Research Article

Acceptance Sampling Plans from Life Tests Based on Percentiles of Half Normal Distribution

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The design of acceptance sampling plans is developed under truncated life testing based on the percentiles of half normal distribution. The minimum sample size necessary to ensure the specified life percentile is obtained under a given consumer's risk. The operating characteristic values of the sampling plans as well as the producer's risk are presented. The results are illustrated by examples.

1. Introduction

Acceptance sampling is concerned with inspection and decision making regarding lots of products and constitutes one of the oldest techniques in quality control. If the lifetime of the product represents the quality characteristics of interest, the acceptance sampling is as follows: a company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot and the relevant quality characteristic of the units in the sample is inspected. On the basis of the information in the sample, a decision is made regarding lot disposition. Traditionally, when the life test indicates that the mean life of products exceeds the specified one, the lot of products is accepted, otherwise it is rejected. Accepted lots are put into production, while rejected lots may be returned to the vendor or may be subjected to some other lot disposition actions. For the purpose of reducing the test time and cost, a truncated life test may be conducted to determine the sample size to ensure a certain mean life of products when the life test is terminated at a time t_0 , and the number of failures does not exceed a given acceptance number c .

A common practice in life testing is to terminate the life test by a predetermined time t_0 and note the number of failures. One of the objectives of these experiments is to set a lower confidence limit on the mean life. It is then to establish

a specified mean life with a given probability of at least p^* which provides protection to consumers. The test may be terminated before the time is reached or when the number of failures exceeds the acceptance number c in which case the decision is to reject the lot.

Studies regarding truncated life tests can be found in Epstein [1], Sobel and Tischendorf [2], Goode and Kao [3], Gupta and Groll [4], Gupta [5], Fertig and Mann [6], Kantam and Rosaiah [7], Baklizi [8], Wu and Tsai [9], Rosaiah and Kantam [10], Rosaiah et al. [11], Tsai and Wu [12], Balakrishnan et al. [13], Srinivasa Rao et al. [14], Srinivasa Rao et al. [15], Aslam et al. [16], and Srinivasa Rao et al. [17]. All these authors designed acceptance sampling plans based on the mean life time under a truncated life test. In contrast, Lio et al. [18] considered acceptance sampling plans for percentiles using truncated life tests and assuming Birnbaum-Saunders distribution. Srinivasa Rao and Kantam [19] developed similar plans for the percentiles of log-logistic distribution.

Normal distribution is the most preferred distribution in statistical studies. But for life test models it is not suitable distribution because of its domain $[-\infty, +\infty]$. Half normal distribution is an increasing failure rate (IFR) model which is most useful in reliability studies. Because of this IFR nature with domain $[0, +\infty]$ we are motivated to study this distribution.

In this paper, acceptance sampling plans are developed for percentiles of half normal distribution life test and are given in Section 2. Operating characteristic (OC) and producer's risk are given in Section 3. Examples based on real fatigue life data sets are provided for an illustration in Section 4. The paper is closed with summary and conclusions in Section 5.

2. Acceptance Sampling Plans

The probability density function (pdf) of a half normal distribution is given by

$$p(x) = \frac{2\theta}{\pi} e^{-x^2\theta^2/\pi}, \quad x \geq 0. \quad (1)$$

Its cumulative distribution function (cdf) is

$$P(x) = \text{erf}\left(\frac{\theta x}{\sqrt{\pi}}\right), \quad x \geq 0, \quad (2)$$

where erf is an error function and θ is a parameter. Assume that the life time of a product follows half normal distribution with σ as scale parameter. Its cumulative distribution function $F(\cdot)$ is given by

$$F(t) = \text{erf}\left(\frac{\theta(t/\sigma)}{\sqrt{\pi}}\right), \quad t \geq 0, \quad \sigma > 0. \quad (3)$$

Given $0 < q < 1$, the 100th percentile is given by

$$t_q = \sigma \frac{\sqrt{\pi}}{\theta} \text{erf}^{-1}(q). \quad (4)$$

Substituting σ in (3) in the scaled form we get

$$F(t) = \text{erf}\left(\frac{\theta(t/t_q)(\sqrt{\pi}/\theta) \text{erf}^{-1}(q)}{\sqrt{\pi}}\right) \quad (5)$$

$$\Rightarrow \text{erf}(\delta_0 \text{erf}^{-1}(q)),$$

where $\delta_0 = t/t_q$. Here erf^{-1} is simulated by

$$\text{erf}^{-1}(z) = \frac{1}{2} \sqrt{\pi} \left[z + \frac{\pi}{12} z^3 + \frac{7\pi^2}{480} z^5 + \frac{127\pi^3}{40320} z^7 + \frac{4369\pi^4}{5806080} z^9 \dots \right]. \quad (6)$$

