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Research Article

Gravitational Force between the Black Hole and Light Particle in XRBs

Dipo Mahto,[1] Md Shams Nadeem,[2] Mahendra Ram,[3] and Kumari Vineeta[4]

[1] *Department of Physics, Marwari College, Bhagalpur, India*
[2] *University Department of Physics, T.M.B.U., Bhagalpur, India*
[3] *Department of Physics, G.L.A. College, Daltonganj, India*
[4] *Department of Education, S.M. College, Bhagalpur, India*

Correspondence should be addressed to Dipo Mahto; dipomahto@hotmail.com

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The present research paper derives a formula for gravitational force acting between the black hole and light particle passing near the radius of event horizon of black holes and calculates also their values of different test black holes existing in only X-ray binaries (XRBs).

1. Introduction

The English physicist Isaac Newton proposed Universal law of Gravitation in 1687, which states that every particle in the universe exerts a force on every particle along the line joining their centres. The magnitude of the force is directly proportional to the product of the masses of two particles and inversely proportional to the square of distance between them [1], which was successively explained by the observation on the planetary movements made by the German astronomer Kepler (1571–1630). It works perfectly well in the world of ordinary experience and has dominated for about 250 years. It, however, shows its shortcoming when explaining the unusual orbit of Mercury around the sun. It breaks down when the gravitational forces get very strong or involving bodies moving at speed near that of light (http://library.thinkquest.org/C007571/english/printback.htm).

In 1915, Albert Einstein demonstrated better theory of gravitation on the basis of general relativity, which has overcome the limitations of Newton's law of universal gravitation [2].

In 1997, Lerner discussed the problem of the deflection of light in a medium with varying refractive index applied to the motion of light in a weak Schwarzschild gravitational field [3].

In 1999-2000, Mario presented a theory which introduces new unknown relationships that may shed new light on the nature of matter. This theory allows the calculation of the gravitational constant (G) with a precision comparable to the other atomic constants, gives a direct relation between mass and charge of the electron without the need of the ubiquitous "classical electron radius," and generates a second fine structure constant while also offering the disconcerting possibility of an antigravitational force [4].

In 2013, Ng and Raymond Ooi analysed the gravitational force due to a pulsed Bessel beam and its effect on the probe pulse. They found that the Bessel beam generates gravitational repulsive forces at small distances and attractive forces at large distance. These forces can be coherently controlled in a medium by introducing a slow light effect through electromagnetic induced transparency [5].

In the present work, we have derived a formula for gravitational force acting between the black hole and light particle passing near the radius of event horizon of black holes and thereafter calculated their values of different test black holes existing only in X-ray binaries (XRBs).

2. Theoretical Discussion

Isaac Newton proposed universal laws of Gravitation which states that

$$F = G\frac{m_1 m_2}{r^2}, \tag{1}$$

where m_1 and m_2 are the mass of any two bodies in universe, r is the distance between them, G is gravitational constant, and F is the force of attraction acting between m_1 and m_2.

We want to calculate the force of attraction acting between the super-massive body like black holes and light particles. Now applying (1), we have

$$F = G\frac{Mm}{R_{\text{bh}}^2}, \tag{2}$$

where M and R_{bh} denote the mass and radius of event horizon of black holes and m is the mass of light particle.

The mass-energy equivalence relation can be applied with the mass of light particle and we have

$$E = mc^2, \tag{3}$$

where involving parameters have their usual meaning. Or

$$m = \frac{E}{c^2}. \tag{4}$$

According to quantum theory of radiation, we have

$$E = h\nu, \tag{5}$$

where h is Planck constant, ν is the frequency of radiation having value equal to c/λ, and λ is the wavelength of radiation, that is, electromagnetic wave, specially visible wave, because electromagnetic radiation with a wavelength between approximately 400 nm and 700 nm is directly detected by the human eye and perceived as visible light (wikipedia, electromagnetic radiation). Since the invisibility of black holes occurs due to the presence of visible waves, a light adapted eye generally has maximum sensitivity at around 555 nm, in the green region of the optical spectrum (wikipedia, visible spectrum). Using (5) in (4), we have

