Research Article

Fault Detection Filter Design for Stochastic Systems with Mixed Time-Delays and Parameter Uncertainties

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This paper purposes the design of a fault detection filter for stochastic systems with mixed time-delays and parameter uncertainties. The main idea is to construct some new Lyapunov functional for the fault detection dynamics. A new robustly asymptotically stable criterion for the systems is derived through linear matrix inequality (LMI) by introducing a comprehensive different Lyapunov-Krasovskii functional. Then, the fault detection filter is designed in terms of linear matrix inequalities (LMIs) which can be easily checked in practice. At the same time, the error between the residual signal and the fault signal is made as small as possible. Finally, an example is given to illustrate the effectiveness and advantages of the proposed results.

1. Introduction

Stochastic systems have strong practical relevance in mechanical systems, economics, systems with human operators, and other engineering areas. Meanwhile, the filtering problem has many applications in the areas of signal processing, signal estimation, pattern recognition, and many practical control systems. Therefore, the filter design problems and the stochastic systems have become important areas of research and received great attention over the past few years [1–6].

With the rising demand for higher safety and increasing demand for higher performance in the modern industries, the research on fault detection for dynamic systems has received more and more attention during the past two decades, like model-based schemes [7], knowledge-based approaches [8], and signal-based methods [9], and so forth. Particularly, the problem of model-based fault detection has been an active research area among them. The basic objective of model-based fault detection is to construct the residual generator, and to determine the residual evaluation function and the threshold. An alarm of fault will be generated, when the value of the evaluation function is greater than the stated threshold. However, randomly occurring nonlinearities and

the existence of unknown inputs may seriously affect the performance of model-based fault detection systems. Therefore, robustness issue plays an important role in the application of model-based fault detection schemes. Increasing the robustness of residual to unknown inputs and modelling errors and enhancing the sensitivity to faults are of prime importance in designing a model-based fault detection system. Among different methods for fault detection, the model-based approach has been widely used in recent years. So far, the problem of fault detection has been thoroughly investigated for a variety of systems including uncertain systems [2, 10–14], time-delay systems [15–17], and Markovian jump linear systems [18].

Besides, time-delay is one of the major sources of instability and poor performance of a practical control system. And many results on stochastic systems with time delays have been reported in the literature [10–12] and the references therein. There is a need to discuss the distributed delays that occur very often in practical systems. The engineering significance of distributed delays has been widely recognized, and a number of corresponding results have been published; see, for example, [15]. The distributed delays in the discrete-time setting, on the other hand, have received little attention. It is well known that nonlinearities exist universally in practical

systems, and therefore nonlinear control has been an ever hot topic in the past few decades.

As far as we know, the delay-dependent criteria on fault detection filter design for delayed stochastic systems with parameter uncertainties have not been fully studied, which is still open. Motivated by the above discussion and in order to obtain less conservative results, we choose an appropriate new Lyapunov functional and establish a new integral inequality in the stochastic setting. What is more, because we have carefully considered the ranges for the time-varying delays, our criteria can be applicable to both fast and slow time-varying delays. The stability criteria obtained are in terms of linear matrix inequalities (LMIs) which can be checked efficiently via the LMI toolbox. Finally, a numerical example is also given to demonstrate the effectiveness and advantages of our theoretical results.

Notations. The notations used throughout the paper are fairly standard. The superscript “ T ” stands for matrix transposition; R^n denotes the n -dimensional Euclidean space; $R^{n \times m}$ is the set of all $n \times m$ real matrices; the notation $P > 0$ means that P is a positive definite matrix; I and 0 represent identity matrix and zero matrix, respectively; and $\text{diag}(\cdot)$ denotes the diagonal matrix. The vector norm is taken to be Euclidean, and the matrix norm is the corresponding induced one. $E\{\cdot\}$ denotes the expectation operator. $\mathcal{L}_2[0, +\infty)$ is the space of square-integrable vector function over $[0, +\infty)$. In symmetric block matrices, we use an asterisk ($*$) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulation and Some Preliminaries

The simplest and most fundamental case considered is the problem of quadratic stabilization for the following system:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k), \end{aligned} \quad (1)$$

where $x(k) \in R^n$ is the state and $u(k)$ is the control input. Then, we add discrete time-delays $\tau_i(k)$, ($i = 1, 2, \dots, q$), while d ($d = 1, 2, \dots, \infty$) describe the distributed time-delays. In addition, we are motivated to study the fault detection problem for a class of discrete-time systems involving stochastic time-delay, such that we consider the following stochastic time-delay system:

$$\begin{aligned} x(k+1) &= Ax(k) + A_1 \sum_{i=1}^q \alpha_i(k) x(k - \tau_i(k)) \\ &\quad + \beta(k) A_2 \sum_{d=1}^{\infty} \mu_d x(k-d) + \gamma(k) g(k, x(k)) \\ &\quad + D_1 w(k) + Gf(k) \end{aligned}$$

$$y(k) = Cx(k) + D_2 w(k) + Hf(k)$$

$$x(k) = \phi(k), \quad \forall k \in \mathbb{Z}^-, \quad (2)$$

where $x(k) \in R^n$ is the state; $y(k) \in R^m$ is the process output; $w(k) \in R^p$ is the unknown input which belongs to $\mathcal{L}_2[0, \infty)$; $f(k) \in R^l$ is the fault to be detected; A, A_1, A_2, C, D_1, D_2 , and G are real known constant matrices with compatible dimensions. The discrete time-delay $\tau_i(k)$ ($i = 1, 2, \dots, n$) is a time-varying differentiable function satisfying $\tau_m \leq \tau_i(k) \leq \tau_M$, while d ($d = 1, 2, \dots, \infty$) denotes the discrete time-delays, and the constant $\mu_d \geq 0$ satisfies the following convergence conditions $\bar{\mu} := \sum_{d=1}^{\infty} \mu_d \leq \sum_{d=1}^{\infty} d\mu_d < \infty$, $\hat{\mu} := \sum_{d=1}^{\infty} \mu_d^2 < \infty$.

Assumption 1. The nonlinear function $g(k, x(k)) \in R^n$ satisfies the following condition: for any $x, y \in R$, ($x \neq y$),

$$\hat{g}_i^- \leq \frac{g_i(x) - g_i(y)}{x - y} \leq \hat{g}_i^+, \quad (3)$$

where $\hat{g}_i^-, \hat{g}_i^+$ ($i = 1, 2, \dots, n$) are constants.

