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Research Article

On the Divergenceless Property of the Magnetic Induction Field

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Maxwell's equations beautifully describe the electromagnetic fields properties. In what follows we will be interested in giving a new perspective to divergence-free Maxwell's equations regarding the magnetic induction field: $\nabla \cdot \vec{B}(\vec{r}, t) = 0$. To this end we will consider some physical aspects of a system consisting of massive nonrelativistic charged particles, as sources of an electromagnetic field (e.m.) propagating in free space. In particular the link between conservation of total momentum and divergence-free condition for the magnetic induction $\vec{B}$ field will be deeply investigated. This study presents a new context in which the necessary condition for the divergence-free property of the magnetic induction field in the whole space, known as solenoidality condition, directly comes from the conservation of total momentum for the system, that is, sources and field. This work, in general, leads to results that leave some open questions on the existence, or at least the observability, of magnetic monopoles, theoretically plausible only under suitable symmetry assumptions as we will show.

1. Introduction

The elegant description of the electromagnetic field given by Maxwell's equations is a universally accepted milestone in physics. Many text books describe these equations in detail, together with their relative applications [1]. In what follows we will essentially discuss the Maxwell equation regarding the divergence-free property of the magnetic induction field, in order to give a new interpretation of it. We will consider a system formed by massive, nonrelativistic charged particles as moving sources of the electromagnetic field propagating in a homogeneous, isotropic, and linear space.

The main idea concerning the link between total momentum [2] and solenoidality of $\vec{B}$ field has been suggested by the structure of Lorentz's equation in which magnetic induction acts perpendicularly to the particle's velocities such that no work variation occurs. In an isolated system, total momentum is a constant of motion [3, 4]; starting from this invariance we will deduce the necessary condition for the solenoidality of $\vec{B}$.

This work leaves some open questions on the existence, or at least the observability, of magnetic monopoles; the reader interested in these topics can refer to the extensive bibliography in the literature for details [5–7]. Anyway this paper will discuss special symmetries in which the assumption of nonsolenoidality for the magnetic induction field and the consequently existence of magnetic monopoles could subsist consistently with the conservation of the total momentum. In the present paper we will assume as a starting hypothesis that the magnetic sources are absent or, if present, their effects on the dynamics are smaller than the ones generated by electric sources. From a classical standpoint this is a reasonable hypothesis since magnetic monopoles have never been observed.

2. Charged Particles Moving in Free Space

Let us consider the physical problem in which a finite number of discrete moving sources, consisting of massive and nonrelativistic point-like charges, create an electromagnetic field. The entire space can be thought as homogeneous, isotropic, and linear. Each particle of mass m_α [kg] charge q_α [C] and position $\vec{r}_\alpha(t)$ [m] and nonrelativistic velocity

$\vec{v}_\alpha(t) = d\vec{r}_\alpha(t)/dt$ [m/s] is subject to the following Newton-Lorentz equation [1]:

$$m_\alpha \frac{d^2\vec{r}_\alpha(t)}{dt^2} = q_\alpha \left[\vec{E}\left(\vec{r}_\alpha(t), t\right) + \vec{v}_\alpha(t) \times \vec{B}\left(\vec{r}_\alpha(t), t\right)\right], \quad (1)$$

where $\vec{E}$ [V/m] is the electric field and $\vec{B}$ [T] is the magnetic induction field (linked to the magnetic field $\vec{H} = \vec{B}/\mu$ [A/m] where $\mu = \mu_r\mu_0$; $\mu_0 = 4\pi\cdot 10^{-7}$ [H/m] is the vacuum magnetic constant and μ_r is the relative permeability). Equation (1) is the evolution equation for our system (charged particles and e.m. field). Total momentum for this system is

$$\vec{P}_{\mathrm{TOT}}(t) = \sum_\alpha m_\alpha \frac{d\vec{r}_\alpha(t)}{dt} + \int_\Gamma \vec{D}(\vec{r}, t) \times \vec{B}(\vec{r}, t)\, d\Gamma, \quad (2)$$