$$m = \frac{h\nu}{c^2} \tag{6}$$

or

$$m = \frac{h}{c\lambda}. \tag{7}$$

The previous equation is using in (2), we have

$$F = G\frac{Mh}{c\lambda R_{\text{bh}}^2}. \tag{8}$$

The Black hole possesses an event horizon (a one-way membrane) that casually isolates the "inside" of the Black hole from the rest of the universe. The radius of the event horizon of a nonspinning BH given by the Schwarzschild radius can be obtained as [6]

$$R_{\text{bh}} = \frac{2GM}{c^2}. \tag{9}$$

The radius of the event horizon of a spinning BH given by the Schwarzschild radius can be obtained as [6]

$$R'_{\text{bh}} = \frac{GM}{c^2}. \tag{10}$$

Putting the values of (9) and (10) in to (8) and solving, we have

$$\begin{aligned} F &= \frac{hc^3}{4GM\lambda}, \\ F' &= \frac{hc^3}{GM\lambda}. \end{aligned} \tag{11}$$

Equation (11) represent the gravitational force acting on light particle due to nonspinning and spinning black holes.

The three fundamental constants of nature—the speed of light (c), Planck's constant (h), and Newton's gravitational constant (G)—are present in (11). The Planck's constant (h) governs the law of quantum world. The speed of light (c) is the cornerstone of the special theory of relativity. The fact that light is an electromagnetic wave travelling at the speed (c) is very important consequences of Maxwell's equations for electromagnetic field. In general relativity, Newton's gravitational constant G has entirely new meaning. For Newton, G is the constant of proportionality that appears in inverse square law of gravitation, while for Einstein, G is a constant that determines the degree to which a given distribution of matter warps space and time. In this new conception, space time was no longer a spectator of events but itself, a dynamical participant that changed in response to the amount of matter present. It was no longer flat and Euclidean but curved in much the same way as the surface of the earth is round and curved. This curvature of space time is, according to Einstein, the origin of gravity [7].

The surface gravity (κ) of black holes for Schwarzschild case is given by the following equation [8, 9]:

$$\kappa = \frac{1}{4M}. \tag{12}$$

The term M stands for the mass of black holes. From (12), it is clear that the surface gravity of black hole is inversely proportional to its mass and the different black holes will have different surface gravity. The greater the mass of the black holes, the smaller the surface gravity and vice versa. The surface gravity (κ) has the same role in the black hole mechanics as the temperature in the ordinary laws of thermodynamics. The zeroth law of classical black hole mechanics states that the surface gravity (κ) of a black hole is constant on horizon [8, 9] and the surface gravity (κ) can be thought of roughly as the acceleration at horizon of black hole [7].

For convenience, we will use $G = h = c = 1$, in our research work; then (11) is transformed as

$$F = \frac{1}{4M\lambda}, \tag{13}$$

$$F' = \frac{1}{M\lambda}. \tag{14}$$

With the help of (12), equations (13) and (14) can be expressed in terms of surface gravity as follows:

$$F = \frac{\kappa}{\lambda}, \tag{15}$$

$$F' = \frac{4\kappa}{\lambda}. \tag{16}$$

As we have discussed previously, the surface gravity (κ) of a black hole is constant on horizon. Hence for the region of event horizon of black holes, (15) and (16) can be written as

$$F\alpha\frac{1}{\lambda}. \tag{17}$$

The previous relation shows that the force of attraction acting between black hole and light particle is inversely proportional to the wavelength of electromagnetic wave coming towards the event horizon of black holes. Hence the electromagnetic radiation of longer wavelengths is attracted more lesser than that of others.

The situation is rather different for charged black holes. For this, let us consider Reissner-Nördstrom geometry, describing a static electrically charged black hole with the following line element:

$$ds^2 = f(r)\,dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right), \tag{18}$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{19}$$

$$r = M \pm \sqrt{M^2 - Q^2}. \tag{20}$$

Here the parameter r denotes two possible horizon called outer and inner horizons for sign (+) and (−), respectively [9, 10].

