Research Article

Fuzzy Logic-Based Techniques for Modeling the Correlation between the Weld Bead Dimension and the Process Parameters in MIG Welding

Y. Surender and Dilip Kumar Pratihari

Soft Computing Lab., Mechanical Engineering Department, Indian Institute of Technology, Kharagpur 721 302, India

Correspondence should be addressed to Dilip Kumar Pratihari; dkpra@mech.iitkgp.ernet.in

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Fuzzy logic-based techniques have been developed to model input-output relationships of metal inert gas (MIG) welding process. Both conventional and hierarchical fuzzy logic controllers (FLCs) of Mamdani type have been developed, and their performances are compared. The conventional FLC suffers from the curse of dimensionality for handling a large number of variables, and a hierarchical FLC was proposed earlier to tackle this problem. However, in that study, both the structure and knowledge base of the FLC were not optimized simultaneously, which has been attempted here. Simultaneous optimization of the structure and knowledge base is a difficult task, and to solve it, a genetic algorithm (GA) will have to deal with the strings having varied lengths. A new scheme has been proposed here to tackle the problem related to crossover of two parents with unequal lengths. It is interesting to observe that the conventional FLC yields the best accuracy in predictions, whereas the hierarchical FLC can be computationally faster than others but at the cost of accuracy. Moreover, there is no improvement of interpretability by introducing a hierarchical fuzzy system. Thus, there exists a trade-off between the accuracy obtained in predictions and computational complexity of various FLCs.

1. Introduction

Arc welding process plays a vital role in manufacturing. Despite the widespread use of arc welding for joining the metals, total automation of the process is yet to be achieved, and it is so due to the fact that the physics of the problem is not fully understood and quantified. Metal inert gas (MIG) welding is one of the most commonly used arc welding processes, due to its low initial cost and high productivity. To have the better knowledge and control of MIG welding process, it is necessary to determine its input-output relationships. MIG welding is a complex process involving multiple variables. The quality of weld bead is decided by its mechanical properties, which are dependent on its both metallurgical properties and bead geometry, which, in turn, depend on a number of input process parameters, such as welding speed, voltage, wire feed rate, gas flow rate, nozzle-plate distance, torch angle, surface tension of the molten metal, and electromagnetic force. Several approaches had been developed by various researchers to

predict bead geometry in welding. Those approaches include theoretical studies, statistical regression analysis, and soft computing-based approaches.

Rosenthal [1] studied temperature distributions in an infinite plate, due to a moving point heat source, considering the conduction mode of heat transfer. He did not relate his study with the prediction of weld-bead geometry. Later on, Roberts and Wells [2] theoretically estimated the weld-bead width by considering the conduction mode of heat transfer, after making a number of assumptions. It might be difficult to model a welding process analytically due to its complexity and inherent nonlinearity.

Various investigators had tried to model the process using statistical regression analysis also. In this connection, the studies of Yang et al. [3], Murugan et al. [4, 5], and Ganjigatti et al. [6, 7] are worth mentioning. In conventional statistical regression analysis, input-output relationship is determined response-wise (i.e., one at a time). Thus, it might not be able to capture the dynamics of the process fully. Moreover, some of

the responses could be dependent on each other, which cannot be determined using this approach.

Soft computing-based approaches [8] had also been used to model the input-output relationships of welding processes by various researchers. Li et al. [9] developed a control system for gas metal arc welding (GMAW) process, where an artificial neural network received online arc voltage data and predicted the mode of metal transfer and assessed whether the operating regime was appropriate for producing good quality welds. An attempt was also made by Juang et al. [10] using back-propagation neural networks to establish input-output relationships of a tungsten inert gas (TIG) welding process. Later on, Amarnath and Pratihari [11] could successfully establish input-output relationships of the TIG welding process in both forward and reverse directions using radial basis function neural networks (RBFNNs). Experimental data are generally associated with imprecision and uncertainty. Realizing the fact that fuzzy logic can deal with the imprecision and uncertainty, Wu et al. [12] developed a fuzzy logic system for process monitoring and quality evaluation in the GMAW by examining the welding voltage and current distributions. The system was capable of detecting common disturbances in the GMAW process by measuring the voltage and current. Hong et al. [13] designed a neurofuzzy controller, which was coupled with a vision-based system that could stabilize the weld pool, online. Ganjigatti and Pratihari [14] suggested an approach for automatic design of fuzzy logic controller (FLC) that could predict bead geometry in MIG welding. Singh and Gill [15] developed a model to predict tensile strength of friction welded GI pipes using an adaptive neurofuzzy inference system (ANFIS). Several attempts were made to utilize the fuzzy and/or neurofuzzy systems for modeling of various manufacturing processes. However, those systems suffered from the well-known problem of *curse of dimensionality*. The number of rules increases exponentially with the number of input variables and linguistic terms used to represent those variables. This, in turn, leads to an increase in computer processing time and memory requirement for the storage. To overcome this problem, Raju et al. [16] proposed a hierarchical structure, in which a higher dimensional FLC [17] was considered to be a combination of some lower dimensional fuzzy logic systems connected in a hierarchical fashion to reduce the number of rules to a linear function of system variables. Later on, Cheong and Lai [18] considered another hierarchical structure, which was not similar to that proposed by Raju et al. [16]. They designed a symmetrical rule base, and the data base was optimized using an evolutionary algorithm known as differential evolution. Lin et al. [19] automated the design of Raju's hierarchical FLC (HFLC) to control water level in an advanced boiling water reactor using the steepest descent method. To the best of the authors' knowledge, no significant study has been reported to model input-output relationships of welding process using the HFLC.

Building a hierarchical fuzzy system is a difficult task. This is so due to the fact that we need to define both the architecture of the system (i.e., modules, input variables of each module, and interactions among the modules) and the rules of each module. Chen et al. [20] developed an automatic method of evolving hierarchical Takagi-Sugeno fuzzy systems

TABLE 1: Input welding parameters and their ranges.

Sl. no.	Input parameters	Units	Notation	Range
1	Welding speed	cm/min	S	25–46
2	Arc voltage	volts	V	26–31
3	Wire feed rate	mm/min	F	6–8
4	Gas flow rate	lit/min	G	14–21
5	Nozzle to plate distance	mm	D	15–23
6	Torch angle	degree	A	70–110

(TS-FS). The hierarchical structure was evolved using a probabilistic incremental program evolution (PIPE) with some specific instructions. They considered the structure optimization in a sequential manner, and the rule parameters embedded in the structure were tuned using an evolutionary programming (EP).

