solution is improved by an analytical method by Mir and Imam [7]. The approach is then tested on five instances from the literature.

The remaining parts of the paper are organized as follows. The next section describes the formulation of the facility layout problem chosen for our work. Section 3 describes accruing an initial placement. In Section 4, the analytical method is described, and the approach is compared to other approaches in the literature. In Section 5, the proposed initial solution is compared with random initial solution. Finally, Section 6 provides a summary and conclusion.

FIGURE 1: An MPG (solid lines) and its correspondent block layout (dashed lines).

FIGURE 2: An example of straight line drawing.

FIGURE 3: Steps of the proposed approach.

2. Problem Formulation

In this paper, we label the facilities $1, 2, \dots, N$, where N is the total number of facilities. Facilities are assumed to be rectangles with fixed shape. The notation is given as follows:

(x_i, y_i) coordinates of the center of facility i

L_i length of facility i

W_i width of facility i

f_{ij} the total cost of flow per unit distance between two facilities i and j

TABLE 1: Results for test problem #1.

Program	Cost function value
TOPOPT (Imam and Mir, 1989) [5]	794
VIP-PLANOPT (2006) [8]	692
GOT	752.7

TABLE 2: Results for test problem #2.

Program	Best design
Topopt (Mir and Imam, 1989) [5]	1320.72
FLOAT (Imam and Mir, 1993) [6]	1264.94
HOT (Imam and Mir, 2001) [7]	1225.40
VIP-PLANOPT (2006) [8]	1157
GOT	1302

TABLE 3: Results for test problem #3.

Program	Cost function value
HOT (Mir and Imam, 2001) [7]	80794.24
VIP-PLANOPT (2006) [8]	78224.7
GOT	76882.3

TABLE 4: Results for test problem #4.

Program	Cost function value
HOT (Mir and Imam, 2001) [7]	558556.2
VIP-PLANOPT (2006) [8]	538193.1
GOT	527094.1

d_{ij} distance between the centers of the facilities i and j .
 d_{ij}^* could be one of the following three distance norms.

(1) Euclidean distance:

$$d_{ij} = \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)^{1/2}. \quad (1)$$

(2) Squared Euclidean distance:

$$d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2. \quad (2)$$

(3) Rectilinear distance:

$$d_{ij} = |y_i - y_j| + |x_i - x_j|. \quad (3)$$

The requirement for problem is that the facilities must not overlap each other. The area of overlap is defined as follows:

$$A_{ij} = \lambda_{ij} (\Delta X_{ij}) (\Delta Y_{ij}), \quad (4)$$

where

$$\Delta X_{ij} = \frac{L_i + L_j}{2} - |x_i - x_j|,$$

$$\Delta Y_{ij} = \frac{W_i + W_j}{2} - |y_i - y_j|, \quad (5)$$

$$\lambda_{ij} = \begin{cases} 1, & \text{for } \Delta X_{ij} \geq 0, \Delta Y_{ij} \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

TABLE 5: For test problem #4, the coordinates of facilities obtained by GOT are given below. The value of the cost function for this layout is 527094.1.

Facility	X	Y
1	17.599	22.158
2	18.514	32.933
3	29.485	18.634
4	35.526	20.582
5	14.798	22.65
6	26.425	25.388
7	12.213	13.282
8	19.435	27.466
9	32.119	15.319
10	24.743	8.862
11	16.231	2.711
12	25.533	17.621
13	14.274	20.352
14	22.791	3.889
15	14.467	14.812
16	38.012	16.94
17	9.735	11.276
18	24.609	27.663
19	20.685	20.578
20	19.866	22.579
21	15.832	8.421
22	8.866	31.495
23	22.402	17.335
24	35.551	9.125
25	16.95	19.378
26	36.104	25.31
27	16.328	21.843
28	13.208	26.161
29	22.439	14.732
30	14.052	13.065
31	28.21	21.934
32	16.436	12.282
33	24.467	11.365
34	23.103	32.489
35	23.283	24.968
36	19.444	14.708
37	32.927	16.699
38	11.708	28.857
39	15.14	29.793
40	28.164	5.174
41	32.08	8.476
42	29.636	13.049
43	26.373	31.127
44	8.693	15.785
45	16.05	26.859
46	31.965	26.099
47	29.361	9.586
48	33.822	12.885

TABLE 5: Continued.