The space-time (18) describes a black hole; that is, there is a horizon, when $M > Q$. For $M < Q$ there is no horizon and the space-time has a naked singularity. The case $M = Q$ is called an external black hole. For Reissner-Nördstrom black holes, the temperature is still given by $T = h(\kappa/2\pi)$ with surface gravity κ calculated from the metric (18). For $M \gg Q$, the temperature reduces to the Schwarzschild result. However, as $M \rightarrow Q$ the surface gravity $\kappa \rightarrow 0$, with $\kappa = 0$ for $M = Q$. Therefore, the temperature vanishes for an external black hole [8].

On the basis of the previous explanation, it may be concluded from (15) that the gravitational force acting between a static electrically charged black hole and light particle is zero for the case with $\kappa = 0$ for $M = Q$; that is, no light particle is attracted towards the static electrically charged black hole in the case with $\kappa = 0$ for $M = Q$.

Equation (14) holds for spinning black holes; hence surface gravity κ in this case is given by the Kerr solution [12]:

$$\kappa = \frac{\left(M^4 - J_H^2\right)^{1/2}}{2M\left\{M^2 + \left(M^4 - J_H^2\right)^{1/2}\right\}}. \tag{21}$$

Each black hole is characterized by just three numbers: mass M, spin parameter a_* defined such that the angular momentum of the BH is $a_* GM^2/c$, and electric charge Q [6]. That is,

$$J_H = \frac{a_* GM^2}{c}. \tag{22}$$

Using $G = h = c = 1$, the previous equation can be written as

$$J_H = a_* M^2. \tag{23}$$

The radius is smaller in the case of spinning BH, tending to GM/c^2 as $a_* \rightarrow 1$; then (23) yields as

$$J_H = M^2. \tag{24}$$

Equation (21) becomes by the use of previous equation

$$\kappa = 0. \tag{25}$$

This means that, for maximally spinning black holes, the surface gravity becomes zero and temperature should vanish as discussed in the case of a static electrically charged black hole.

Actually, an astrophysical BH is not likely to have any significant electric charge because it will usually be rapidly neutralised by surrounding plasma. Therefore, the BH can be fully characterised by measuring just two parameters, M and a_* of which the latter is constrained to lie in the range 0 for non-spinning BH to 1 for maximally spinning BHs [6]. Hence only mass of black holes are mainly responsible for characterising BH. So we, here, shall calculate the force of attraction acting between the super-massive body like black holes and light particles by using the mass of different test nonspinning and spinning black holes.

There are two categories of black holes classified on the basis of their masses clearly very distinct from each other, with very different masses $M \sim 5$–$20\ M_\odot$ for stellar – mass black holes in X-ray binaries and $M \sim 10^6$–$10^{9.5}\ M_\odot$ for super-massive black holes in Galactic Nuclei [6, 9].

3. Geodesic Structure

According to Newton's laws the "natural" trajectory of a particle which is not being acted upon by any external force is a straight line. In GR, since gravity manifests itself as space time curvature, these "natural" straight line trajectories generalise to curved paths known as geodesics. These are defined physically as the trajectories followed by freely falling particles, that is, particles which are not being acted upon by any nongravitational external force. Geodesics are defined mathematically as space-time curves that parallel transport their own tangent vectors. For metric spaces, we can also define geodesics as external paths in the sense that along the geodesic between two events E_1, and E_2, the elapsed proper time is an extremum, that is [13]

$$\delta \int_{E_1}^{E_2} d\tau = 0. \tag{26}$$

According to Einstein's theory of general relativity, particles of negligible mass travel along geodesic in the space time. In uncurved space time, far from a source of gravity, these geodesics correspond to straight lines; however, they may deviate from straight lines when the space time is curved. The equation for the geodesic lines is [11]

$$\frac{d^2 x^\mu}{dq^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{dq} \frac{dx^\lambda}{dq}, \tag{27}$$

where Γ represents the Christoffel symbol and the variable q parameterizes the particle's path through space time; its so-called world line. The Christoffel symbol depends only on the metric tensor ($g_{\mu\nu}$), or rather on how it changes with position.