where $\vec{D} = \varepsilon\vec{E}$ [C/m^2] is the electric displacement vector and $\varepsilon = \varepsilon_r\varepsilon_0$ ($\varepsilon_0 = 8.854 \cdot 10^{-12}$ [C^2/(N m^2)] is the vacuum dielectric constant, and ε_r is the relative permittivity). In our assumptions, electric and magnetic fields are generated by the previously mentioned moving point-like charges by means of the volumetric density of charge $\rho = \sum_\alpha q_\alpha \delta(r - r_\alpha(t))$ [C/m^3] (since $\nabla \cdot \vec{D} = \rho$) and the superficial density of electrical current $\vec{J} = \sum_\alpha q_\alpha v_\alpha(t)\delta(r - r_\alpha(t))$ [A/m^2] (since $\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$). By using (1) and the expressions of $\rho(r)$ and $\vec{J}(r)$ just defined, (2) can be recasted as follows:

$$\vec{P}_{\mathrm{TOT}}(t) = \int_\Gamma \left(\vec{p}_{\mathrm{mech.}} + \vec{p}_{\mathrm{e.m.}}\right) d\Gamma, \quad (3)$$

where the mechanical momentum density $\vec{p}_{\mathrm{mech.}}$ is linked to the first term on right-hand side of (2) by

$$\frac{\partial \vec{p}_{\mathrm{mech.}}}{\partial t} = \rho\vec{E} + \vec{J} \times \vec{B}, \quad (4)$$

while the electromagnetic momentum density $\vec{p}_{\mathrm{e.m.}}$ is the second term on right-hand side of the same equation (2). The latter is the Minkowski definition for the e.m. momentum density [8]. We have chosen this definition because, in our opinion, with respect to the other possible Abraham definition [9, 10], according to [11], the macroscopic description of the e.m. stress tensor, introduced by Minkowski, is conceptually better than the vacuum-like definition offered by Abraham. More details about this choice can be found in [12] and in references therein. Before proceeding further, it is important to point out that the expression of the e.m. momentum density $\vec{p}_{\mathrm{e.m.}} = \vec{D} \times \vec{B}$, used in (2), is independent by the assumption of existence or nonexistence of magnetic monopoles and currents, but it is an intrinsic property of the electromagnetic field [13], known as a consequence of charge symmetry of Maxwell equations [14]. In the appendix, for reader convenience, a simplified vectorial proof of the previously mentioned property is reported. The integral in (2) can be extended to any closed volume, large enough but finite, so as to include all e.m. sources and the e.m. field itself. Such a closed volume, called Γ, is in the general case not symmetric with respect to the reference system. We discuss this last assertion in the next subsection. Γ volume encloses an isolated system in which total momentum is conserved. Stating this for Γ it means that total momentum is an invariant; in other words (2) becomes a constant of motion and is no more time dependent.

The time derivative of total momentum (3) is, by using (4),

$$\begin{aligned}\frac{d\vec{P}_{\mathrm{TOT}}(t)}{dt} &= \int_\Gamma \left[\frac{\partial \vec{p}_{\mathrm{mech.}}}{\partial t} + \frac{\partial \vec{p}_{\mathrm{e.m.}}}{\partial t}\right] d\Gamma \\ &= \int_\Gamma \left[\rho\vec{E} + \vec{J} \times \vec{B} + \left(\frac{\partial \vec{D}}{\partial t} \times \vec{B} + \vec{D} \times \frac{\partial \vec{B}}{\partial t}\right)\right] d\Gamma.\end{aligned} \quad (5)$$

Inserting the Maxwell equations $\nabla \times \vec{H} = \vec{J} + \partial\vec{D}/\partial t$ and $\nabla \times \vec{E} = -\partial\vec{B}/\partial t$ in (5) and rearranging the terms we easily obtain

$$\begin{aligned}\frac{d\vec{P}_{\mathrm{TOT}}(t)}{dt} &= \int_\Gamma \left[\rho\vec{E} + \frac{1}{\mu}(\nabla \times B) \times \vec{B} - \frac{\partial \vec{D}}{\partial t} \times \vec{B} + \frac{\partial \vec{D}}{\partial t}\right. \\ &\qquad \left. \times \vec{B} - \frac{1}{\varepsilon}\vec{D} \times \left(\nabla \times \vec{D}\right)\right] d\Gamma \\ &= \int_\Gamma \left[\rho\vec{E} - \frac{1}{\mu}\vec{B} \times (\nabla \times B) - \frac{1}{\varepsilon}\vec{D} \times \left(\nabla \times \vec{D}\right)\right] d\Gamma \\ &= \int_\Gamma \left[\rho\vec{E} - \frac{1}{\mu}\vec{B}\left(\nabla \cdot \vec{B}\right) - \frac{1}{\varepsilon}\vec{D}\left(\nabla \cdot \vec{D}\right)\right] d\Gamma \\ &= -\frac{1}{\mu}\int_\Gamma \vec{B}(\vec{r}, t)\left(\nabla \cdot \vec{B}(\vec{r}, t)\right) d\Gamma.\end{aligned} \quad (6)$$