Facility	X	Y
49	19.394	25.043
50	25.902	21.954
51	4.234	16.534
52	19.539	30.211
53	5.587	11.801
54	13.399	32.831
55	28.771	16.071
56	30.974	22.774
57	22.453	22.117
58	12.35	23.138
59	21.725	18.944
60	30.419	31.854
61	24.388	18.897
62	19.587	36.974
63	19.943	12.839
64	24.632	15.019
65	5.951	32.233
66	16.918	14.346
67	23.818	6.408
68	19.581	18.447
69	35.191	31.263
70	0.927	21.347
71	32.644	19.464
72	9.487	20.325
73	39.749	21.788
74	18.485	16.27
75	15.581	36.344
76	6.345	20.203
77	13.484	23.21
78	18.527	20.734
79	11.773	20.295
80	19.329	2.663
81	4.471	22.79
82	4.788	27.961
83	27.51	36.612
84	26.306	15.776
85	4.545	25.257
86	23.545	35.892
87	10.746	37.712
88	7.405	6.656
89	12.021	6.684
90	29.14	27.123
91	23.367	29.856
92	14.87	16.629
93	11.961	17.137
94	16.126	18.34
95	22.426	16.075
96	19.729	10.752
97	19.707	7.63
98	23.824	21.511
99	8.257	27.817
100	9.179	24.151

TABLE 6: Results for test problem #5.

Program	Cost function value
VIP-PLANOPT (2006) [8]	1084451
GOT	1062080

The value of overlap area A_{ij} is a nonnegative number. A_{ij} will be zero only if there is no overlapping between facilities i and j . The objective is to minimize material handling costs. So, the problem can be stated as follows:

$$\begin{aligned} \text{minimizing cost} &= \sum_{i=1}^{N-1} \sum_{j=i+1}^N f_{ij} d_{ij} \\ \text{subject to } A_{ij} &= 0 \quad \forall i, j, i < j. \end{aligned} \quad (6)$$

The constraint ensures that facilities do not overlap. A similar formulation also can be found in [7].

3. Obtaining an Initial Solution

The initial solution is obtained by graph theoretic facility layout approaches (GTFLP) and graph drawing algorithms. The following subsection describes obtaining an MPG. Section 3.2 describes the drawing of the MPG on the plane.

3.1. Generating a Maximal Planar Graph. In GTFLP, facilities are represented by vertices, and flow (adjacency desirability) between them is represented by weighted edges. Created graph is called adjacency graph. Graph theory is particularly useful for the facility layout problems, because graphs easily enable us to capture the adjacency information and model the problem. A review of graph theory applications to the facility layout problem can be found in [9, 10]. GTFLP consists of two stages. At the first stage, the adjacency graph is converted to a maximal planar graph (MPG). In the second stage, a block layout is constructed from the MPG. The second stage is not our concern here. For more details, we refer to [11–15]. Figure 1 shows an MPG and its correspondent block layout.

Many heuristic and metaheuristic methods for obtaining an MPG have been suggested [16–21]. In this paper, we use from the greedy heuristic [16]. It is conceptually simple and creates high weighted MPGs [16]. This heuristic has a simple instruction: the edges are sorted in nonincreasing order of weight. Each edge is tested in turn and accepted as part of the MPG unless it makes the graph nonplanar. So, the heuristic needs planarity testing. Boyer and Myrvold [22] developed a simplified $O(n)$ planarity testing algorithm. We use this algorithm for planarity testing. In the worst case, $O(n^2)$ edges are considered, and for each edge, the Boyer and Myrvold test is called. Hence, the approach results in a complexity of $O(n^3)$.

3.2. Drawing Maximal Planar Graph on the Plane. Graph drawing, as a branch of graph theory, applies topology and geometry to derive two-dimensional representations of graphs. A graph drawing algorithm reads as input a combinatorial description of a graph G and produces as output

TABLE 7: For test problem #5, the coordinates of facilities obtained by GOT are given below. The layout cost is 1062080.