4. Results and Discussion

In the present work, we have derived a formula for gravitational force acting between the black hole and light particle passing near the radius of event horizon of black holes applying Newton's laws of gravitation ($F = Gm_1m_2/r^2$) with the mass-energy equivalence relation ($E = mc^2$) and quantum theory of radiation ($E = h\nu$) and the work is further extended to calculate their values of different test black holes existing in only X-ray binaries (XRBs). To know the nature of gravitational force acting between the black hole and light particle passing near the radius of event horizon with the wavelength, graphs have been plotted between the following:

(i) the radius of the event horizon (R_{bh}) of different test non-spinning black holes and their corresponding values of gravitational force acting between the black hole and light particle passing near the radius of event horizon of black holes in XRBs (Figure 1),

(ii) the radius of event horizon (R'_{bh}) of different test spinning black holes and their corresponding values of gravitational force acting between the black hole and light particle passing near the radius of event horizon of black holes in XRBs (Figure 2).

From Figures 1 and 2, it is clear that the gravitational force acting between the black holes and light particle decreases gradually with increase of the radius of the event horizon of different test nonspinning and spinning black holes for a given wavelength of radiation. From the data available in Tables 1 and 2, it is also clear that the spinning black hole of the same mass has more gravitational force than that of non-spinning black holes.

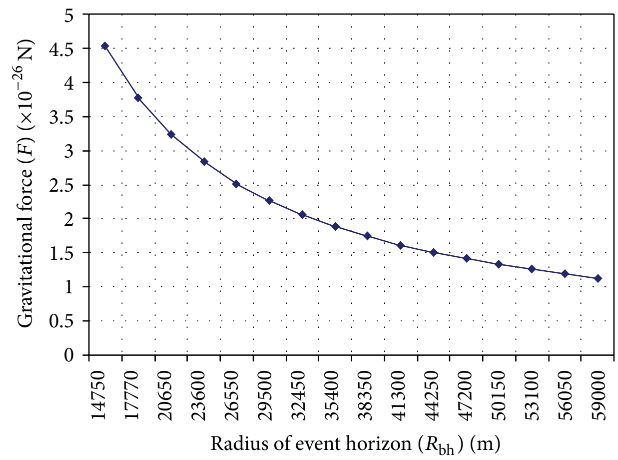

Figure 1: The graph plotted between the radius of event horizon and gravitational force between different test nonspinning black holes and light particles in XRBs.

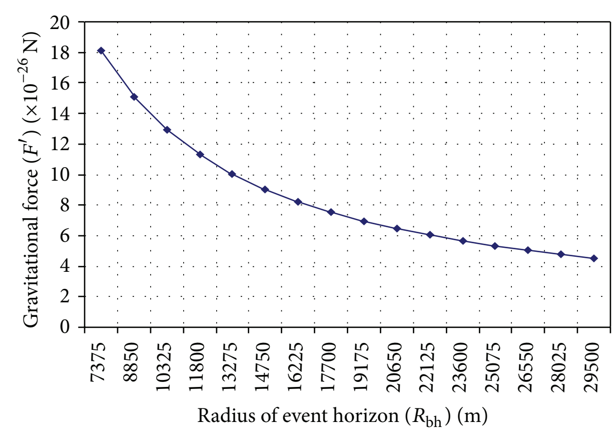

Figure 2: The graph plotted between the radius of event horizon and gravitational force between different test spinning black holes and light particles in XRBs.

Equations (13) and (14) also justify the previous facts. Equations (15) and (16) show that the light particle (light wave) of shorter wavelength has attracted more than that of longer wavelength for constant surface gravity.

The present work is also concerned with the force acting upon a body deviated from its geodesic path due to its being at rest in a gravitational field. General relativity provides a consistent no-force explanation of gravitational interaction of bodies following geodesic paths. As the nonresistant motion of a particle is regarded as inertial particle whose world line is geodesic moving by inertia. In both special and general relativity a particle whose world line is not geodesic is prevented from moving by inertia and therefore is subjected to an inertial force [14].