In the present work, both conventional FLCs and HFLCs of Mamdani type have been developed and their performances are tested for modeling of MIG welding process. A novel approach has been developed for simultaneous optimization of the structure and knowledge base of the HFLCs using a genetic algorithm (GA) [21]. The performances of the developed approaches have been compared in terms of accuracy in predictions and computational time.

The paper has been organized as follows: Section 2 gives a brief description of the MIG welding process considered in the present study; the tools and techniques utilized in the present study have been discussed in Section 3; the developed approaches are explained in Section 4; results are stated and discussed in Section 5; some concluding remarks are made in Section 6 and the scope for future work is suggested in Section 7.

2. MIG Welding Process

MIG welding, one of the most popular arc welding processes, uses a consumable metal electrode, and the molten metal is protected from the atmosphere utilizing the shielding of an inert gas like argon or helium. The weld bead geometry can be represented by its width (BW), height (BH), and penetration (BP), and it is dependent on the following input parameters: welding speed (S), voltage (V), wire feed rate (F), gas flow rate (G), nozzle-plate distance (D), torch angle (A), and others. Figure 1 displays the experimental set-up used for conducting experiments on the MIG welding process. The notations of input parameters and their ranges are shown in Table 1. Figure 2 shows a typical weld bead.

Experiments were conducted previously to collect input-output data according to a full-factorial statistical design [22]. Bead-on-plate welding was carried out on steel plate of 20 mm thickness. Welding current was varied in the range of 3–500 A, and a reverse polarity was used during the experiments. It is to be noted that argon was used as the shielding gas during the welding. It is also important to mention that mainly a spray mode of metal transfer took place during the welding. There were six input process parameters, and two

FIGURE 1: Photograph of the set-up of MIG welding process [22].

levels had been considered for each of them. Thus, there was a set of $2^6 = 64$ combinations of input variables. The weld-bead samples were cut, surface ground using belt grinder, and polished utilizing various grades of sandpaper. The specimens were again polished using aluminum oxide initially and then utilizing diamond paste and cloth in a polishing machine. The polished specimens were cleaned with alcohol and macroetched using 2% nital solution to reveal the geometry of the weld bead and heat-affected zone. Photograph of each macro-etched sample was taken, and the image was converted into a pdf. The measurement of bead geometry was carried out, and the measured dimensions were also compared with the results obtained using a microscope to test the accuracy of the measurement. Regression analysis has been carried out to determine nonlinear response equations, as given below:

$$\begin{aligned} BH &= \frac{10^{2.97885} \times F^{0.318995} \times D^{0.101367}}{S^{0.389096} \times V^{1.20688} \times G^{0.007758} \times A^{0.264379}}, \\ BW &= \frac{V^{1.55667} \times F^{0.402256} \times G^{0.0211693} \times A^{0.112076}}{10^{1.05147} \times S^{0.326531} \times D^{0.113891}}, \\ BP &= \frac{V^{1.29320} \times F^{0.781304} \times G^{0.112076}}{10^{0.843747} \times S^{0.252858} \times D^{0.0726984} \times A^{0.0726984}}. \end{aligned} \quad (1)$$

It is interesting to observe from the previous equations that weld bead-geometric parameters are mainly dependent on the selection of welding speed and arc voltage/current, which exactly match with the experience of the welders.

2.1. Training Data Collection. One thousand input-output data have been generated using the previous nonlinear regression equations by varying the input process parameters at random within their respective ranges. It is to be noted that sixty-four anchor data points (collected through real experiments and with the help of which regression analysis is carried out) have also been considered along with the previous mentioned one thousand data during the training of FLCs. Thus, the training set consists of $1000 + 64 = 1064$ data points.

FIGURE 2: A typical weld bead [22] (BH: bead height, BP: bead penetration, and BW: bead width).

FIGURE 3: A schematic view of the HFLC developed by Raju et al. [16].

2.2. Test Data Collection. In order to check the performances of the developed fuzzy logic techniques, twenty-seven test cases (collected through the real experiments) have been considered [22] (refer to Appendix A).

3. Tools and Techniques Used

In this section, the tools and techniques used to solve the problem related to input-output modeling of MIG welding process are going to be discussed.

To establish input-output relationships of the said process, Mamdani type of FLC [17] has been adopted. It consists of four modules, namely, identification of variables, fuzzification, inference engine, and defuzzification. Interested readers may refer to [8] for a detailed description of the Mamdani type of FLC. It is important to mention that if there are n input variables of the process to be controlled and m linguistic terms are used to represent each variable, there will be a total of m^n rules of the FLC. Center of area method has been utilized in the present work for defuzzification. It is also interesting to observe that the number of rules increases with the number of variables and that of linguistic terms used to represent them. Thus, computational complexity of the FLC will be more for the higher values of n and m .

To overcome the above difficulty of the conventional FLC, a hierarchical fuzzy logic controller (HFLC) [16] was

FIGURE 4: A schematic view of a genetic-fuzzy system.

developed, in which a higher dimensional FLC is considered to be a combination of some lower dimensional fuzzy logic systems (FLSs) connected in a hierarchical fashion. Figure 3 shows the schematic view of the hierarchical FLC developed by Raju et al. [16]. In their method, two inputs are fed to a subcontroller called FLS, and the produced output along with the third input is passed to another FLS. This process continues until all the input variables are utilized. Here, only two inputs are considered for each FLS. If each variable is expressed using m linguistic terms, m^2 rules are to be designed for each FLS. As there are $(n-1)$ such FLSs, $m^2 (n-1)$ (in place of m^n) rules are necessary to develop the FLC completely, which clearly indicates that the number of rules of the HFLC increases linearly with the number of input variables n .

In Raju's HFLC [16], the inputs should be passed in such a way that the most important variable is considered at the first level and the second most important variable is put at the next level, and so on. However, it is not necessary that the number of inputs at each subcontroller should always be equal to two. The inputs may vary from two to n variables (i.e., total number of inputs). If all the n variables are passed at a time, then it becomes the conventional FLC. Thus, the HFLC structure is not unique, and one may be in dilemma

in choosing a particular type of HFLC structure. The chosen structure may not be optimal also for a given problem. Therefore, the performance of the HFLC depends on (i) structure of the HFLC chosen for modeling, (ii) knowledge base of the subcontrollers, (iii) the order, in which the inputs are passed to the subcontrollers. However, designing a suitable hierarchical FLS is a difficult task. It is so, because both the architecture of the system (i.e., modules, input variables of each module, and interactions among the modules) and the rules of each module are to be defined.