Facility	X	Y
1	28.617	19.513
2	25.371	22.082
3	27.683	32.088
4	6.488	13.531
5	17.53	34.357
6	10.131	16.929
7	29.906	25.226
8	28.243	36.473
9	14.372	31.298
10	37.502	22.299
11	32.586	4.846
12	19.349	13.249
13	3.314	23.588
14	46.96	19.354
15	21.953	8.539
16	34.13	25.537
17	33.636	32.8
18	37.21	17.564
19	12.603	23.713
20	21.763	21.428
21	16.261	37.283
22	27.453	25.185
23	25.213	15.36
24	27.111	41.844
25	21.208	24.959
26	33.287	38.471
27	25.882	19.125
28	43.87	14.147
29	27.342	6.265
30	2.831	26.3
31	18.328	31.432
32	38.33	7.761
33	22.131	27.038
34	7.166	26.77
35	18.909	46.302
36	15.123	34.383
37	28.974	22.602
38	18.728	4.656
39	33.526	8.657
40	12.176	32.318
41	16.539	39.732
42	28.383	39.485
43	15.09	23.899
44	32.571	12.559
45	41.221	40.78
46	24.335	27.843
47	2.516	30.62
48	37.452	25.455

TABLE 7: Continued.

Facility	X	Y
49	40.189	36.294
50	12.672	36.825
51	34.282	27.349
52	19.126	15.005
53	36.395	41.322
54	31.473	40.034
55	30.991	34.983
56	38.702	27.03
57	28.176	12.73
58	29.219	45.218
59	25.113	10.766
60	11.021	27.029
61	22.158	14.239
62	22.543	17.582
63	12.348	21.322
64	13.684	43.96
65	8.569	36.149
66	8.828	33.239
67	10.933	12.373
68	24.203	26.047
69	12.647	7.071
70	29.789	27.362
71	6.492	38.642
72	18.394	17.993
73	4.815	17.754
74	24.465	45.646
75	24.704	24.448
76	48.912	31.164
77	19.063	11.088
78	27.843	16.379
79	35.271	29.217
80	21.659	35.429
81	15.303	28.087
82	32.599	31.07
83	3.311	20.836
84	44.02	33.76
85	43.843	9.862
86	16.694	23.899
87	20.628	17.23
88	19.088	35.563
89	48.998	25.308
90	33.559	46.176
91	31.162	30.86
92	24.82	36.607
93	44.673	24.381
94	18.385	27.694
95	30.086	10.858
96	33.289	15.156
97	30.399	17.188
98	15.52	19.071
99	36.518	32.041
100	19.439	40.84
101	22.932	4.334

TABLE 7: Continued.

Facility	X	Y
102	27.279	28.155
103	31.849	19.046
104	11.48	19.243
105	7.31	21.826
106	38.088	12.744
107	22.892	30.522
108	14.917	13.713
109	6.995	30.065
110	40.466	30.376
111	41.964	19.47
112	30.3	14.954
113	19.953	27.768
114	27.316	2.155
115	35.983	36.293
116	44.374	28.853
117	22.621	41.026
118	31.99	20.775
119	11.309	39.88
120	41.379	24.172
121	31.589	26.996
122	18.031	8.178
123	18.549	22.682
124	31.64	23.551
125	33.452	23.294

a drawing of G . A graph has infinitely many different drawings. For a review of various graphs drawing algorithms, refer to [23]. We use algorithm of Chrobak and Payne [24] to form a straight line drawing, of the MPG. In such a drawing, each edge is drawn using a straight line segment. The algorithm draws vertices in an MPG to integer coordinates in a $(2N - 4) \times (N - 2)$ grid. Figure 2 shows an example of straight line drawing.

For acquiring an initial solution, each vertex is replaced by its correspondent facility. In a feasible solution, facilities have no overlaps. For this reason, the coordinates of facilities can be multiplied by maximum dimensions of all facilities (width and length). This operation increases distance between facilities and makes the solution feasible. For the case of circular facilities, the diameter of circle can be considered as maximum dimensions.

4. Improving Initial Solution and Comparing

To evaluate the performance, the initial solution is improved by an analytical method by Mir and Imam [7]. In this method, the convergence is controlled by carrying out the optimization using concept of “magnified envelop blocks.” The dimensions of the blocks are determined by multiplying the dimensions of the facilities with a “magnification factor.” The optimization is then carried out for these envelop blocks rather than the actual facilities. The analytical method searches the optimum placements of each envelop block in

TABLE 8: The value of cost function in GOT initial solution and the best value of random placements.