Hence the force acting on a particle which follows the geodesic path is not gravitational but inertial, while in the case of Newton's laws of gravitation, the force is always attractive as we have discussed for the black hole and light particle.

Table 1: Gravitational force between non-spinning black holes and light particles in XRBs. Data: mass of sun ($M_\odot$) = 1.99×10^{30} kg.

Sl. number	Mass of BHs (M)	(R_s = 2950 M/$M_\odot$) (in metre)	Wavelength of light (λ)	Gravitational force (F) in Newton
1	5 $M_\odot$	14750	555×10^{-9} m	4.527×10^{-26}
2	6 $M_\odot$	17700	555×10^{-9} m	3.773×10^{-26}
3	7 $M_\odot$	20650	555×10^{-9} m	3.234×10^{-26}
4	8 $M_\odot$	23600	555×10^{-9} m	2.829×10^{-26}
5	9 $M_\odot$	26550	555×10^{-9} m	2.515×10^{-26}
6	10 $M_\odot$	29500	555×10^{-9} m	2.263×10^{-26}
7	11 $M_\odot$	32450	555×10^{-9} m	2.058×10^{-26}
8	12 $M_\odot$	35400	555×10^{-9} m	1.886×10^{-26}
9	13 $M_\odot$	38350	555×10^{-9} m	1.741×10^{-26}
10	14 $M_\odot$	41300	555×10^{-9} m	1.617×10^{-26}
11	15 $M_\odot$	44250	555×10^{-9} m	1.509×10^{-26}
12	16 $M_\odot$	47200	555×10^{-9} m	1.414×10^{-26}
13	17 $M_\odot$	50150	555×10^{-9} m	1.331×10^{-26}
14	18 $M_\odot$	53100	555×10^{-9} m	1.257×10^{-26}
15	19 $M_\odot$	56050	555×10^{-9} m	1.191×10^{-26}
16	20 $M_\odot$	59000	555×10^{-9} m	1.131×10^{-26}

Table 2: Gravitational force between spinning black holes and light particles in XRBs.

Sl. Number	Mass of BHs (M)	(R_s = 1475 M/$M_\odot$) (in metre)	Wavelength of light (λ)	Gravitational force (F) in Newton
1	5 $M_\odot$	7375	555×10^{-9} m	18.108×10^{-26}
2	6 $M_\odot$	8850	555×10^{-9} m	15.092×10^{-26}
3	7 $M_\odot$	10325	555×10^{-9} m	12.936×10^{-26}
4	8 $M_\odot$	11800	555×10^{-9} m	11.316×10^{-26}
5	9 $M_\odot$	13275	555×10^{-9} m	10.060×10^{-26}
6	10 $M_\odot$	14750	555×10^{-9} m	9.052×10^{-26}
7	11 $M_\odot$	16225	555×10^{-9} m	8.232×10^{-26}
8	12 $M_\odot$	17700	555×10^{-9} m	7.544×10^{-26}
9	13 $M_\odot$	19175	555×10^{-9} m	6.964×10^{-26}
10	14 $M_\odot$	20650	555×10^{-9} m	6.464×10^{-26}
11	15 $M_\odot$	22125	555×10^{-9} m	6.036×10^{-26}
12	16 $M_\odot$	23600	555×10^{-9} m	5.656×10^{-26}
13	17 $M_\odot$	25075	555×10^{-9} m	5.324×10^{-26}
14	18 $M_\odot$	26550	555×10^{-9} m	5.028×10^{-26}
15	19 $M_\odot$	28025	555×10^{-9} m	4.764×10^{-26}
16	20 $M_\odot$	29500	555×10^{-9} m	4.524×10^{-26}

5. Conclusions

During the study of present research work, we can draw the following conclusions such as the following.

(i) The gravitational force acting between the black holes and light particle is inversely proportional to the wavelength of radiation for constant surface gravity.

(ii) The spinning black hole of the same mass has more gravitational force than that of non-spinning black holes.

(iii) The light particle (light wave) of shorter wavelength has attracted more than that of longer wavelength for constant surface gravity.

(iv) The spinning black holes having spin $a_* \rightarrow 1$ have zero surface gravity.

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