This last equality gives rise to the heralded result. Observing (6), if total momentum $\vec{P}_{\mathrm{TOT}}(t)$ is conserved then the following stationary condition holds: $d\vec{P}_{\mathrm{TOT}}(t)/dt = 0$; therefore the right-hand side of (6) must vanish, too. Such a term must vanish for "any" choice of the volume, so that it remains to adopt the cancellation property of multiplication applied to the function $\vec{B}(\vec{r}, t)(\nabla \cdot \vec{B}(\vec{r}, t))$. Since the magnetic induction field $\vec{B}$ is generally a nonnull function, this field must be solenoidal, that is, $\nabla \cdot \vec{B}(\vec{r}, t) = 0$. In the next sections we will show in some details that the just obtained conclusion regarding the divergenceless of vector $\vec{B}$ is not unique.

2.1. Symmetry Considerations of Integral's Domains. Some simple considerations about symmetry properties of the integral domains Γ in (2)–(4) deserve to be better investigated. To this end let us consider the system of massive charged particles, described in the previous section, symbolically depicted in Figure 1 as striped (positives) and black (negatives) dots. In Figure 1 Center of Mass (CM) of particles is symbolically shown. As discussed in advance, if Γ volume is large enough the system enclosed in $\partial\Gamma$ is isolated (no external forces). For this motivation CM will be in a state of inertial motion with respect to any fixed reference system R.

Since the involved particle's velocities are nonrelativistics, CM could have a nonrelativistic (or zero), constant velocity

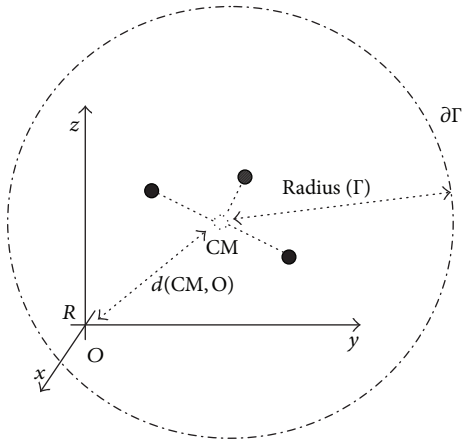

Figure 1: Region Γ which encloses an isolated system is represented. Striped and black dots symbolically represent positive and negative massive point-like charges. Center of Mass (CM) is shown together with a fixed reference system R and its origin O. Distance between origin O and CM is reported as d(CM,O).

with respect to the origin O of reference system R. Since velocity of CM v_{CM} is nonrelativistic, $v_{\mathrm{CM}}/c \ll 1$.

If Δt is a suitable observation time of the process under consideration, the closed system hypothesis entails that the following condition $Radius(\Gamma) > c\Delta t$ must hold. Depending on how large the observation time Δt is, $Radius(\Gamma)$ has to be large, too. With reference to Figure 1 we can imagine the Γ region (symmetrization principle) as given by two contributions: symmetric part Γ_{S} (with respect to the origin O) and antisymmetric part Γ_{AS}. In the special case in which the distance between CM and the origin O is such that $d(\mathrm{CM}, \mathrm{O}) \ll Radius(\Gamma)$ it happens that $\Gamma_{\mathrm{AS}} \to 0$, and in this case the region Γ is essentially totally symmetric with respect to the origin O, that is, $\Gamma \simeq \Gamma_{\mathrm{S}}$. But since (2)–(5) must be valid for "any" closed region Γ, they must still be valid even if $d(\mathrm{CM}, \mathrm{O}) \sim Radius(\Gamma)$, that is, if the distance between CM and the origin O is of the same order of magnitude of $Radius(\Gamma)$. In this hypothesis neither Γ_{S} nor Γ_{AS} can be considered as a negligible contribution to the volume Γ. Summarizing, Γ region does not have any particular "a priori" given symmetry with respect to the assigned reference system R.