A common practice in life testing is to terminate the life test by a predetermined time t , to require the probability of rejecting a bad lot to be at least p^* , and to have the maximum permissible number of bad items to accept the lot equal to c . The acceptance sampling plan for percentiles under a truncated life test is to set up the minimum sample size n for a given acceptance number c such that the consumer's risk, the probability of accepting a bad lot, does not exceed $1 - p^*$. A bad lot means that the true 100th percentile, t_q , is below a specified percentile, t_q^0 . Thus the probability p^* is a confidence level in the sense that the chance of rejecting a bad lot with

$t_q < t_q^0$ is at least equal to p^* . Therefore, for given p^* , the proposed acceptance sampling plan can be characterized by the triplet $(n, c, \delta_0) = (n, c, t/t_q^0)$, where $\delta = t/t_q^0$.

We consider large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* , t_q^0 , and c the smallest positive integer n required to assert that $t_q > t_q^0$ must satisfy the relation

$$\sum_{i=0}^c p_0^i (1 - p_0)^{n-i} \leq 1 - p^*, \quad (7)$$

where $p_0 = F_t(\delta_0)$ is the probability of a failure during the time $t = \delta t_q^0$ for the specified 100qth percentile of lifetime t_q^0 . The value p_0 depends only on $\delta_0 = t/t_q^0$. Since $\partial F_t(\delta)/\partial \delta > 0$, it is an increasing function of δ . Accordingly, we have

$$F_t(\delta) \leq F_t(\delta_0) \iff \delta \leq \delta_0 \quad (8)$$

or equivalently

$$F(t; t_q) < F(t; t_q^0) \iff t_q \geq t_q^0. \quad (9)$$

The smallest sample size n satisfying (9) can be obtained for any given q , t/t_q^0 , or p^* . In contrast, only the input values t/σ , p^* are needed to calculate the smallest sample size n . In particular, if we take $q = 0.50$, the value of n would become "the smallest sample size required to test that the population median life exceeds a given specified value." Half normal distribution is a skewed distribution; for our present skewed population the median is a more appropriate average for decision making about the quality of the life than population mean. Thus we may conclude that population median based sampling plans of half normal distribution model are more economical than those based on population mean with respect to sample size. To save space, only the results of small sample sizes for $q = 0.50$; $p^* = 0.75, 0.90, 0.95, 0.99$; $c = 0(1)10$; $\delta = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0$ are displayed in Table 1.

3. Operating Characteristic of the Sampling Plan and Producer's Risk

The operating characteristic (OC) function L of the sampling plan $(n, c, t/t_q^0)$ is the probability of accepting a lot as a function of $p = F_t(\delta)$ with $\delta_0 = t/t_q$. It is given as

$$L(p) = \sum_{i=0}^c \binom{n}{i} p^i (1 - p)^{n-i}. \quad (10)$$

Therefore, we have

$$p = F_t(\delta) = F\left(\frac{t}{t_q^0} \frac{1}{d_q}\right), \quad (11)$$

where $d_q = t_q/t_q^0$. Using (10) the OC values can be obtained for any sampling plan $(n, c, t/t_{0.5}^0)$ (Table 5). To save space we

TABLE 1: Minimum sample sizes necessary to assert the median life of a product.