An FLC does not have an inbuilt optimizer, and consequently, a proper training is to be provided to it with the help of some training scenarios and an optimization tool, say GA [21]. The performance of an FLC depends on its knowledge base consisting of data base (which carries information related to membership function distributions of the variables) and rule base. The GA has been utilized by a number of investigators to improve the performance of the FLC, and the developed approaches are popularly known as the genetic-fuzzy systems [23]. Figure 4 shows the schematic view of the genetic-fuzzy system, in which a batch mode of training has been adopted with the help of a large number of scenarios using a GA.

4. Developed Approaches

To model input-output relationships of the MIG welding process, six approaches based on fuzzy logic technique (as discussed previously) have been developed, and their performances are compared among themselves.

Approach 1: GA-Based Tuning of Rule Base and Data Base of Manually Constructed Conventional FLC [24]. Fuzzy logic- (FL-) based controller has been developed to model input-output relationships in MIG welding process. There are six inputs, namely, welding speed (S), voltage (V), wire feed rate (F), gas flow rate (G), nozzle-plate distance (D), and torch angle (A), and three outputs such as bead height (BH), bead width (BW), and bead penetration (BP) of the process. Figure 5 shows the genetic-fuzzy system developed to model the MIG welding process.

Manually constructed membership function distributions of the input and output variables for MIG welding process are shown in Figure 6. For simplicity, the shape of the membership function distributions has been assumed to be triangular in nature. The range of each variable has been divided and expressed using three linguistic terms such as L: low, M: medium, and H: high. Thus, there are $3^6 = 729$ rules or input combinations. The rule base is designed manually based on the designer's knowledge of the process. A typical rule of the FLC will look as follows.

If S is H AND V is L AND F is M AND G is M AND D is M AND A is M then BH is M, BW is L, BP is L.

A binary-coded GA has been used [21]. Ten bits are assigned to represent each of the nine variables carrying base-width information of the triangular membership function distributions (represented using b values). Thus, $9 \times 10 = 90$ bits are required to represent all the real variables. Moreover, 729 bits are utilized to represent 729 rules (where 1 and 0 represent the presence and absence of a rule, resp.). The GA-string is, thus, $90 + 729 = 819$ bits long. Such a GA-string representation is shown in Figure 7.

Here, b values indicate the half base-width values of isosceles and base-width of right angled triangles of the input and output variables. During optimization, the b values for welding speed, arc voltage, wire feed rate, gas flow rate, nozzle-to-plate distance, torch angle, bead height, bead width, and bead penetration have been varied in the ranges of (8.05, 12.15), (2.00, 3.20), (0.40, 0.90), (1.80, 3.20), (2.20, 3.80), (10.05, 20.15), (0.885, 1.455), (2.30, 3.90), and (0.64, 1.18), respectively. The fitness of a GA-string f is calculated as the average of absolute mean deviation in prediction for the entire set of training scenarios, as given below:

$$f = \frac{1}{N} \sum_{i=1}^N \left(\frac{1}{n'} \sum_{j=1}^{n'} (|T_{ij} - O_{ij}|) \right), \quad (2)$$

where T_{ij} and O_{ij} represent the target and predicted values, respectively, of j th output for i th training scenario; N indicates the number of training scenarios; n' denotes the number of outputs.

The FLC has been trained using a GA offline with the help of a batch mode of training involving 1064 scenarios. The GA starts with a population of binary strings (i.e., solutions) created at random. The population of solutions is then modified using the operators, namely, reproduction, crossover, and mutation [8], to hopefully create the better solutions. A tournament selection scheme has been utilized, in which t strings (equal to tournament size) are selected from the population at random and the best one in terms of fitness value is picked. Thus, a mating pool will be created to participate in the crossover. In this operation, there is an exchange of properties between two parent strings and consequently, two children strings will be created. In the present work, a uniform crossover scheme has been adopted. The possibility of occurring crossover is checked at each bit position with a probability of 0.5 for the head appearing. If the head appears at a particular bit position, there will be an exchange of bits between the two parents, and consequently, two children will be created. Moreover, a bit-wise mutation has been implemented to bring a local change over the current solution.

Approach 2: Automatic Design of FLC Using a GA [24]. In Approach 1, the GA is used to optimize the manually designed Knowledge Base (KB) of the FLC. However, it might be difficult sometimes to design the KB beforehand, due to a lack of knowledge of the process. An attempt has been made to design the FLC automatically using the GA in this approach. The GA-string carries information of the bases of triangular membership function distributions, presence, or absence of the rules along with the consequent parts. It is to be noted that there are three outputs for each set of input variables, and two bits are used to represent each output (such as 00 for L, 01 and 10 for M, and 11 for H). Thus, six bits are utilized to denote consequent parts of a rule. A typical GA-string will look as in Figure 8.

During optimization, the ranges of b values have been kept the same, as those used in Approach 1, and the FLC is trained with the help of the same set of 1064 training scenarios used earlier. The fitness of the GA-string has been determined in the same way, as it has been done in Approach 1.

Approach 3: GA-Based Tuning of Rule Base and Data Base of the Manually Constructed HFLC Structure Proposed by Raju et al. [16]. The schematic diagram of HFLC as proposed by Raju et al. [16] is shown in Figure 3. As mentioned earlier, the input variables are considered at different levels according to their contributions towards the output. The order of importance of the input variables has been kept the same as that in Ganjigatti [22]. It is important to mention that a significance test is conducted during statistical regression analysis to identify the significant and insignificant input variables. Moreover, the order of importance of significant input variables can be decided from either normal probability plot or Pareto-chart of effects on response. Interested readers may refer to [25] for a detailed description of the significance test. With respect to bead height (BH), it is found to be as follows: S , V , A , F , D , and G . Thus, the contributions of welding speed S and gas flow rate G are seen to be the maximum and minimum,

FIGURE 5: Genetic-fuzzy system to model MIG welding process.

FIGURE 6: Manually constructed membership function distributions of the input-output variables; MIG welding process.

FIGURE 7

FIGURE 8

FIGURE 9

respectively, towards the mentioned response. Similarly, the orders of importance of the input variables are coming out to be V, S, F, D, A , and G and A, V, S, F, G , and D for the outputs BW and BP, respectively. As the mentioned order of importance is not found to be the same for all the responses, a separate HFLC is constructed for each output/response. As there are six inputs, each HFLC will consist of five sub-FLCs.

In HFLC, each sub-FLC has two inputs represented by three linguistic terms each, and thus, a total of $5 \times 3^2 = 45$ rules are considered for one response. For three responses, the number of rules is coming out to be equal to $3 \times 45 = 135$. Thus, in the designed HFLC, the total number of rules is reduced from 729 to 135. The rule base is designed manually based on the knowledge of the process the designer has.