n	GOT initial solution	The best random initial solution
10	1945	2437.3
11	2122	2758.5
12	2368	3531.7
13	4919	5764
14	5514	7024
15	5538	7449
16	8116	10287.7
17	8468	12207.5
18	10648	15304
19	13943	19119.3
20	14265	20094
21	17199	22491.2
22	16610	26370
23	22128	30749.2
24	25533	35508
25	27585	37968.4
26	29929	43803.6
27	36255	51850
28	41483	57050.5
29	47543	65109.2
30	57830	73705.8
31	61408	82058.4
32	63687	86436
33	59970	94066.4
34	82721	110195.2
35	76220	104940
36	92426	124186.1
37	95386	125468
38	87532	150060
39	93708	146793.8
40	118266	180538
41	141363	183911
42	104263	195458.7
43	152188	204207.3
44	163529	239253
45	166360	237051.8
46	167002	251049.3
47	189027	290085
48	226097	305900
49	234811	324500.3
50	238855	337198.9
51	264842	358480.5
52	251009	358666.7
53	233805	370262.4
54	298806	404104.4
55	284543	431091.9
56	344755	481656.9
57	371962	493986.4

TABLE 8: Continued.

n	GOT initial solution	The best random initial solution
58	344044	510976
59	375321	503358.9
60	373118	538554.9
61	370817	573555.9
62	457275	628602.9
63	544350	688184.6
64	530408	708518.3
65	502322	690275.2
66	526230	741481.1
67	556568	773325.9
68	639735	867242.4
69	578533	860229.8
70	643592	914437.3
71	589557	893978.8
72	670866	950592.5
73	736669	1058290.5
74	660395	1039023
75	749795	1115704.6
76	835418	1180300
77	727648	1157821.9
78	852689	1147676
79	971135	1273329.8
80	920522	1294614.8
81	882645	1420567
82	1084711	1500037.8
83	1072241	1436365.1
84	1072132	1606669.3
85	1154018	1552139.2
86	1150925	1756302.3
87	1164220	1608156.3
88	1230008	1726587.4
89	1379479	1913688.3
90	1351210	1970899.3
91	1275975	1895126.4
92	1360771	2114230
93	1520228	1969614.1
94	1542740	2125200
95	1581145	2230222.7
96	1640792	2305569.3
97	1486796	2362639.4
98	1645889	2365109.3
99	1607866	2490733.7
100	2073979	2635666.4

the direction of steepest descent which is opposite to the gradient direction. The sizes of the envelop blocks are then reduced, and the optimization process is repeated for the second phase. The number of optimization phases is equal to the magnification factor number for the envelop blocks. In the last optimization phase, the dimensions of the envelop blocks become equal to the actual facilities. For more detail, we refer to Mir and Imam [7].

TABLE 9: Summary of the results.

Problem	Number of facilities	Best result by other methods	GOT	Cost reduction
#1	8	692.5	752.7	-60.2
#2	20	1157	1302	-145
#3	50	78224.7	76882.3	1342.4
#4	100	538193.1	527094.1	11099
#5	125	1084451	1062080	22371

So, the proposed approach for solving a facility layout problem can be summarized as follows.

Step 1: encapsulating facilities in envelop blocks (multiplying the dimensions of facilities by a magnification factor).

Step 2: obtaining an MPG.

Step 3: drawing the MPG on the plane and obtaining an initial solution.

Step 4: improving initial solution by analytical method.

Figure 3 shows summary of these steps.

The proposed approach was coded using the VB.NET programming language in a program named GOT (Graph optimization technique). Five test problems were run. For all test problems, results were obtained on a PC with Intel T5470 processor. The results were compared with the previously published papers and commercial software VIP-PLANOPT 2006. VIP-PLANOPT is a useful layout software package that can generate near-optimal layout [25]. For more details about VIP-PLANOPT, see Engineering Optimization Software [8]. VIP-PLANOPT results were obtained from the software user's manual. The results are presented in the following sections.

4.1. Test Problem #1. This problem of 8 facilities was introduced by Imam and Mir [5]. Figure 4 shows the steps for accruing the initial solution. Figure 4(a) shows the flow matrix and dimension of facilities. All dimensions and cost matrix elements are integer-valued numbers ranging between 1 and 6. There are several pairs of facilities with no flow between them. Distance norm is squared Euclidean. The greedy heuristic generates the edges lists of MPG as shown in Figure 4(b).

The straight line drawing algorithm gives the coordinates of vertices. The drawing is shown in Figure 4(c). Then, each vertex is replaced by its correspondent facility. The coordinates are multiplied by maximum dimensions of facilities (width and length), and finally, the initial layout design is shown in Figure 4(d).