p^*	c	$t/t_{0.5}^0$									
		0.1	0.2	0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
0.75	0	26	13	6	4	3	3	2	2	2	1
0.75	1	50	25	12	8	6	5	3	3	2	2
0.75	2	72	36	18	12	9	7	5	4	4	3
0.75	3	94	47	23	16	12	10	7	5	5	4
0.75	4	116	58	29	19	15	12	8	7	6	5
0.75	5	137	68	34	23	17	14	10	8	7	6
0.75	6	158	79	39	26	20	16	11	9	8	8
0.75	7	179	89	45	30	23	18	13	11	9	9
0.75	8	200	100	50	33	25	21	14	12	10	10
0.75	9	220	110	55	37	28	23	16	13	12	11
0.75	10	241	120	60	40	31	25	18	14	13	12
0.90	0	42	21	10	7	5	4	2	2	2	2
0.90	1	71	35	17	11	8	7	4	3	3	2
0.90	2	98	48	24	16	12	9	6	5	4	4
0.90	3	123	61	30	20	15	12	8	6	5	5
0.90	4	147	73	36	24	18	14	10	8	6	6
0.90	5	171	85	42	28	21	17	11	9	8	7
0.90	6	194	96	48	32	24	19	13	10	9	8
0.90	7	217	108	53	35	27	21	15	12	10	9
0.90	8	240	119	59	39	29	24	16	13	11	10
0.90	9	262	130	65	43	32	26	18	14	12	11
0.90	10	284	141	70	47	35	28	20	16	14	12
0.95	0	55	27	13	8	6	5	3	2	2	2
0.95	1	87	43	21	14	10	8	5	4	3	3
0.95	2	115	57	28	18	13	11	7	5	4	4
0.95	3	142	70	34	23	17	13	9	7	6	5
0.95	4	168	83	41	27	20	16	11	8	7	6
0.95	5	193	96	47	31	23	18	12	10	8	7
0.95	6	218	108	53	35	26	21	14	11	9	8
0.95	7	242	120	59	39	29	23	16	12	11	10
0.95	8	266	132	65	43	32	26	18	14	12	11
0.95	9	289	143	71	47	35	28	19	15	13	12
0.95	10	312	155	77	51	38	30	21	16	14	13
0.99	0	84	41	20	13	9	7	4	3	2	2
0.99	1	121	59	29	19	14	11	7	5	4	3
0.99	2	154	76	37	24	17	14	9	7	5	5
0.99	3	184	91	44	29	21	17	11	8	7	6
0.99	4	212	105	51	33	25	19	13	10	8	7
0.99	5	240	119	58	38	28	22	15	11	9	8
0.99	6	267	132	65	42	31	25	16	13	10	9
0.99	7	293	145	71	47	35	27	18	14	12	10
0.99	8	319	158	77	51	38	30	20	15	13	12
0.99	9	345	171	84	55	41	33	22	17	14	13
0.99	10	370	183	90	59	44	35	24	18	15	14

present the OC values for sampling plans with $q = 0.50$; $p^* = 0.75, 0.90, 0.95, 0.99$; $\delta = 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5, 3.0$; $t_{0.5}/t_{0.5}^0 = 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0$; for $c = 1$ and $c = 5$ which are given in Table 2 and Table 3, respectively.

The producer's risk is defined as the probability of rejecting the lot when $t_q > t_q^0$. For a given value of the producer's risk, say γ , we are interested in knowing the value of d_q to ensure that the producer's risk is less than or equal to γ if a sampling plan $(n, c, t/t_q)$ is developed at a specified confidence level p^* . Thus, one needs to find the smallest value d_q according to (10) as

$$\sum_{i=0}^c \binom{n}{i} p^i (1-p)^{n-i}, \quad (12)$$

where

$$p = F\left(\frac{t}{t_q^0} \frac{1}{d_q}\right). \quad (13)$$

To save space, based on the sampling plans $(n, c, t/t_q^0)$ given in Table 1 the minimum ratios of $d_{0.5}$ for the acceptability of a lot at the producer's risk of $\gamma = 0.05$ are presented in Table 4.

4. Illustrative Examples

In this section, we consider two examples with real data sets to illustrate the proposed acceptance sampling plans.

4.1. Example 1. The first data refers to software reliability presented by Wood [20]. The data set was reported in hours as 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. The confidence level is assumed for this acceptance sampling plan only if the lifetimes are from half normal distribution with shape parameter σ . So in order to apply this example for our tables we have to confirm the goodness of fit of half normal distribution to the data in the example. This is done through the well-known QQ-plot method, and the value we get is $R = 0.9287$. Hence we conclude that the half normal distribution provides a reasonable goodness of fit for the data.

Suppose that the experimenter would like to establish the unknown 50th percentile life time for the software mentioned above to be at least 300 h and the life test would be ended at 600 h which should have led to the ratio $t/t_{0.5}^0 = 2.0$. Thus, with $c = 1$ and $p^* = 0.75$, the experimenter may take from Table 1 the sample size n which must be at least 3; thus the sampling plan to the data is given by

$$\left(n, c, \frac{t}{t_{0.5}^0}\right) = (3, 1, 2.0). \quad (14)$$

Since there is one item with a failure time less than or equal to 600 h in the given sample of 9 observations, the lot is accepted as the result indicates that the 50th percentile life time $t_{0.5}$ is at least 300 h with a confidence level of $p^* = 0.75$.

The OC values for acceptance sampling plan in (14) and confidence $p^* = 0.75$, for a half normal distribution, may be taken from Table 2.

The producer's risk is almost equal to 0.3683 ($= 1 - 0.6317$), when the true percentile is greater than or equal to 2.0 times the specified 50th percentile.

TABLE 2: OC values of sampling plans.