3. Divergenceless Property of Magnetic Induction Field

In Section 2 we show that since the system under consideration is isolated, the magnetic induction field $\vec{B}$ must be solenoidal. By reversing this proposition, "ceteris paribus," if $\nabla \cdot \vec{B}(\vec{r}, t) \neq 0$ necessarily entails that $d\vec{P}_{\mathrm{TOT}}(t)/dt \neq 0$. This result shows that, *in general,* the existence of magnetic monopoles infringes the conservation principle of the total momentum for an isolated system. We used italics in writing *in general* because the existence of magnetic monopoles (i.e., $\nabla \cdot \vec{B}(\vec{r}, t) \neq 0$) can still remain consistent with the conservation principle of total momentum in our hypothesis (i.e., $d\vec{P}_{\mathrm{TOT}}(t)/dt = 0$) provided that particular symmetry conditions on $\vec{B}(\vec{r}, t)$ field and over the Γ region hold. In fact, with the exception of the trivial case $\vec{B}(\vec{r}, t) = 0$, the second term in (5) can vanish despite $\vec{B}(\vec{r}, t) \neq 0$ and $\nabla \cdot \vec{B}(\vec{r}, t) \neq 0$, but the integration domain Γ must be completely symmetric with respect to the origin O of the reference system (see Figure 1), that is, $\Gamma \simeq \Gamma_{\mathrm{S}}$, and the field $\vec{B}(\vec{r}, t)$ must also possess a well-defined parity. In this case, as a matter of fact, the divergence $\nabla \cdot \vec{B}(\vec{r}, t)$ also has a well-defined parity (converse to the parity of $\vec{B}(\vec{r}, t)$) in such a way that the product $\vec{B}(\vec{r}, t)(\nabla \cdot \vec{B}(\vec{r}, t))$ has an odd parity. So, if the right-hand side of (5) consists in an integral of an odd function over a symmetrical domain, it vanishes notwithstanding the field $\vec{B}(\vec{r}, t)$ and its divergence is different from zero.

The mechanical counterpart of momentum density, as obtained from Lorentz equation (1), together with the electromagnetic component of the total momentum, forms a constant of motion in two cases:

(a) $\nabla \cdot \vec{B}(\vec{r}, t) = 0$ or

(b) $\nabla \cdot \vec{B}(\vec{r}, t) \neq 0$ with $\vec{B}(\vec{r}, t)$ having a well-defined parity, and the observation domain Γ is symmetric.

The first case (a) is an expected result since the expression of the used Lorentz equation requires this as an implicit assumption. It is just considering the former case (a) that we do the assertion whereby the necessary condition for the divergenceless property of the magnetic induction field in the whole space directly comes from the conservation of total momentum for the total system (particles and field). The latter case (b) allowed us to extend the validity of the presented conservation scheme to the larger case of existence of magnetic monopoles (with appropriate restrictions on the observation domain). However, the rigorously converse proof, that is, to assert that the condition $\nabla \cdot \vec{B}(\vec{r}, t) \neq 0$, where $\vec{B}(\vec{r}, t)$ has a well-defined parity (that it means to make as an assumption the existence of magnetic monopoles), entails the conservation of total momentum it was not explicitly showed in detail in this work since it is only a particular consequence of a more general conservation property of the electromagnetic energy-momentum tensor valid in the presence of magnetic sources [14]. In fact, if we postulate the existence of magnetic monopoles ρ_m (and currents $\vec{J}_m$), Lorentz equation must be symmetrized (adding the terms that include magnetic forces $[\rho_m H(\vec{r}, t) - \vec{J}_m(r, t) \times \vec{D}(\vec{r}, t)]$), but the total momentum is still a conserved quantity. This is true without any restriction on the magnetic induction field $\vec{B}$; therefore it is still valid for the particular case in which the field $\vec{B}$ has a well-defined parity. In [14] a simple demonstration of the total momentum conservation in the special case of existence of magnetic sources as an initial assumption can be found. Here we emphasize again that in the present treatise we built a theory without postulating the existence of magnetic monopoles, that is, "how if" they are absent or their effect is smaller with respect to the one of the electric sources.