Assumption 2. The stochastic variables $\alpha_i(k)$ ($i = 1, 2, \dots, n_1$), $\beta(k)$, and $\gamma(k)$ and mutually uncorrelated Bernoulli distributed white sequences that account for, respectively, the phenomena of randomly occurring discrete time-delays, distribute time-delays, and nonlinearities. A natural assumption on the sequences $\alpha_i(k)$ ($i = 1, 2, \dots, n_1$), $\beta(k)$, and $\gamma(k)$ is made as follows:

$$\begin{aligned} \text{Prob}\{\alpha_i(k) = 1\} &= \mathbb{E}\{\alpha_i(k)\} = \bar{\alpha}_i, \\ \text{Prob}\{\alpha_i(k) = 0\} &= 1 - \bar{\alpha}_i, \\ \text{Prob}\{\beta(k) = 1\} &= \mathbb{E}\{\beta(k)\} = \bar{\beta}, \\ \text{Prob}\{\beta(k) = 0\} &= 1 - \bar{\beta}, \\ \text{Prob}\{\gamma(k) = 1\} &= \mathbb{E}\{\gamma(k)\} = \bar{\gamma}, \\ \text{Prob}\{\gamma(k) = 0\} &= 1 - \bar{\gamma}. \end{aligned} \quad (4)$$

In a networked environment, it is quite common that the measurements $y(k)$ of the system are quantized during the signal transmission. Let us denote the quantizer as $h(\cdot) = [h_1(\cdot), h_2(\cdot), \dots, h_m(\cdot)]^T$ which is symmetric, that is, $h_j(-v) = -h_j(v)$, $j = 1, \dots, m$.

The map of the quantization process is $\hat{y}(k) = h(y(k)) = [h_1(y^{(1)}(k)), h_2(y^{(2)}(k)), \dots, h_m(y^{(m)}(k))]^T$. In this paper, we are interested in the logarithmic static and time-invariant quantizer. For each, the set of quantization levels is described by $\mathcal{U}_j = \{\pm \hat{\mu}_i^{(j)}, \hat{\mu}_i^{(j)} = \chi_j^j \hat{\mu}_0^{(j)}, i = 0, \pm 1, \pm 2, \dots\} \cup \{0\}$, $0 < \chi_j < 1$, $\hat{\mu}_0^{(j)} > 0$, and each of the quantization levels corresponds to a segment, such that the quantizer maps the whole segment

to this quantization level. According to [19], the quantizer is given by

$$h_j(y^{(j)}(k)) = \begin{cases} \hat{\mu}_i^{(j)}, & \frac{1}{1+\delta_j}\hat{\mu}_i^{(j)} \leq y^{(j)}(k) \leq \frac{1}{1-\delta_j}\hat{\mu}_i^{(j)} \\ 0, & y^{(j)}(k) = 0 \\ -h_j(-y^{(j)}(k)), & y^{(j)}(k) < 0, \end{cases} \quad (5)$$

where $\delta_j = (1 - \chi_j)(1 + \chi_j)$, such that $h_j(y^{(j)}(k)) = (I + \Delta_k)y^{(j)}(k)$ with $|\Delta_k^{(j)}| \leq \delta_j$; the measurements with quantization effect can be expressed as

$$\begin{aligned} \hat{y}(k) &= (I + \Delta_k)y(k) \\ &= (I + \Delta_k)Cx(k) + (I + \Delta_k)D_2\omega(k) \\ &\quad + (I + \Delta_k)Hf(k), \end{aligned} \quad (6)$$

with $\Delta_k = \text{diag}(\Delta_k^{(1)}, \dots, \Delta_k^{(m)})$.

Consider the following full-order fault detection filter for system (2):

$$\begin{aligned} \hat{x}(k+1) &= A_f\hat{x}(k) + B_f\hat{y}(k), \\ r(k) &= C_f\hat{x}(k) + D_f\hat{y}(k), \end{aligned} \quad (7)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the filter state vector, $r(k) \in \mathbb{R}^l$ is the so called residual that is compatible with the fault vector $f(k)$, and A_f, B_f , and C_f are appropriately dimensioned filter matrices to be designed.

From (2), (5), and (7), we can obtain the following filtering error system:

$$\begin{aligned} \tilde{x}(k+1) &= (\tilde{A} + \Delta A)\tilde{x}(k) \\ &\quad + \sum_{i=1}^q (\tilde{A}_{1i} + \tilde{A}_{1i}(k))\tilde{x}(k - \tau_i(k)) \\ &\quad + (\tilde{A}_2 + \tilde{A}_2(k))\sum_{d=1}^{\infty} \mu_d \tilde{x}(k-d) \\ &\quad + (\tilde{\gamma} + \tilde{\gamma}(k))\tilde{I}g(k, x(k)) + (\tilde{D} + \Delta D)v(k) \\ \tilde{r}(k) &= (\tilde{C} + \Delta C)\tilde{x}(k) + (\tilde{D}_f + \Delta D_f)v(k) \\ \tilde{x}(k) &= \tilde{\phi}(k), \quad \forall k \in [-\tau_M, 0], \end{aligned} \quad (8)$$

with

$$\begin{aligned} \tilde{x}(k) &= [x^T(k), \hat{x}^T(k)]^T, \\ \tilde{\phi}^T(k) &= [\phi^T(k), 0^T], \\ \tilde{r}(k) &= r(k) - f(k), \quad v(k) = [\omega^T(k), f^T(k)]^T, \\ \tilde{A} &= \begin{pmatrix} A & 0 \\ B_f C & A_f \end{pmatrix}, \quad \Delta A = \begin{pmatrix} 0 & 0 \\ B_f \Delta_k C & 0 \end{pmatrix}, \\ \tilde{A}_{1i} &= \begin{pmatrix} \tilde{\alpha}_i A_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{A}_{1i}(k) = \begin{pmatrix} \tilde{\alpha}_i(k) A_1 & 0 \\ 0 & 0 \end{pmatrix}, \\ \tilde{A}_2 &= \begin{pmatrix} \tilde{\beta} A_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{A}_2(k) = \begin{pmatrix} \tilde{\beta}(k) A_2 & 0 \\ 0 & 0 \end{pmatrix}, \\ \tilde{I} &= \begin{pmatrix} I \\ 0 \end{pmatrix}, \quad \tilde{D} = \begin{pmatrix} D_1 & G \\ B_f D_2 & B_f H \end{pmatrix}, \\ \Delta D &= \begin{pmatrix} 0 & 0 \\ B_f \Delta_k D_2 & B_f \Delta_k H \end{pmatrix}, \quad \tilde{C} = (D_f C \quad C_f), \\ \Delta C &= (D_f \Delta_k C \quad 0), \quad \tilde{D}_f = (D_f D_2 \quad D_f H - I), \\ \Delta D_f &= (D_f \Delta_k D_2 \quad D_f \Delta_k H), \quad \tilde{\alpha}_i(k) = \alpha_i(k) - \bar{\alpha}_i, \\ \tilde{\beta}(k) &= \beta(k) - \bar{\beta}, \quad \tilde{\gamma}(k) = \gamma(k) - \bar{\gamma}. \end{aligned} \quad (9)$$