Besides the rules, the performances of an HFLC used to model one response depend on the membership function distributions of six inputs, four intermediate outputs, and one final output; that is, $6 + 4 + 1 = 11$ real variables. Ten bits are assigned to represent each of the 33 variables (for three responses). The first 330 bits carry the information of 33 real variables, and the next 135 bits represent the rule base for three approaches. The GA-string is 465 bits long, which is shown in Figure 9.

During optimization, the ranges of b values for optimizing the system variables have been kept the same as before. The half base-width values of the triangles used for intermediate outputs have been varied in the range of (0.40–0.50). The fitness of the GA-string has been calculated in the same way as discussed previously. The HFLC has been trained with the help of the same set of scenarios considered in the previous approaches.

Approach 4: Automatic Design of HFLC Proposed by Raju et al. [16] Using a GA. An attempt is made to design the sub-FLCs automatically using a GA. In this approach, the task of designing good KB of the sub-FLCs is given to the GA, and it tries to find an optimal KB through search. As discussed in Approach 3, 45 rules are used to determine each of the three responses. Thus, there are a total of $3 \times 45 = 135$ rules. Two bits are utilized to represent the consequent part of each rule. With the two bits, there could be four possibilities such as 00, 01, 10, and 11. If the sum of two bits is found to be equal to 0 (i.e., $0 + 0 = 0$), it indicates L, that means the consequent L is represented by 00. Similarly, both 01 and 10 will denote the consequent M, and 11 represents the consequent H. Thus, the GA-string will consist of 465 (as in Approach 3) $+ 3 \times 45 \times 2 = 735$ bits. A particular GA-string can be represented as given in Figure 10.

During optimization, the ranges of b values have been kept the same, as those used in Approach 3, and the FLC is trained using the same 1064 training scenarios. The fitness of the GA-string has been determined in the same way, as discussed previously.

Approach 5: Structure Optimization and Automatic Design of HFLC Using a GA, Considering Accuracy in Predictions as the Fitness. In this approach, the structure of the FLC has not been kept as a fixed one. The structures investigated may vary from the hierarchical structure proposed by Raju et al. [16] to the structure of conventional FLC. For the MIG welding process involving six input variables and for a fixed order of importance of the variables, sixteen hierarchical structures may be possible to design. The tree structure representation is used for generating the structure of the HFLC randomly.

Here, the GA-string carries information of the structure and knowledge base of the HFLC. It is important to mention that the number of inserted intermediate variables and that of the rules depend on structural information of the HFLC. Therefore, the length of GA-string may vary in the population of solutions. The first five bits represent the structure of an HFLC, and based on that, the number of bits required for representing the data base and rule base is determined. For example, let us consider a structure of $\{2, 2, 3, 2, 0\}$.

It indicates that there are four subcontrollers (FLSs) with 2, 2, 3, and 2 inputs, respectively. For one response, ten b values are required to represent the data base of the HFLC. Thus, a total of $3 \times 10 = 30$ real b values are to be specified for determining three responses. Moreover, a total of $3^2 + 3^2 + 3^3 + 3^2 = 54$ rules are to be considered for determining each response. Thus, there are $3 \times 54 = 162$ rules in the HFLC used for determining three responses. As two bits are utilized to denote the consequent of each rule (as done in Approach 4), $162 \times 2 = 324$ bits are required to represent the consequent of all of them. It is to be noted that ten bits are used to denote each b value. Thus, the GA-string will consist of $5 + 30 \times 10 + 3 \times 54 + 3 \times 54 \times 2 = 791$ bits. The ranges of system variables have been kept the same with those considered in the previous approaches. However, the variables representing the span of membership function distributions of intermediate outputs have been varied in the range of (0.001, 1.001). The string representation for this approach is shown in Figure 11.

The fitness of the GA-string has been defined in the same way, as done in the previous approaches. The HFLC has been trained with the help of the same set of 1064 scenarios. As the GA-strings contained in the population of solutions may have varied lengths, it will be difficult to implement the conventional crossover operator. Thus, a special type of crossover operator is to be implemented. In the present work, a crossover operator suitable for variable string lengths has been proposed, and its performance has been tested. The proposed crossover operator has been explained below.

Simultaneous optimization of the structure and KB of an HFLC lead to a problem, in which the GA will have to handle strings having varied lengths. In such a situation, the conventional crossover operators cannot be used. Let us suppose that the following two parent strings are participating in the crossover to create two children strings by exchanging their properties (see Figure 12).

The first five bits representing the structure of FLC will not participate in the crossover. Let us also assume that Parents 1 and 2 have L_1 and L_2 bits, respectively, and $L_1 < L_2$. Thus, Parent 2 contains $(L_2 - L_1)$ more bits than Parent 1 does. An additional substring of length $(L_2 - L_1)$ will be created for Parent 1, which is nothing but the complementary substring of $(L_2 - L_1)$ bits counted from the right-hand side of Parent 2. Thus, the participating parents in the crossover will look like as shown in Figure 13.

It is to be noted that the bits representing the structure will not take part in the crossover and for the remaining bits; a uniform crossover of 0.5 probability will be used for the exchange of properties. Due to the crossover, Child 1 and Child 2 will be created from the Parent 1 and Parent 2, respectively. After the crossover is done, $(L_2 - L_1)$ bits will be deleted from the right-hand side of Child 1. Let us assume that uniform crossover takes place at 7th, 10th, 13th, 16th, and 18th positions in the above parent strings. Using the above principle, the children strings are found to be as shown in Figure 14.

Approach 6: Structure Optimization and Automatic Design of HFLC Using a GA, Considering Both Accuracy and Compactness as the GA-Fitness. This approach differs from Approach 5

FIGURE 10

FIGURE 11

TABLE 2: Differences and similarities of six approaches.

Approach 1	It deals with conventional Mamdani type FLC of fixed structure. A GA is used to tune RB and DB of the manually constructed FLC. Accuracy in predictions has been considered as the fitness of a GA-string.
Approach 2	Conventional Mamdani type FLC of fixed structure has been dealt. A GA is utilized for automatic design of DB and RB of the FLC. Accuracy in predictions has been considered as the fitness of a GA-string.
Approach 3	It deals with Hierarchical FLC of fixed structure. A GA has been used to tune RB and DB of the manually constructed HFLC. Accuracy in predictions has been considered as the fitness of a GA-string.
Approach 4	Hierarchical FLC of fixed structure has been dealt. A GA is utilized for automatic design of DB and RB of the HFLC. The fitness of a GA-string is calculated as the accuracy in predictions.
Approach 5	An FLC of variable structure (may vary from HFLC to conventional FLC) has been considered in this approach. A GA has been used for structure optimization and automatic design of FLC. Accuracy in predictions has been considered as the fitness of a GA-string.
Approach 6	It is the same with Approach 5, but both accuracy in predictions and compactness of the structure have been considered as the fitness of a GA-string.