Before summarizing all the novelties of the presented interpretation about the solenoidality of $\vec{B}$ field, the entire

logical structure of this study deserves to be briefly tackled: in order to obtain the conservation of total momentum (6) we used the expression of the Lorentz force valid in the case of only electric sources $\vec{f}_e$ (see (1)). From a physical standpoint this is true only if the magnetic sources affect the dynamics in a weak way (with respect to the electric sources), as formally showed in the appendix, which is precisely the aim of the present paper. This is not a restriction because up to now magnetic monopoles have never been observed. It is exactly in this framework that we propose a possible way of observing magnetic monopoles, only by using nonrelativistic and classical particles and fields system.

4. Conclusions

Therefore, the possibility of having nonnull divergence of the magnetic induction field (and consequently the existence of the magnetic monopoles) can be deduced starting from first principles and applying just the classical electrodynamics. So far, the hypothesis of magnetic monopoles was born and has been discussed only within the quantum-relativistic physics. In fact, many experiments have been dedicated to the discovery of such monopoles with techniques inspired from string theory, to particle physics, until to astrophysics [6, 7]. According to the authors, the necessary condition for the observability of these monopoles, which is the homogeneous distribution of sources, could collapse at limit in those conditions of singularity assumed by the cosmological theory in the instants just before the Big Bang. A conceptual experiment could be proposed tending to recover those original conditions.

Concluding, the particular parity conditions discussed previously regarding the field $\vec{B}(\vec{r},t)$ and the domain Γ are true from a mathematician's point of view, since the integral in (2) is null for an even or an odd field defined in the symmetric domain Γ; anyway it remains indeterminate from a physician's view point, having still to prove, if the solution deduced as true from mathematics must be rejected as absurd or may have also a real meaning in physics. Finally it leaves open the question even for a philosopher of nature and science. Indeed, assuming that the existence of magnetic monopole would be experimentally verified, it would remain to justify theoretically, but even logically and so philosophically, the reasonableness of the natural subsistence only in preferential symmetries, including the radial one, for that magnetic induction field.

Appendix

A. Electromagnetic Momentum Density

In what follows we show that the expression of the electromagnetic momentum

$$\vec{p}_{\text{e.m.}} = \vec{D} \times \vec{B} \tag{A.1}$$

is independent from the assumption of existence/absence of magnetic monopoles.

In order to demonstrate this assertion we consider the simplified deduction of the electromagnetic momentum density as presented in [15], considering the ordinary case of absence of magnetic monopoles and the special case of a postulated existence of magnetic monopoles.

A.1. Ordinary Electrodynamics (Absence of Magnetic Monopoles). In presence of electric sources and currents, the Maxwell equations are

$$\nabla \cdot \vec{D} = \rho, \tag{A.2a}$$

$$\nabla \cdot \vec{B} = 0, \tag{A.2b}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{A.2c}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \tag{A.2d}$$

The microscopic Lorentz force density in this case is (subscript "e" stands for "electric-only" sources) $\vec{f}_e$, in an infinitesimal volume $d\Gamma$, and can be written as

$$\vec{f}_e = \rho\vec{E} + \vec{J} \times \vec{B}. \tag{A.3}$$

The evaluation of the e.m. momentum density can be easily done considering field sources generated only by the field itself [15]; by using (A.2a) and (A.2c) with some simple algebra we obtain

$$\vec{f}_e = \vec{E}\left(\nabla\vec{D}\right) + \left(\nabla \times \vec{H}\right) \times \vec{B} + \left(\nabla \times \vec{E}\right) \times \vec{D} - \frac{\partial}{\partial t}\left(\vec{D} \times \vec{B}\right). \tag{A.4}$$

If we add $\vec{H}(\nabla\vec{B})$ to the right-hand side of (A.4), (A.2b) holds, and it is like adding nothing. But with this simple trick, as (A.4) must be integrated (for obtaining the macroscopic Lorentz equation), the terms $\vec{E}(\nabla\vec{D}) + (\nabla \times \vec{H}) \times \vec{B} + (\nabla \times \vec{E}) \times \vec{D} + \vec{H}(\nabla\vec{B})$ can be neglected because they are a divergence that gives no contribution to the integral [15]. The only remaining term $-\partial t(\vec{D} \times \vec{B})$ is essential. Since $\vec{f}_e = (\partial/\partial t)\vec{p}_{\text{e.m.}}$, (A.1) is proved.