In order to conduct the stability analysis for the above systems, it is necessary to make the following definitions and lemmas.

Definition 3. Consider the system (8) with $v(k) = 0$; it is said to be mean-square exponentially stable if for any initial conditions, there exist a $\delta > 0$ and $0 < \alpha < 1$, such that $E\{|\tilde{x}(k)|^2\} \leq \delta \alpha^k \sup_{i \in \mathbb{Z}^-} E\{|\tilde{\phi}(i)|^2\}$, $k \geq 0$. Moreover, if $\lim_{t \rightarrow \infty} E\{|\tilde{x}(k)|^2\} = 0$, then the system is said to be robust asymptotically mean-square stable.

Definition 4. Given a scalar $\gamma > 0$, the error system (8) is said to be mean-square robustly exponentially stable with the attenuation level γ if the error system (8) with $v(k) = 0$ is mean-square robustly exponentially stable, and under zero initial condition, the following is true for all nonzero $v(k)$:

$$\begin{aligned} \sum_{k=0}^{+\infty} E\{\tilde{r}^T(k)\tilde{r}(k)\} &\leq \gamma^2 \sum_{k=0}^{+\infty} E\{v^T(k)v(k)\} \\ (\text{i.e., } \|e\|_{E_\infty} &\leq \gamma \|v\|_2, \quad \forall w(k) \in \mathcal{L}_2[0, +\infty)). \end{aligned} \quad (10)$$

Correspondingly, system (7) is said to be stochastic $\mathcal{L}_2 - \mathcal{L}_\infty$ filter of system (2).

We further adopt a residual evaluation stage including an evaluation function $J(k)$ and a threshold J_{th} of the following form: $J(k) = \sum_{h=0}^k r(h)r(h)^{1/2}$ where L denotes the maximum time step of the evaluation function. Based on (16), the occurrence of faults can be detected by comparing $J(k)$ with

J_{th} according to the following rule: $J(k) > J_{th}$ (with faults) alarm; $J(k) \leq J_{th}$ no faults.

Remark 5. The parametric uncertainties ΔA , ΔD , ΔC , and ΔD_f satisfy

$$\begin{bmatrix} \Delta A & \Delta D \\ \Delta C & \Delta D_f \end{bmatrix} = \begin{bmatrix} 0 \\ B_f \\ D_f \end{bmatrix} F(k) [\Lambda C, 0, \Lambda D_2, \Lambda H], \quad (11)$$

where $\Lambda = \text{diag}\{\delta_1, \delta_2, \dots, \delta_m\}$ and $F(k) = \Delta_k \Lambda^-$, and $F(k)$ satisfies $F^T(k)F(k) \leq I$.

Lemma 6. Let M , N , and F be real matrices of appropriate dimensions with $F^T F \leq I$. Then, for any scalar $\epsilon > 0$, one has

$$MFN + (MFN)^T \leq \epsilon^{-1} MM^T + \epsilon NN^T. \quad (12)$$

Lemma 7 (Schur complement). Given constant matrices S_1 , S_2 , and S_3 , where $S_1 = S_1^T$ and $S_2 = S_2^T$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0. \quad (13)$$

Lemma 8 (Liu et al. [5]). For any constant matrix $M \in R^{n \times n}$, $M = M^T > 0$, $x_i \in R^n$, and constants $a_i > 0$ ($i = 1, 2, \dots$). If the series concerned is convergent, then one has

$$\begin{aligned} & - \left(\sum_{i=1}^{\infty} a_i \right) \sum_{i=1}^{\infty} a_i x_i^T M a_i x_i^T \\ & \leq - \left(\sum_{i=1}^{\infty} a_i x_i \right)^T M \left(\sum_{i=1}^{\infty} a_i x_i \right). \end{aligned} \quad (14)$$

3. Main Results

In this section, we will investigate a sufficient condition on the performance analysis for the filter error system (8). And then, the solution to the fault detection filter design for the system (2) is listed as follows.

Theorem 9. For nominal system of (8) with given filter parameters and the index $\gamma > 0$, the fault detection dynamics is robustly exponentially stable in mean square, if there exist constant $l > 0$, positive matrices P , Q , Q_i ($i = 1, 2, \dots, q$), and any matrices U with appropriate dimensions, such that the following LMIs hold:

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 0 & \cdots & 0 & 0 & \Gamma_{14} & 0 & \bar{C}^T & \bar{A}^T P \\ * & \bar{\alpha}_1 (1 - \bar{\alpha}_1) \bar{A}_1^T P \bar{A}_1 - Q_1 & \cdots & 0 & 0 & 0 & 0 & 0 & \bar{A}_{11}^T P \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_q (1 - \bar{\alpha}_q) \bar{A}_1^T P \bar{A}_1 - Q_q & 0 & 0 & 0 & 0 & \bar{A}_{1q}^T P \\ * & * & * & * & -\frac{Q}{\bar{\mu}} & 0 & 0 & 0 & \bar{A}_2^T P \\ * & * & * & * & * & \Gamma_{44} & 0 & 0 & \bar{\gamma} \bar{I}^T P \\ * & * & * & * & * & * & \Gamma_{55} & \bar{D}_f^T & \bar{D}^T P \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -P \end{bmatrix} < 0, \quad (15)$$