Parent 1:	2	2	2	2	2	1	0	1	0	1	1								
Parent 2:	2	2	2	3	0	0	1	1	0	0	1	1	1	0	1	1	0	1	1

FIGURE 12

Parent 1 ^I :	2	2	2	2	2	1	0	1	0	1	1	0	0	1	0	0	1	0	0
Parent 2 ^I :	2	2	2	3	0	0	1	1	0	0	1	1	1	0	1	1	0	1	1

FIGURE 13

Child 1:	2	2	2	2	2	1	1	1	0	0	1								
Child 2:	2	2	2	3	0	0	0	1	0	1	1	1	0	0	1	0	0	0	1

FIGURE 14

in terms of the definition of fitness function only. Here, both accuracy in predictions and number of rules of the HFLC have been considered in the fitness function. It is important to mention that equal weightage has been given to both the accuracy in predictions and compactness of the HFLC. The fitness of the GA-string f has been calculated as given below:

$$f = 0.5 \times \frac{1}{N} \sum_{i=1}^N \left(\frac{1}{n'} \sum_{j=1}^{n'} (|T_{ij} - O_{ij}|) \right) + 0.5 \times \frac{R}{R_{\max}}, \dots, \quad (3)$$

where R indicates the number of rules present in the HFLC and $R_{\max}$ is the maximum number of rules to be present in the conventional FLC. The other terms carry usual meaning, as explained previously.

The differences and similarities of the previously six approaches have been highlighted in Table 2.

5. Results and Discussion

The performances of the developed approaches have been compared in predicting input-output relationships of MIG welding process, as discussed below.

As the performance of a GA depends on its parameters, a careful study has been carried out to determine the set of optimal parameters. During the parametric study, only one parameter has been varied at a time, keeping the others fixed. A uniform crossover with probability $p_c = 0.5$ has been utilized in the present study. Initially, the values of population size and maximum number of generations have been kept fixed to 120 and 160, respectively. Figure 15(a) shows the variations of fitness with the probability of mutation p_m . The GA yields the minimum value of the fitness corresponding to $p_m = 0.0005$.

Thus, p_m has been kept fixed to 0.00055 for the remaining part of the parametric study. Population size has been varied in the range of (100, 320), after keeping the values of p_m and the maximum number of generations fixed at 0.00055 and 160, respectively (refer to Figure 15(b)). The minimum value of the fitness is found to occur corresponding to the population size of 220. The GA is then run for a large number of generations, corresponding to $p_m = 0.00055$ and a population size of 220. It is found to converge to a fixed minimum value of the fitness at 560 generations (refer to Figure 15(c)). From the previous study, the optimum values of mutation probability, population size, and number of generations are found to be equal to 0.00055, 220, and 560, respectively. The optimized membership function distributions of the variables,

FIGURE 15: Results of GA-parametric study: (a) fitness versus probability of mutation, (b) fitness versus population size, and (c) fitness versus number of generations.

as obtained using this approach, are shown in Figure 16. It is to be noted that only 141 rules are found to be good by the GA (refer to Appendix B) from a total of 729 rules in Approach 1.

The performance of the optimized controller has been tested on 27 test cases, for making predictions of the responses, namely, BH, BW, and BP. The predicted values have been compared to the respective target (experimental) values of the responses to determine percent deviation in prediction, if any. It is to be noted that a slightly better prediction has been obtained for BH, compared to the other two responses. Moreover, an underprediction has been observed for the response BP. It might have happened due to the fact that besides some experimental errors, the parameters like surface tension of the molten metal, electro-magnetic force, and others (which have not been considered in the present study) have significant influence on the bead-geometric parameters. Moreover, linear membership function distributions have been considered here for simplicity. However, nonlinear membership function distributions might have yielded a slightly better prediction. The CPU time has been determined by passing mentioned said 27 test cases for 100 times. It has been found to be equal to 2.25 seconds for this approach.

Similarly, the optimized KBs of the FLCs have been obtained using Approaches 2 through 6. Comparisons have been made of their predicted values with their respective

target values of the responses. Moreover, their computational complexities have been determined in terms of CPU time. The following subsections deal with the previous comparisons, in detail. In Approach 5, where the structure of the HFLC has been optimized considering the accuracy in predictions of the fitness, the evolved structure is found to be as that of a conventional FLC. Thus, it has been proved that the conventional FLC can provide the best accuracy in predictions. In Approach 6, the structure of the HFLC has been optimized by putting equal weightage to both accuracy and its compactness. The evolved structure using Approach 6 for prediction of BH is seen to have a form of (2, 2, 4, 0, 0), as shown in Figure 17. The optimized rule base is found to contain only 63 rules in Approach 6.

It is important to mention that input variables are chosen at different levels of an HFLC according to their contributions towards an output. The most significant variables are passed through the first subFLC, that is, FLS_1 . It is also interesting to observe that Approach 6 has evolved an optimal structure of the HFLC for the prediction of BH, in which the sequence of input variables has turned out to be the same with that mentioned in Section 4 (i.e., Approach 3).

5.1. Comparison of Approaches 1 through 6 in Terms of % Deviation in Predictions. The performances of the above six approaches have been compared in terms of % deviation in

FIGURE 16: Optimized membership function distributions of the input and output variables of the FLC: Approach 1.

FIGURE 17: Evolved structure of HFLC used to predict BH: Approach 6.

predictions of different responses. Figures 18(a), 18(b), and 18(c) show the plots of percentage deviations in predictions of BH, BW, and BP, respectively.

For BH and BW, the values of % deviation in predictions are found to lie on both the positive and negative sides for all the approaches. In case of BH, the large % deviations are observed for 1st, 6th, and 26th test cases, and the same has been obtained for the 6th and 9th cases for predicting BW by all the approaches. It might have come due to simplicity of the developed model and some experimental errors. It is to be noted that all the approaches have underpredicted the BP values for almost all the test cases. It might have happened due to the fact that the parameters, such as surface tension of the molten metal and electro-magnetic forces, have been neglected for simplicity of the modeling.