A.2. Special Electrodynamics Containing Magnetic Sources. If we postulate the existence of magnetic monopoles (ρ_m) and currents ($\vec{J}_m$) together with the electric sources terms ρ and $\vec{J}$, the Maxwell equations are

$$\nabla \cdot \vec{D} = \rho, \tag{A.5a}$$

$$\nabla \cdot \vec{B} = \rho_m, \tag{A.5b}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \tag{A.5c}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{J}_m. \tag{A.5d}$$

The microscopic Lorentz force in this case can be written as [13, 14]

$$f = \rho\vec{E} + \vec{J} \times \vec{B} + \rho_m\vec{H} - \vec{J}_m \times \vec{D}. \tag{A.6}$$

By using (A.5a), (A.5b), (A.5c), and (A.5d) we obtain

$$\begin{aligned}\vec{f} = {} & \vec{E}\left(\nabla\vec{D}\right) + \left(\nabla \times \vec{H}\right) \times \vec{B} + \left(\nabla \times \vec{E}\right) \\ & \times \vec{D} + \vec{H}\left(\nabla\vec{B}\right) - \frac{\partial}{\partial t}\left(\vec{D} \times \vec{B}\right).\end{aligned} \tag{A.7}$$

In this case we have to add nothing in order to obtain a total divergence term in (A.7). The first four terms in right-hand side of (A.7) are not essential (because their integral contributions vanish), so we proved that (A.1) is still valid in the presence of magnetic monopoles. In (A.6) the existence of the only magnetic sources (without considering electric sources terms) is not relevant because the associated Lorentz force expression can always be recasted as in (A.3) by a Charge Rotation [14].

A.3. Links between the Two Expressions of Lorentz Forces. From a mathematical point of view we can write, from (A.3) and (A.6),

$$\lim_{\substack{\rho_m \to 0 \\ \vec{J}_m \to 0_v}} \vec{f} = \vec{f}_e, \tag{A.8}$$

that is, in the limit of absence of magnetic sources the Lorentz force obviously coincides with $\vec{f}_e$, given by (A.3). From a physical standpoint this condition can be explicated as

$$\text{if } \frac{\rho_m}{\rho} \ll 1 \text{ and } \frac{\left|\vec{J}_m\right|}{\left|\vec{J}\right|} \ll 1 \text{ then } \vec{f} \simeq \vec{f}_e, \tag{A.9}$$

which means that we can still consider $\vec{f}_e$ as the Lorentz force in the case $\nabla \cdot \vec{B} \neq 0$, but the magnetic sources must have smaller effects on the dynamics with respect to the electric sources.

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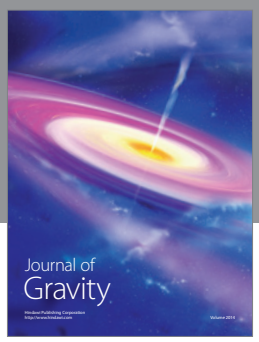

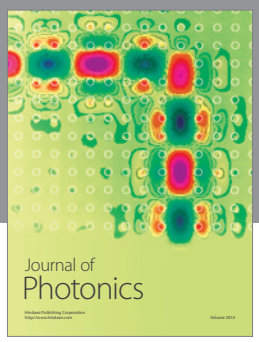

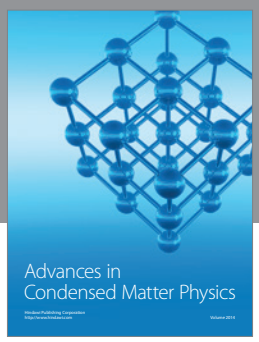

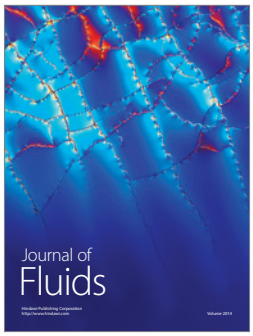

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