$$\Gamma_{11} = -P + \sum_{i=1}^q (\tau_M - \tau_m + 1) Q_i + \bar{\mu} Q + \bar{\mu} \bar{\beta} (1 - \bar{\beta}) \bar{A}_2^T P \bar{A}_2 - \begin{bmatrix} U G_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_{14} = \begin{bmatrix} U G_2 \\ 0 \end{bmatrix}, \quad \Gamma_{44} = \bar{\gamma} (1 - \bar{\gamma}) \bar{I}^T P \bar{I} - U, \quad \Gamma_{55} = -\gamma^2 I, \quad \bar{A}_1 = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} A_2 & 0 \\ 0 & 0 \end{bmatrix}, \quad G_1 = \text{diag} \{ \bar{g}_1^- \bar{g}_1^+, \dots, \bar{g}_n^- \bar{g}_n^+ \},$$

$$G_2 = \text{diag} \left\{ \frac{\bar{g}_1^- + \bar{g}_1^+}{2}, \dots, \frac{\bar{g}_n^- + \bar{g}_n^+}{2} \right\}.$$

Proof. For simplicity, let

$$\begin{aligned} h_1(k) &= \tilde{A}\tilde{x}(k) + \sum_{i=1}^q \tilde{A}_{1i}\tilde{x}(k - \tau_i(k)) \\ &\quad + \tilde{A}_2 \sum_{d=1}^{\infty} \mu_d \tilde{x}(k-d) + \tilde{\gamma} \tilde{I}g(k, x(k)) + \tilde{D}v(k) \\ h_2(k) &= \sum_{i=1}^q \tilde{A}_{1i}(k) \tilde{x}(k - \tau_i(k)) \\ &\quad + \tilde{A}_2(k) \sum_{d=1}^{\infty} \mu_d \tilde{x}(k-d) + \tilde{\gamma}(k) \tilde{I}g(k, x(k)). \end{aligned} \quad (16)$$

Then, we have the following equation:

$$\tilde{x}(k+1) = h_1(k) + h_2(k). \quad (17)$$

For this system, we construct the following Lyapunov functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k), \quad (18)$$

with

$$\begin{aligned} V_1(k) &= \tilde{x}^T(k) P \tilde{x}(k), \\ V_2(k) &= \sum_{i=1}^q \sum_{j=k-\tau_i(k)}^{k-1} \tilde{x}^T(j) Q_i \tilde{x}(j) \\ V_3(k) &= \sum_{i=1}^q \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{l=k+j}^{k-1} \tilde{x}^T(l) Q_i \tilde{x}(l) \\ V_4(k) &= \sum_{d=1}^{\infty} \sum_{i=k-d}^{k-1} \tilde{x}^T(i) \{ \mu_d Q + \mu_d^2 \bar{\beta} (1 - \bar{\beta}) \tilde{A}_2^T P \tilde{A}_2 \} \tilde{x}(i). \end{aligned} \quad (19)$$

Taking the difference of the functional along the solution of the system, we obtain

$$\begin{aligned} \mathbb{E} \{ \Delta V(k) \} &= \sum_{i=1}^4 \mathbb{E} \{ \Delta V_i(k) \} \\ \mathbb{E} \{ \Delta V_1(k) \} &= \mathbb{E} \{ \mathbb{E} \{ V_1(k+1) \mid \tilde{x}_k \} - V_1(k) \} \\ &= \mathbb{E} \{ \tilde{x}^T(k+1) P \tilde{x}(k+1) - \tilde{x}^T(k) P \tilde{x}(k) \} \end{aligned}$$

$$\begin{aligned} &= h_1^T(k) P h_1(k) + \mathbb{E} \{ 2h_1^T(k) P h_2(k) \} \\ &\quad + \mathbb{E} \left\{ \sum_{i=1}^q \sum_{j=1}^q \tilde{x}^T(k - \tau_i(k)) \right. \\ &\quad \times \tilde{A}_{1i}^T(k) P \tilde{A}_{1j}(k) \tilde{x}(k - \tau_j(k)) \\ &\quad + \sum_{d_1=1}^{\infty} \left[\sum_{d_2=1}^{\infty} \mu_{d_1} \mu_{d_2} \tilde{x}^T(k - d_1) \right. \\ &\quad \times \tilde{A}_2^T(k) P \tilde{A}_2(k) \tilde{x}(k - d_2) \left. \right] \\ &\quad \left. + \tilde{\gamma}(k)^2 g^T(k, x(k)) \tilde{I}^T P \tilde{I} g(k, x(k)) \right\} \\ &\quad - \tilde{x}^T(k) P \tilde{x}(k). \end{aligned} \quad (20)$$

It is obvious that

$$\begin{aligned} \mathbb{E} \{ 2h_1^T(k) P h_2(k) \} &= 0, \\ \mathbb{E} \{ \tilde{A}_{1i}^T(k) P \tilde{A}_{1j}(k) \} &= \begin{cases} 0 & i \neq j \\ \bar{\alpha}_i (1 - \bar{\alpha}_i) \tilde{A}_1^T P \tilde{A}_1 & i = j, \end{cases} \\ \mathbb{E} \left\{ \sum_{i=1}^q \sum_{j=1}^q \tilde{x}^T(k - \tau_i(k)) \tilde{A}_{1i}(k) P \tilde{A}_{1j}(k) \tilde{x}(k - \tau_j(k)) \right. \\ &\quad + \sum_{d_1=1}^{\infty} \sum_{d_2=1}^{\infty} \mu_{d_1} \mu_{d_2} \tilde{x}^T(k - d_1) \tilde{A}_2^T(k) P \tilde{A}_2(k) \tilde{x}(k - d_2) \\ &\quad \left. + \tilde{\gamma}(k)^2 g^T(k, x(k)) \tilde{I}^T P \tilde{I} g(k, x(k)) \right\} \\ &= \sum_{i=1}^q \tilde{x}^T(k - \tau_i(k)) \bar{\alpha}_i (1 - \bar{\alpha}_i) \tilde{A}_1^T P \tilde{A}_1 \tilde{x}(k - \tau_i(k)) \\ &\quad + \sum_{d=1}^{\infty} \tilde{x}^T(k-d) \bar{\beta} (1 - \bar{\beta}) \mu_d^2 \tilde{A}_2^T P \tilde{A}_2 \tilde{x}(k-d) \\ &\quad + g^T(k, x(k)) \bar{\gamma} (1 - \bar{\gamma}) \tilde{I}^T P \tilde{I} g(k, x(k)). \end{aligned} \quad (21)$$