The solution is improved by the analytical technique. Figure 5 shows the final layout. The cost function value for this layout is 752.7, and the running time is 0.4 second. Table 1 shows the results obtained by the other approaches. The best solution for this problem is obtained by VIP-PLANOPT 2006.

4.2. Test Problem #2. This problem of 20 unequal area facilities was introduced by Imam and Mir [6]. The data consist of only integer values. The dimensions of the facilities are between 1 and 3. The elements of the cost matrix are integers between 0 and 5. The distance norm is rectilinear. The final layout obtained by GOT is shown in Figure 6. The layout cost is 1302, and the running time is 0.6 second. Table 2 compares the results obtained by GOT with the results available in the literature. VIP-PLANOPT 2006 has the lowest value of the cost function.

4.3. Test Problem #3. This is a problem of 50 facilities randomly generated by VIP-PLANOPT 2006. The dimensions of the facilities are decimal numbers between 1 to 6. The elements of the cost matrix are all integers between 1 and 10. The distance norm is Euclidean. The results are shown in Table 3. The best published result has a cost of 78224.7, whereas GOT produces a final layout with a cost of 76882.3 only in 15.1 seconds. Figure 7 shows the final layout.

4.4. Test Problem #4. This is a randomly generated large size problem of 100 facilities. The dimensions of the facilities are decimal numbers between 1 and 6. The cost matrix elements are integers between 1 and 10. The distance norm is rectilinear. The results are shown in Table 4. GOT obtained the cost function value of 527094.1 in 74.3 seconds. This value is about 2% below the cost function value of VIP-PLANOPT 2006. The coordinates of the facilities for the layout obtained by GOT are given in Table 5.

4.5. Test Problem #5. This is a large size problem of 125 facilities randomly generated by VIP-PLANOPT 2006. The dimensions of facilities are real numbers between 1 and 6, and elements of the cost matrix are integers between 1 and 10. The distance norm is rectilinear. The results are shown in Table 6. GOT obtained the cost function value of 1062080 in 129.6 seconds. This value is about 2% below the cost function value of VIP-PLANOPT 2006. The coordinates of the facilities for the layout obtained by GOT are given in Table 7.

5. Comparing GOT Initial Solution with Random Initial Solution

To compare the proposed initial solution (GOT initial solution) with random initial solution, a set of test problems ($n = 10, 11, 12, \dots, 100$) were generated. The facilities dimensions were 1×1 , and flow matrices were randomly generated between 0 and 10. For acquiring a random initial solution,

FIGURE 4: Obtaining an initial solution for test problem #1.

FIGURE 5: Final layout for test problem #1.

FIGURE 6: Final layout for test problem #2.

facilities were randomly placed in a $(2n - 4) \times (n - 2)$ integer grid. For each test problem, 20 random placements were found. Table 8 shows the value of cost function in GOT initial solution and the best value found by random placements. Figures 8 and 9 shows these results graphically. The results demonstrated significant improvement in cost function.

6. Summary and Conclusion

An initial solution has been presented for the layout design of facilities on a continuous plane. The technique consists of

two stages. In the first stage, a maximal planar graph (MPG) is obtained. In the second stage, the vertices of MPG are drawn on the plane by graph drawing algorithms. Then, vertices are replaced by facilities. Hence, an initial solution is obtained. To evaluate the performance, this initial solution has been applied in rectangular facility layout problem and improved by an analytical method by Mir and Imam [7].

The approach has been tested on five instances from the literature. Table 6 shows the Summary of the results, and Figure 8 shows the cost reduction by the technique. For the large size problems involving 50, 100, and 125 facilities,

FIGURE 7: Final layout for test problem #3.

FIGURE 8: Comparisons of cost function value in GOT initial solution with the best value of random placements.

the layout costs values are better than those obtained by the previously published techniques. As shown in Table 9, the results demonstrate effectiveness of the technique, especially for large size problems.

This paper introduced a simple technique for obtaining a good initial solution. The technique, with some modification, can be applied in facility layout approaches that use a randomly generated initial solution. In future researches, it would be interesting to analyze the influence of MPG and graph drawing algorithm on the solution. The results can be further improved by using a metaheuristic such as GRASP [21] and Tabu search [20] for generating a high weighted MPG.

FIGURE 9: Cost reduction by using GOT.

Conflict of Interests

The authors declare that they have no conflict of interests.

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