5.2. Comparison of Approaches 1 through 6 in Terms of Average RMS Deviation in Predictions. This subsection compares the performances of the developed approaches in terms of average root mean squared (RMS) deviation in prediction of

FIGURE 18: Plots of percentage deviations (obtained using all six approaches) in predictions of (a) BH, (b) BW, and (c) BP.

TABLE 3: Comparison of Approaches 1 through 6 in terms of computational complexity.

Approaches	No. of optimized rules Response-wise			CPU time (27 cases for 100 runs)
	BH	BW	BP	
Approach 1	141	141	141	2250 ms
Approach 2	227	227	227	2740 ms
Approach 3	26	21	30	57.5 ms
Approach 4	17	21	28	55.6 ms
Approach 5	251	251	251	2820 ms
Approach 6	63	53	59	402 ms

the responses. Figure 19 displays RMS deviation values in prediction of individual and combined outputs, as obtained by all six approaches.

It is interesting to note that Approach 5 has slightly outperformed other approaches in terms of RMS deviation in prediction of the responses. As no effort is made for manual design of the rule base, this approach might be interesting also. Moreover, its performance could be further improved by considering asymmetric and nonlinear membership function distributions. Approach 6 has performed better than Approaches 3 and 4 and has showed good interpolation capability. It is also important to note that both Approaches 5 and 6 have outperformed Approach 4, that is, the approach proposed by Raju et al. [16].

5.3. Comparison of Approaches 1 through 6 in Terms of Computational Complexity. Computational complexities of the previous approaches have been compared in terms of CPU time values. The processing time of an approach depends on the architecture of FLC, number of rules present in the rule base, and others. Table 3 displays the number of good

TABLE 4: Twenty-seven test cases used in this study.

Run no.	<i>S</i>	<i>V</i>	<i>F</i>	<i>G</i>	<i>D</i>	<i>A</i>	BH	BW	BP
1	40	27	6.4	17	18	92	2.61	9.46	1.72
2	30	28	6.8	18	17	87	3.45	9.44	2.33
3	40	29	6.6	19	19	84	2.72	10.02	2.39
4	35	27	6.6	19	17	87	3.33	8.74	2.23
5	40	29	6.8	16	17	87	2.58	10.29	2.55
6	40	27	6.8	17	16	96	2.83	10.88	2.55
7	35	29	6.4	17	16	96	2.94	9.78	2.39
8	30	27	6.6	16	16	96	3.36	9.11	2.44
9	30	29	6.4	17	16	96	2.91	8.87	2.09
10	35	27	6.8	19	17	87	3.31	9.20	2.24
11	30	27	6.4	17	17	87	3.54	8.24	2.30
12	40	28	6.8	17	17	87	3.22	8.70	2.03
13	40	28	6.4	18	16	96	2.66	9.13	2.38
14	40	27	6.4	17	17	87	3.04	8.44	2.12
15	30	29	6.8	16	17	87	3.22	10.08	2.51
16	30	28	6.6	17	17	87	3.40	9.52	2.26
17	35	27	6.4	16	19	84	3.00	8.94	2.23
18	35	29	6.6	17	17	87	2.89	10.32	2.67
19	35	28	6.4	18	16	96	2.88	9.88	1.96
20	35	29	6.8	19	17	87	2.95	10.76	2.68
21	30	28	6.8	16	16	96	3.11	9.97	2.26
22	35	28	6.4	19	17	87	2.97	9.93	2.21
23	35	28	6.6	16	18	92	2.89	9.93	2.10
24	40	29	6.4	17	18	92	2.56	10.87	2.25
25	30	29	6.6	19	16	96	3.01	10.47	2.21
26	30	27	6.8	17	17	87	3.76	9.00	2.18
27	40	28	6.6	17	16	96	2.78	9.46	2.24

rules identified by all the response-wise approaches and their computational complexities measured in terms of CPU time values.

The previous comparison shows that there is a large reduction in the size of rule base and CPU time for predicting various outputs in case of HFLCs (except in Approach 5, where the optimized HFLC moves towards the optimized conventional FLC). On the other hand, the conventional FLC has performed slightly better than HFLC in terms of accuracy in predictions, due to more number of optimized rules. Thus, there might be a trade-off between good accuracy of conventional FLC and low computational complexity of HFLC. Our aim is also to develop a compact FL-based technique, which should be able to make the predictions as accurately as possible. It can be posed as a multiobjective optimization problem, as explained below. It is important to mention that there will be no improvement of interpretability by introducing a hierarchical fuzzy system.

5.4. Multiobjective Optimization Using NSGA-II [26]. An attempt has been made to determine a Pareto-optimal front of solutions using an algorithm named NSGA-II [26]. Figure 20 displays the obtained Pareto-optimal front of solutions. The algorithm has identified three distinct and discontinuous regions denoted by Zones A, B, and C (refer to Figure 20).

It is interesting to note that Zone A shows nondominated solutions obtained by the conventional FLC (i.e., Approaches 1 and 2). Zone B consists of the solutions yielded by the proposed HFLC (i.e., Approach 6), and Zone C contains the solutions of HFLC structure proposed by Raju et al. [16] (i.e., Approach 3).

6. Concluding Remarks

To model input-output relationships of MIG welding process, both conventional and hierarchical FLCs have been developed, and their performances are evaluated. Out of six approaches, two are related to conventional FLC and the remaining four belong to HFLCs. Conventional FLCs are found to yield better accuracy in predictions compared to the HFLCs. On the other hand, HFLC proposed by Raju et al. [16] and its automatic implementation (i.e., Approaches 3 and 4) are found to be the most compact ones (hence, it has the least computational complexity) but at the cost of accuracy in predictions. However, the HFLC proposed by the authors (i.e., Approach 6) is able to yield some results, which can be considered as the trade-off between the above two extreme conditions. It is important to mention that there is no improvement of interpretability in the developed hierarchical fuzzy systems.

TABLE 5: Optimal rule base of conventional FLC obtained using Approach 1.