Then, we have the following equations:

$$\begin{aligned} \mathbb{E} \{ \Delta V_1(k) \} &= h_1^T(k) P h_1(k) - \tilde{x}^T(k) P \tilde{x}(k) \\ &\quad + \sum_{i=1}^q \tilde{x}^T(k - \tau_i(k)) \bar{\alpha}_i (1 - \bar{\alpha}_i) \\ &\quad \times \tilde{A}_1^T P \tilde{A}_1 \tilde{x}(k - \tau_i(k)) \\ &\quad + \sum_{d=1}^{\infty} \tilde{x}^T(k-d) \bar{\beta} (1 - \bar{\beta}) \mu_d^2 \tilde{A}_2^T P \tilde{A}_2 \tilde{x}(k-d) \\ &\quad + g^T(k, x(k)) \bar{\gamma} (1 - \bar{\gamma}) \tilde{I}^T P \tilde{I} g(k, x(k)) \end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{\Delta V_2(k)\} &= \mathbb{E} \{\mathbb{E} \{V_2(k+1)\} - V_2(k)\} \\
&= \mathbb{E} \left\{ \sum_{i=1}^q \sum_{j=k+1-\tau_i(k+1)}^k \tilde{x}^T(j) Q_i \tilde{x}(j) \right. \\
&\quad \left. - \sum_{i=1}^q \sum_{j=k-\tau_i(k)}^{k-1} \tilde{x}^T(j) Q_i \tilde{x}(j) \right\} \\
&= \sum_{i=1}^q \mathbb{E} \left\{ \tilde{x}^T(k) Q_i \tilde{x}(k) \right. \\
&\quad \left. - \tilde{x}^T(k - \tau_i(k)) Q_i \tilde{x}(k - \tau_i(k)) \right. \\
&\quad \left. + \sum_{j=k-\tau_i(k+1)+1}^{k-1} \tilde{x}^T(j) Q_i \tilde{x}(j) \right. \\
&\quad \left. - \sum_{j=k-\tau_i(k)+1}^{k-1} \tilde{x}^T(j) Q_i \tilde{x}(j) \right\} \\
&\leq \sum_{i=1}^q \mathbb{E} \left\{ \tilde{x}^T(k) Q_i \tilde{x}(k) \right. \\
&\quad \left. - \tilde{x}^T(k - \tau_i(k)) Q_i \tilde{x}(k - \tau_i(k)) \right. \\
&\quad \left. + \sum_{j=k-\tau_M+1}^{k-\tau_m} \tilde{x}^T(j) Q_i \tilde{x}(j) \right\},
\end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{\Delta V_3(k)\} &= \mathbb{E} \{\mathbb{E} \{V_3(k+1)\} - V_3(k)\} \\
&= \sum_{i=1}^q \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{l=k+j+1}^k \tilde{x}^T(l) Q_i \tilde{x}(l) \\
&\quad - \sum_{i=1}^q \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{l=k+j}^{k-1} \tilde{x}^T(l) Q_i \tilde{x}(l) \\
&= \sum_{i=1}^q \left((\tau_M - \tau_m) \tilde{x}^T(k) Q_i \tilde{x}(k) \right. \\
&\quad \left. - \sum_{j=k-\tau_M-1}^{k-\tau_m} \tilde{x}^T(j) Q_i \tilde{x}(j) \right),
\end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{\Delta V_4(k)\} &= \mathbb{E} \{\mathbb{E} \{V_4(k+1)\} - V_4(k)\} \\
&= \sum_{d=1}^{\infty} \mathbb{E} \left\{ \sum_{i=k-d+1}^k \tilde{x}^T(i) \right. \\
&\quad \times [\mu_d Q + \mu_d^2 \bar{\beta} (1 - \bar{\beta}) \hat{A}_2^T P \hat{A}_2] \tilde{x}(i) \\
&\quad \left. - \sum_{i=k-d}^{k-1} \tilde{x}^T(i) \right\}
\end{aligned}$$

$$\begin{aligned}
&\times [\mu_d Q + \mu_d^2 \bar{\beta} (1 - \bar{\beta}) \hat{A}_2^T P \hat{A}_2 \tilde{x}(i)] \Big\} \\
&= \tilde{x}^T(k) (\bar{\mu} Q + \bar{\mu} \bar{\beta} (1 - \bar{\beta}) \hat{A}_2^T P \hat{A}_2) \tilde{x}(k) \\
&\quad - \sum_{d=1}^{\infty} \tilde{x}^T(k-d) \mu_d^2 \bar{\beta} (1 - \bar{\beta}) \hat{A}_2^T P \hat{A}_2 \tilde{x} \\
&\quad \times (k-d) - \sum_{d=1}^{\infty} \tilde{x}^T(k-d) \mu_d Q \tilde{x}(k-d).
\end{aligned} \tag{22}$$

By using Lemma 8, we have

$$\begin{aligned}
&- \sum_{d=1}^{\infty} \tilde{x}^T(k-d) \mu_d Q \tilde{x}(k-d) \\
&\leq -\frac{1}{\bar{\mu}} \left(\sum_{d=1}^{\infty} \tilde{x}(k-d) \right)^T Q \left(\sum_{d=1}^{\infty} \tilde{x}(k-d) \right).
\end{aligned} \tag{23}$$

From Assumption 1, $\hat{g}_i^- \leq (g_i(x) - g_i(y))/(x - y) \leq \hat{g}_i^+$, we have $(g(k, x(k)) - \hat{g}_i^- x(k))(g(k, x(k)) - \hat{g}_i^+ x(k)) \leq 0$.

It can be deduced that there exists $\mathbf{U} = \text{diag}[u_1, u_2, \dots, u_n] > 0$ such that

$$\begin{aligned}
&\sum_{i=1}^n u_i \begin{bmatrix} x(k) \\ g(k) \end{bmatrix}^T \begin{bmatrix} \hat{g}_i^- \hat{g}_i^+ e_i e_i^T & -\frac{\hat{g}_i^- + \hat{g}_i^+}{2} e_i e_i^T \\ -\frac{\hat{g}_i^- + \hat{g}_i^+}{2} e_i e_i^T & e_i e_i^T \end{bmatrix} \\
&\times \begin{bmatrix} x(k) \\ g(k) \end{bmatrix} \\
&= \begin{bmatrix} x(k) \\ g(k) \end{bmatrix}^T \begin{bmatrix} U G_1 & -U G_2 \\ -U G_2 & U \end{bmatrix} \begin{bmatrix} x(k) \\ g(k) \end{bmatrix} \leq 0,
\end{aligned} \tag{24}$$

where e_i denotes the unit column vector having one element on its i th row and zeros elsewhere.

Now, we are ready to prove the exponential stability of the system (8) with $v(k) = 0$. Combining (16)–(24), we can easily see that

$$\mathbb{E} \{\Delta V(k)\} \leq \mathbb{E} \left\{ \zeta_1^T(k) (\Gamma_1 + \Delta^T P \Delta) \zeta_1(k) \right\}, \tag{25}$$

where

$$\zeta_1(k) = \begin{bmatrix} \tilde{x}^T(k) & \tilde{x}^T(k - \tau_1(k)) & \tilde{x}^T(k - \tau_2(k)) & \cdots & \tilde{x}^T(k - \tau_q(k)) & \sum_{d=1}^{\infty} \mu_d x(k-d) & g^T(k) \end{bmatrix}^T,$$

$$\Gamma_1 = \begin{bmatrix} \phi_{11} & 0 & \cdots & 0 & 0 & 0 & 0 \\ * & \bar{\alpha}_1(1 - \bar{\alpha}_1)\hat{A}_1^T P \hat{A}_1 - Q_1 & \cdots & 0 & 0 & 0 & 0 \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_q(1 - \bar{\alpha}_q)\hat{A}_1^T P \hat{A}_1 - Q_q & 0 & 0 & 0 \\ * & * & * & * & -\frac{Q}{\mu} & 0 & 0 \\ * & * & * & * & * & \bar{\gamma}(1 - \bar{\gamma})\tilde{I}^T P \tilde{I} - II & 0 \end{bmatrix} \quad (26)$$

$$\Delta = [\bar{A} \quad \bar{A}_{11} \quad \cdots \quad \bar{A}_{1q} \quad \bar{A}_2 \quad \bar{\gamma}\tilde{I}].$$

If $\Gamma_1 + \Delta^T P \Delta < 0$, we can obtain that $\Gamma_2 < 0$ by Schur complements (Lemma 7), where

$$\Gamma_2 = \begin{bmatrix} \phi_{11} & 0 & \cdots & 0 & 0 & 0 & \bar{A}^T P \\ * & \bar{\alpha}_1(1 - \bar{\alpha}_1)\hat{A}_1^T P \hat{A}_1 - Q_1 & \cdots & 0 & 0 & 0 & \bar{A}_{11}^T P \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_q(1 - \bar{\alpha}_q)\hat{A}_1^T P \hat{A}_1 - Q_q & 0 & 0 & \bar{A}_{1q}^T P \\ * & * & * & * & -\frac{Q}{\mu} & 0 & \bar{A}_2^T P \\ * & * & * & * & * & \bar{\gamma}(1 - \bar{\gamma})\tilde{I}^T P \tilde{I} - II & \bar{\gamma}\tilde{I}^T P \\ * & * & * & * & * & * & -P \end{bmatrix}. \quad (27)$$

Then, the system (8) with $v(k) = 0$ is mean-square robustly asymptotically stable. Furthermore, along the same line of the proof for Theorem 1 in [15], the exponential stability of system (8) can be confirmed on the mean-square sense.

Next, we will establish the performance of the filtering error system (8) under the zero initial condition. To this end, we introduce

$$J(k) = E \sum_{k=0}^{\infty} \{ \tilde{r}^T(k) \tilde{r}(k) \} - \gamma^2 \sum_{k=0}^{\infty} E \{ v^T(k) v(k) \}. \quad (28)$$

And, it is obvious to see that

$$\begin{aligned} J(n) &= E \sum_{k=0}^n \{ \tilde{r}^T(k) \tilde{r}(k) \} \\ &\quad - \gamma^2 \sum_{k=0}^n E \{ v^T(k) v(k) \} \\ &\quad + E \sum_{k=0}^n \{ V(k+1) - V(k) \} - V(n+1) + V(0) \\ &\leq E \sum_{k=0}^n \{ \tilde{r}^T(k) \tilde{r}(k) \} - \gamma^2 \sum_{k=0}^n E \{ v^T(k) v(k) \} \\ &\quad + E \sum_{k=0}^n \{ \Delta V(k) \}. \end{aligned} \quad (29)$$

Along the same line as the proof of the stability of system (7), $J(n) \leq E \sum_{k=0}^n \{ \zeta_2^T(k) \Gamma \zeta_2(k) \}$, where

$$\zeta_2(k) = \begin{bmatrix} \tilde{x}^T(k) & \tilde{x}^T(k - \tau_1(k)) & \tilde{x}^T(k - \tau_2(k)) & \cdots & \tilde{x}^T(k - \tau_q(k)) & \sum_{d=1}^{\infty} \mu_d x^T(k-d) & g^T(k) & v^T(k) \end{bmatrix}^T, \quad (30)$$

we can show that $J(n) \leq 0$.

Letting $n \rightarrow \infty$, we obtain

$$\sum_{k=0}^{+\infty} E \{ \tilde{r}^T(k) \tilde{r}(k) \} \leq \gamma^2 \sum_{k=0}^{+\infty} E \{ v^T(k) v(k) \}. \quad (31)$$

This completes the proof. $\square$

Next, we are in a position to deal with the design of the filter output feedback controller for the system (8) and obtain the main result of this paper in the following theorem.