Sl. no.	S	V	F	G	D	A	BH	BW	BP
1	H	L	M	M	M	M	M	L	L
2	M	H	H	M	M	L	H	H	H
3	M	L	M	H	M	L	H	L	H
4	H	M	M	L	M	H	L	L	L
5	H	M	M	M	L	L	L	L	H
6	M	M	M	M	M	M	M	M	M
7	H	H	H	H	H	M	L	H	H
8	M	M	M	L	L	M	M	M	M
9	L	L	M	M	M	L	H	L	H
10	H	H	M	H	H	L	L	L	H
11	M	H	L	H	M	L	L	M	H
12	L	H	M	M	M	L	H	H	H
13	L	H	L	L	L	H	L	H	M
14	L	H	L	H	L	H	H	H	H
15	M	H	L	L	L	H	L	H	L
16	L	H	H	H	H	H	H	H	H
17	L	M	H	M	M	L	H	H	H
18	L	H	H	L	M	L	H	H	H
19	H	L	M	L	L	M	M	L	L
20	H	H	L	H	M	M	L	M	M
21	L	M	H	M	H	L	H	M	H
22	H	M	M	L	L	M	L	L	L
23	L	H	H	L	L	H	M	H	H
24	M	M	L	L	L	M	L	L	L
25	H	M	M	H	H	M	L	L	L
26	L	L	M	M	H	M	H	L	L
27	M	L	M	M	H	H	H	L	L
28	M	L	M	H	H	M	H	L	L
29	M	M	M	H	H	L	H	L	H
30	L	L	H	M	H	L	H	L	H
31	H	M	M	L	L	H	L	M	L
32	L	H	L	H	H	M	L	H	H
33	L	M	M	L	L	M	H	H	H
34	H	M	L	M	M	M	L	L	L
35	H	H	L	L	M	H	L	L	L
36	H	H	M	H	H	M	L	M	M
37	H	L	L	H	H	H	L	L	L
38	L	H	M	M	M	M	M	H	H
39	H	L	L	L	L	L	M	L	L
40	H	H	H	H	M	L	L	H	H
41	H	H	L	M	M	H	L	M	L
42	H	L	H	M	L	L	H	L	H
43	L	L	L	H	H	M	H	L	L
44	L	L	L	L	H	H	H	L	L
45	H	H	M	M	L	L	L	M	H
46	H	L	L	H	H	L	M	L	L
47	H	M	M	M	H	H	L	L	L
48	M	H	M	H	M	L	L	H	H
49	M	L	L	M	M	M	M	L	L

TABLE 5: Continued.

Sl. no.	S	V	F	G	D	A	BH	BW	BP
50	L	L	M	L	L	L	H	L	H
51	L	H	M	L	M	H	M	H	M
52	L	M	M	M	L	L	H	H	H
53	M	H	M	M	L	L	L	H	H
54	H	H	H	M	L	L	L	H	H
55	M	L	M	M	M	L	H	L	M
56	H	L	L	L	L	M	L	L	L
57	H	L	M	H	M	L	M	L	M
58	H	M	M	M	M	M	L	L	L
59	H	H	M	L	L	M	L	M	M
60	M	L	H	H	H	H	H	H	L
61	L	L	M	M	M	M	H	M	M
62	L	L	L	M	H	H	H	L	L
63	L	L	L	M	L	L	H	L	H
64	M	L	L	M	H	H	M	L	L
65	H	L	L	H	L	M	L	L	L
66	L	M	H	H	H	L	H	H	H
67	H	L	M	H	H	M	M	L	L
68	M	L	M	L	L	M	H	L	L
69	H	M	L	M	L	M	L	L	L
70	L	H	L	M	L	H	L	H	H
71	L	H	M	M	H	M	H	H	H
72	H	M	M	M	L	M	L	M	M
73	H	H	M	M	M	L	L	L	H
74	L	M	L	M	M	M	M	M	M
75	M	L	L	L	M	M	H	L	L
76	L	H	M	H	H	M	M	H	H
77	M	L	M	L	L	L	H	L	M
78	L	H	L	M	H	H	L	H	L
79	L	H	M	L	H	H	H	H	L
80	L	L	L	L	H	M	H	L	L
81	H	H	H	M	M	M	L	H	H
82	H	M	H	M	L	L	M	M	H
83	H	L	M	M	L	M	L	L	L
84	H	L	L	M	L	H	L	L	L
85	M	L	M	L	M	H	H	L	L
86	L	H	L	H	L	L	L	H	H
87	M	H	L	M	M	M	L	M	M
88	H	L	H	M	M	M	H	L	L
89	M	L	L	M	M	L	H	L	L
90	H	L	M	M	L	L	M	L	M
91	M	M	L	M	M	M	L	L	L
92	L	H	H	M	L	M	M	H	H
93	M	L	M	L	M	M	H	L	L
94	H	L	H	L	L	L	H	L	M
95	L	M	H	L	L	L	H	H	H
96	H	L	L	L	M	H	L	L	L
97	H	M	L	M	L	L	L	L	M
98	L	H	M	H	M	L	M	H	H
99	L	H	M	M	H	H	M	H	M

TABLE 5: Continued.

Sl. no.	S	V	F	G	D	A	BH	BW	BP
100	H	M	L	H	H	L	L	L	L
101	H	M	M	M	M	L	M	L	M
102	H	L	H	L	H	H	H	L	L
103	L	L	L	L	L	L	H	L	M
104	M	M	M	M	M	L	H	L	H
105	L	H	H	H	L	L	M	H	H
106	M	H	M	L	L	M	L	H	H
107	L	H	M	H	H	L	H	H	H
108	H	L	M	M	M	L	H	L	L
109	H	L	H	H	H	M	H	L	L
110	M	L	L	H	H	H	L	L	L
111	L	H	M	L	L	L	H	H	H
112	M	L	H	M	M	L	H	L	H
113	L	M	M	M	M	M	H	H	H
114	H	H	M	M	H	H	L	M	L
115	M	L	L	L	L	H	L	L	L
116	H	L	L	M	M	L	M	L	L
117	L	H	L	M	M	M	L	H	H
118	M	L	H	L	H	H	H	L	L
119	H	M	M	H	H	L	M	L	M
120	M	M	M	L	H	H	H	L	L
121	L	L	L	M	M	M	H	L	L
122	H	L	M	H	M	M	L	L	L
123	L	L	M	L	H	H	H	L	L
124	H	M	L	M	M	H	L	L	L
125	L	M	M	H	H	M	H	M	M
126	L	M	H	L	H	L	H	M	H
127	L	H	M	M	L	L	H	H	H
128	M	M	M	M	H	M	L	H	L
129	M	H	M	M	M	M	M	H	H
130	H	L	L	L	L	H	L	L	L
131	H	L	M	L	M	M	M	L	L
132	H	M	L	H	M	H	L	L	M
133	H	H	H	M	M	H	L	M	M
134	H	H	H	M	H	L	M	M	H
135	H	H	H	M	H	H	L	H	M
136	M	L	M	H	H	L	H	L	M
137	H	H	M	M	L	M	L	H	H
138	M	L	M	M	M	M	H	L	L
139	L	L	M	L	M	M	H	L	L
140	M	H	L	M	M	H	L	H	L
141	M	L	H	L	L	L	H	L	H

7. Scope of Future Work

In the present work, an attempt has been made to optimize both the structure and the knowledge base of the HFLC. The problems related to forward mapping of MIG welding process have been tackled using the FL-based techniques. Symmetric and linear membership function distributions have been considered to develop Mamdani-type fuzzy inference system. The present work can be extended in the following ways:

FIGURE 19: Comparison of RMS deviations in prediction of individual and combined outputs.