Theorem 10. For given constants $0 \leq \tau_m < \tau_M$, $\varepsilon > 0$, and a scalar $\lambda > 0$, the fault detection filter (7) exists such that error system (8) is robustly asymptotically exponentially stable in mean square, if there exist constant $l > 0$, positive matrices P , R , and R_i ($i = 1, 2, \dots, q$), and any matrices $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, and $\hat{D}_f$ with appropriate dimensions, respectively,

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & \cdots & 0 & 0 & \Sigma_{14} & 0 & \Sigma_{16} & \Sigma_{17} \\ * & \bar{\alpha}_1(1 - \bar{\alpha}_1)\bar{A}_1 - R_1 & \cdots & 0 & 0 & 0 & 0 & 0 & \Sigma_{2,17} \\ * & * & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_q(1 - \bar{\alpha}_q)\bar{A}_1 - R_q & 0 & 0 & 0 & 0 & \Sigma_{2,q7} \\ * & * & * & * & -\frac{R}{\bar{\mu}} & 0 & 0 & 0 & \Sigma_{37} \\ * & * & * & * & * & \Sigma_{44} & 0 & 0 & \Sigma_{47} \\ * & * & * & * & * & * & \Sigma_{55} & \Sigma_{56} & \Sigma_{57} \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & \Sigma_7 \end{bmatrix} < 0, \quad (32)$$

$$\Sigma_{11} = -\begin{pmatrix} X & Y \\ Y & Y \end{pmatrix} + \sum_{i=1}^q (\tau_M - \tau_m + 1) R_i + \bar{\mu}R + \bar{\mu}\bar{\beta}(1 - \bar{\beta})\bar{A}_2^T - \bar{U},$$

$$\bar{U} = \begin{pmatrix} UG_1 & UG_1 \\ UG_1 & UG_1 \end{pmatrix}, \quad \bar{A}_1 = \begin{pmatrix} A_1^T X A_1 & A_1^T X A_1 \\ A_1^T X A_1 & A_1^T X A_1 \end{pmatrix}, \quad \bar{A}_2 = \begin{pmatrix} A_2^T X A_2 & A_2^T X A_2 \\ A_2^T X A_2 & A_2^T X A_2 \end{pmatrix},$$

$$\Sigma_{14} = \begin{bmatrix} UG_2 \\ UG_2 \end{bmatrix}, \quad \Sigma_{16} = [C^T \bar{D}_f^T \quad C^T \bar{D}_f^T + \bar{C}_f^T], \quad \Sigma_{17} = \begin{bmatrix} A^T X + C^T \bar{B}_f^T & A^T Y \\ A^T X + C^T \bar{B}_f^T + \bar{A}_f^T & A^T Y \end{bmatrix}, \quad (33)$$

$$\Sigma_{2,i7} = \begin{bmatrix} \bar{\alpha}_i A_1^T X & \bar{\alpha}_i A_1^T Y \\ \bar{\alpha}_i A_1^T X & \bar{\alpha}_i A_1^T Y \end{bmatrix} \quad (i = 1, 2, \dots, q),$$

$$\Sigma_{37} = \begin{bmatrix} \bar{\beta} A_2 X & \bar{\beta} A_2 Y \\ \bar{\beta} A_2 X & \bar{\beta} A_2 Y \end{bmatrix}, \quad \Sigma_{44} = \bar{\gamma}(1 - \bar{\gamma})X - U, \quad \Sigma_{55} = -\gamma^2 I, \quad \Sigma_{47} = [\gamma X \quad \gamma Y],$$

$$\Sigma_{56} = \begin{bmatrix} D_2^T \bar{D}_f^T \\ H^T \bar{D}_f^T - I \end{bmatrix}, \quad \Sigma_{57} = \begin{bmatrix} D_1^T X + D_2^T \bar{B}_f^T & D_1^T Y \\ G^T X + H^T \bar{B}_f^T & G^T Y \end{bmatrix}, \quad \Sigma_{77} = -\begin{bmatrix} X & Y \\ Y & Y \end{bmatrix}.$$

If (X, Y, Q, Q_i) is a feasible solution of (15), there exist nonsingular matrices U and V satisfying $UV = I - XY^{-1}$. The fault detection filter parameters in the form of (7) are given as follows:

$$A_f = U^{-1} \hat{A}_f Y^{-1} V^{-1}, \quad B_f = U^{-1} \hat{B}_f,$$

$$C_f = \hat{C}_f Y^{-1} V^{-1}, \quad D_f = \hat{D}_f. \quad (34)$$

Proof. The condition in (31) pledges that $\begin{pmatrix} X & Y \\ * & Y \end{pmatrix} > 0$.

This inequality implies that $I - XY^{-1}$ and $X - Y$ are nonsingular by Schur complement. If the condition in (32)

is feasible, there exist nonsingular matrices U, V satisfying $UV = I - XY^{-1}$.

Then, we introduce the following matrices:

$$\Omega_1 = \begin{pmatrix} X & Y \\ U^T & 0 \end{pmatrix}, \quad \Omega_2 = \begin{pmatrix} I & I \\ 0 & VY \end{pmatrix}. \quad (35)$$

Obviously Ω_1, Ω_2 are nonsingular.

Let

$$P \triangleq \Omega_1 \Omega_2^{-1} = \begin{pmatrix} X & U \\ U^T & -U^T Y^{-1} V^{-1} \end{pmatrix}; \quad (36)$$

then $\Omega_2^T P \Omega_2 = \begin{pmatrix} X & Y \\ Y^T & Y \end{pmatrix}$.

Pre- and postmultiplying inequality (32) by $\text{diag}\{\Omega_2^{-1}, k\Omega_2^{-1}, \dots, \Omega_2^{-1}, \Omega_2^{-1}, I, I, I, \Omega_2^{-1}\}$, and letting $\Omega_2^T Q_i \Omega_2 = R_i$,

$\Omega_2^T Q \Omega_2 = R$, we can obtain (8). This completes the proof. $\square$

When there are parameters uncertainties, we have the following corollary.

Corollary 11. For given constants $0 \leq \tau_m < \tau_M$, $\varepsilon > 0$ and a scalar $\lambda > 0$, the fault detection filter (7) exists such that error system (8) is robustly asymptotically exponentially stable in mean square, if there exist constant $l > 0$, $\epsilon > 0$, positive matrices P, R , and R_i ($i = 1, 2, \dots, q$), and any matrices $\widehat{A}_f, \widehat{B}_f, \widehat{C}_f$, and $\widehat{D}_f$ with appropriate dimensions, respectively,