FIGURE 20: Pareto-optimal front of nondominated solutions for multiobjective optimization problems.

- reverse mapping problems (in which the process parameters are expressed as the functions of bead-geometric parameters) related to MIG welding process will be solved in the future by following the same procedure;
- asymmetrical and nonlinear membership function distributions may be considered to improve the accuracy in predictions;
- hierarchical structure related to Takagi- and Sugeno-type fuzzy inference system might be developed to tackle the mentioned problem.

Appendices

A.

See Table 4.

B.

See Table 5.

References

- [1] D. Rosenthal, "Mathematical theory of heat distribution during welding and cutting," *Welding Journal*, vol. 20, no. 5, pp. 220–234, 1941.
- [2] D. K. Roberts and A. A. Wells, "Fusion welding of aluminum alloys," *Welding Journal*, vol. 12, pp. 553–559, 1954.
- [3] L. J. Yang, R. S. Chandel, and M. J. Bibby, "An analysis of curvilinear regression equations for modeling the submerged-arc welding process," *Journal of Materials Processing Tech.*, vol. 37, no. 1–4, pp. 601–611, 1993.
- [4] N. Murugan, R. S. Parmar, and S. K. Sud, "Effect of submerged arc process variables on dilution and bead geometry in single wire surfacing," *Journal of Materials Processing Tech.*, vol. 37, no. 1–4, pp. 767–780, 1993.
- [5] N. Murugan and R. S. Parmar, "Effects of MIG process parameters on the geometry of the bead in the automatic surfacing of stainless steel," *Journal of Materials Processing Tech.*, vol. 41, no. 4, pp. 381–398, 1994.
- [6] J. P. Ganjigatti, D. K. Pratihari, and A. Roychoudhury, "Modeling of the MIG welding process using statistical approaches," *International Journal of Advanced Manufacturing Technology*, vol. 35, no. 11–12, pp. 1166–1190, 2008.
- [7] J. P. Ganjigatti, D. K. Pratihari, and A. R. Choudhury, "Global versus cluster-wise regression analyses for prediction of bead geometry in MIG welding process," *Journal of Materials Processing Technology*, vol. 189, no. 1–3, pp. 352–366, 2007.
- [8] D. K. Pratihari, *Soft Computing*, Narosa, New Delhi, India, 2008.
- [9] X. Li, S. W. Simpson, and M. Rados, "Neural networks for online prediction of quality in gas metal arc welding," *Science and Technology of Welding and Joining*, vol. 5, no. 2, pp. 71–79, 2000.
- [10] S. C. Juang, Y. S. Tarng, and H. R. Lii, "A comparison between the back-propagation and counter-propagation networks in the modeling of the TIG welding process," *Journal of Materials Processing Technology*, vol. 75, no. 1–3, pp. 54–62, 1998.
- [11] M. V. Amarnath and D. K. Pratihari, "Forward and reverse mappings of the tungsten inert gas welding process using radial basis function neural networks," *Proceedings of the Institution of Mechanical Engineers B*, vol. 223, no. 12, pp. 1575–1590, 2009.
- [12] C. S. Wu, T. Polte, and D. Rehfeldt, "A fuzzy logic system for process monitoring and quality evaluation in GMAW," *Welding Journal*, vol. 80, no. 2, supplement, pp. 33–S, 2001.
- [13] L. Hong, L. F. M. Kee, J. W. J. Yu et al., "Vision-based GTA weld pool sensing and control using neuro-fuzzy logic," *Tech. Rep. SIMTECH*, 2000.
- [14] J. P. Ganjigatti and D. K. Pratihari, "Forward and reverse modeling in MIG welding process using fuzzy logic-based approaches," *Journal of Intelligent and Fuzzy Systems*, vol. 19, no. 2, pp. 115–130, 2008.
- [15] J. Singh and S. S. Gill, "Multi-input single output fuzzy model to predict tensile strength of radial friction welded GI pipes," *International Journal of Information and Systems Sciences*, vol. 4, no. 3, pp. 462–477, 2008.
- [16] G. V. S. Raju, J. Zhou, and R. A. Kisner, "Hierarchical fuzzy control," *International Journal of Control*, vol. 54, no. 5, pp. 1201–1216, 1991.
- [17] E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller," *International Journal of Man-Machine Studies*, vol. 7, no. 1, pp. 1–13, 1975.
- [18] F. Cheong and R. Lai, "Designing a hierarchical fuzzy logic controller using the differential evolution approach," *Applied Soft Computing Journal*, vol. 7, no. 2, pp. 481–491, 2007.
- [19] C. Lin, F.-L. Jeng, C.-S. Lee, and R. Raghavan, "Hierarchical fuzzy logic water-level control in advanced boiling water reactors," *Nuclear Technology*, vol. 118, no. 3, pp. 254–262, 1997.
- [20] Y. Chen, B. Yang, A. Abraham, and L. Peng, "Automatic design of hierarchical Takagi-Sugeno type fuzzy systems using evolutionary algorithms," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 3, pp. 385–397, 2007.
- [21] J. H. Holland, *Adaptation in Natural and Artificial Systems*, The University of Michigan Press, Ann Arbor, Mich, USA, 1975.
- [22] J. P. Ganjigatti, *Application of statistical methods and fuzzy logic techniques to predict bead geometry in welding [Ph.D. thesis]*, IIT, Kharagpur, India, 2006.
- [23] O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, and L. Magdalena, "Ten years of genetic fuzzy systems: current framework and new trends," *Fuzzy Sets and Systems*, vol. 141, no. 1, pp. 5–31, 2004.
- [24] D. K. Pratihari, K. Deb, and A. Ghosh, "A genetic-fuzzy approach for mobile robot navigation among moving obstacles," *International Journal of Approximate Reasoning*, vol. 20, no. 2, pp. 145–172, 1999.
- [25] D. C. Montgomery, *Design and Analysis of Experiments*, John Wiley & Sons, New York, NY, USA, 2003.
- [26] N. Srinivas and K. Deb, "Multiobjective optimization using non-dominated sorting in genetic algorithms," *Evolutionary Computation*, vol. 2, no. 3, pp. 221–248, 1994.