$$\bar{\Sigma} = \begin{bmatrix} \Sigma_{11} & 0 & \cdots & 0 & 0 & \Sigma_{14} & 0 & \Sigma_{16} & \Sigma_{17} & 0 & \Sigma_{19} \\ * & \bar{\alpha}_1(1 - \bar{\alpha}_1)\bar{A}_1 - R_1 & \cdots & 0 & 0 & 0 & 0 & 0 & \Sigma_{2,17} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & * & \bar{\alpha}_q(1 - \bar{\alpha}_q)\bar{A}_1 - R_q & 0 & 0 & 0 & 0 & \Sigma_{2,q7} & 0 & 0 \\ * & * & * & * & -\frac{R}{\mu} & 0 & 0 & 0 & \Sigma_{37} & 0 & 0 \\ * & * & * & * & * & \Sigma_{44} & 0 & 0 & \Sigma_{47} & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{55} & \Sigma_{56} & \Sigma_{57} & 0 & \Sigma_{59} \\ * & * & * & * & * & * & * & -I & 0 & \Sigma_{68} & 0 \\ * & * & * & * & * & * & * & * & \Sigma_7 & \Sigma_{78} & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \quad (37)$$

where, $\Sigma_{11}, \Sigma_{16}, \Sigma_{17}, \Sigma_{2,17}, \dots, \Sigma_{2,q7}, \bar{A}_1, \Sigma_{37}, \Sigma_{44}, \Sigma_{47}, \Sigma_{55}, \Sigma_{56}, \Sigma_{57}$, and Σ_7 have been defined in Theorem 9:

$$\Sigma_{19} = \begin{bmatrix} C^T \Lambda^T \\ C^T \Lambda^T \end{bmatrix}, \quad \Sigma_{59} = \begin{bmatrix} D_2^T \Lambda^T \\ H^T \Lambda^T \end{bmatrix}, \quad (38)$$

$$\Sigma_{68} = [\epsilon \widehat{D}_f], \quad \Sigma_{78} = -\begin{bmatrix} \epsilon \widehat{B}_f \\ 0 \end{bmatrix}.$$

If (X, Y, Q, Q_i) is a feasible solution of (15), there exist nonsingular matrices U and V satisfying $UV = I - XY^{-1}$. The fault detection filter parameters in the form of (7) are given as follows:

$$\begin{aligned} A_f &= U^{-1} \widehat{A}_f Y^{-1} V^{-1}, & B_f &= U^{-1} \widehat{B}_f, \\ C_f &= \widehat{C}_f Y^{-1} V^{-1}, & D_f &= \widehat{D}_f. \end{aligned} \quad (39)$$

Proof. When considering the system with uncertain parameters, we need to replace $\widetilde{A}, \widetilde{D}, \widetilde{C}, \widetilde{D}_f$ with $\widetilde{A} + \Delta A, \widetilde{D} + \Delta D, \widetilde{C} + \Delta C, \widetilde{D}_f + \Delta D_f$ in (32). Then, (32) changes into $\Sigma + MF(k)N + N^T F^T(k)M^T < 0$, where $M = (0, 0, \dots, 0, \widehat{D}_f^T, \widehat{B}_f^T, 0)^T$; $N = (\Lambda C, \Lambda C, 0, \dots, 0, \Lambda D_2, \Lambda H, 0, 0, 0)$.

Using Schur complement and lemma, we can derive (37). This completes the proof. $\square$

4. Illustrative Example

In this section, a numerical example will be presented to illustrate the effectiveness of our results.

Example 1. Consider the system (2) with the parameters listed as follows [13]:

$$A = \begin{bmatrix} 0.9719 & -0.0013 \\ -0.0340 & 0.8628 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.14 & 0.2 \\ 0 & 0.2 \end{bmatrix},$$

$$\begin{aligned}
A_2 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, & D_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
D_2 &= [0 \ 0.1], & C &= [1 \ 0.1], \\
G &= [-0.0839 \ 0.0761]^T, & B &= 0, \\
H &= 0, & \bar{\alpha}_1 &= 0.9, & \bar{\alpha}_2 &= 0.7, \\
\bar{\gamma} &= 0.8, & g_1(k, x(k)) &= 0.5x_1(k) \sin(x_2(k)), \\
g_2(k, x(k)) &= 0.5x_2(k) \sin(x_1(k)), & \varepsilon(k) &= 1, \\
G_1 &= \begin{bmatrix} 0.25 & 0 \\ 0 & 0.26 \end{bmatrix}, & G_2 &= \begin{bmatrix} 0.45 & 0 \\ 0 & 0.45 \end{bmatrix}.
\end{aligned} \tag{40}$$

Let the time-varying communication delays satisfy $1 \leq \tau_i(k) \leq 3$ ($i = 1, 2$). For the measurement quantization, the parameters of the logarithmic quantizer are set as $\bar{\mu}_0 = 2$ and $\chi = 0.8$. Then, the fault detection filter parameters can be obtained from Corollary 11 as follows.

For the parameters listed previously, by Corollary 11 in our paper, we can obtain the following feasible filtering parameters:

$$\begin{aligned}
A_f &= \begin{bmatrix} -0.2524 & -0.0325 \\ -0.0325 & -0.1975 \end{bmatrix}, & B_f &= \begin{bmatrix} -3.4252 \\ -0.4137 \end{bmatrix}, \\
C_f &= [0.0065 \ -0.0045], & D_f &= [0.0122],
\end{aligned} \tag{41}$$

and the performance index between the robustness and sensitivity is $\gamma = 3.8382$.

Example 2. To show the usefulness and effectiveness of the designed fault detection filter, let the external disturbance be $w(k) = 0$. Figures 1 and 2 show the residual signal $r(k)$ and evolution of residual function $J(k)$, respectively, when the fault signal $f(k)$ is given as

$$f(k) = \begin{cases} 1, & 40 \leq k \leq 80 \\ 0, & \text{else.} \end{cases} \tag{42}$$

It indicates that the designed filter can detect the fault effectively. In [13], the fault can be detected in 6 time steps after its occurrence. However, from Figure 2, we can see that the fault can be detected in 4 time steps after its occurrence. Therefore, it can be seen that the residual can not only reflect the fault in time but also detect the fault without confusing it with the disturbance $w(k)$.

FIGURE 1: Residual signal without $w(k)$.

FIGURE 2: Evolution of residual function $J(k)$ without $w(k)$.

5. Conclusion

The fault detection filter design for stochastic systems with time-varying delays has been investigated in this research. Based on the Lyapunov functional method, sufficient conditions are obtained to ensure that the error systems are mean-square robustly asymptotically stable, and then the filters are designed in terms of LMIs. Numerical example has been given to illustrate the effectiveness of the proposed main results. The foregoing results have the potential to be useful for the study of stochastic systems.

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