$c = 1$		$t/t_{0.5}^0$									
p^*	n	$\delta_{0.01}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.75	50	0.1	0.2420	0.4605	0.6093	0.7075	0.7741	0.8207	0.8545	0.8796	0.8989
	25	0.2	0.2345	0.4576	0.6090	0.7084	0.7754	0.8221	0.8559	0.8810	0.9001
	12	0.4	0.2408	0.4764	0.6301	0.7281	0.7928	0.8373	0.8690	0.8924	0.9101
	8	0.6	0.2283	0.4759	0.6347	0.7342	0.7989	0.8430	0.8742	0.8970	0.9142
	6	0.8	0.2173	0.4788	0.6425	0.7427	0.8070	0.8502	0.8805	0.9026	0.9191
	5	1.0	0.1875	0.4609	0.6328	0.7373	0.8038	0.8482	0.8793	0.9018	0.9185
	3	1.5	0.2309	0.5616	0.7262	0.8143	0.8662	0.8992	0.9214	0.9370	0.9484
	3	2.0	0.0832	0.4276	0.6317	0.7464	0.8157	0.8602	0.8905	0.9120	0.9272
	2	2.5	0.1752	0.6334	0.7938	0.8680	0.9084	0.9327	0.9484	0.9593	0.9670
	2	3.0	0.0843	0.5920	0.7711	0.8535	0.8983	0.9253	0.9428	0.9548	0.9634
0.90	71	0.1	0.0993	0.2723	0.4274	0.5466	0.6354	0.7019	0.7524	0.7915	0.8221
	35	0.2	0.0980	0.2752	0.4332	0.5534	0.6424	0.7085	0.7586	0.7971	0.8272
	17	0.4	0.0961	0.2833	0.4473	0.5695	0.6583	0.7235	0.7723	0.8095	0.8384
	11	0.6	0.0953	0.2950	0.4654	0.5891	0.6772	0.7409	0.7879	0.8235	0.8509
	8	0.8	0.0958	0.3109	0.4880	0.6124	0.6989	0.7604	0.8053	0.8388	0.8645
	7	1.0	0.0625	0.2634	0.4449	0.5767	0.6698	0.7365	0.7854	0.8221	0.8503
	4	1.5	0.0928	0.3766	0.5734	0.6949	0.7725	0.8243	0.8605	0.8866	0.9061
	3	2.0	0.0832	0.4276	0.6317	0.7464	0.8157	0.8602	0.8905	0.9120	0.9277
	3	2.5	0.0237	0.3441	0.5687	0.7000	0.7805	0.8329	0.8688	0.8942	0.9130
	2	3.0	0.0843	0.5930	0.7711	0.8535	0.8983	0.9253	0.9428	0.9548	0.9634
0.95	87	0.1	0.0484	0.1766	0.3176	0.4390	0.5362	0.6127	0.6730	0.7209	0.7594
	43	0.2	0.0468	0.1773	0.3210	0.4440	0.5420	0.6186	0.6788	0.7264	0.7645
	21	0.4	0.0441	0.1804	0.3302	0.4566	0.5558	0.6325	0.6920	0.7388	0.7760
	14	0.6	0.0377	0.1753	0.3298	0.4595	0.5605	0.6380	0.6978	0.7445	0.7815
	10	0.8	0.0404	0.1948	0.3602	0.4932	0.5935	0.6688	0.7259	0.7699	0.8043
	8	1.0	0.0352	0.1951	0.3671	0.5033	0.6047	0.6798	0.7363	0.7795	0.8131
	5	1.5	0.0354	0.2431	0.4397	0.5795	0.6761	0.7441	0.7933	0.8297	0.8575
	4	2.0	0.0194	0.2436	0.4558	0.6002	0.6969	0.7633	0.8105	0.8450	0.8710
	3	2.5	0.0237	0.3441	0.5687	0.7000	0.7805	0.8329	0.8688	0.8942	0.9130
	3	3.0	0.0054	0.2983	0.5323	0.6726	0.7597	0.8166	0.8557	0.8836	0.9041
0.99	121	0.1	0.0098	0.0662	0.1603	0.2633	0.3593	0.4433	0.5148	0.5749	0.6256
	59	0.2	0.0100	0.0695	0.1679	0.2740	0.3718	0.4565	0.5280	0.5878	0.6378
	29	0.4	0.0086	0.0688	0.1709	0.2807	0.3811	0.4672	0.5392	0.5991	0.6489
	19	0.6	0.0075	0.0694	0.1766	0.2908	0.3938	0.4812	0.5536	0.6133	0.6626
	14	0.8	0.0066	0.0714	0.1851	0.3044	0.4103	0.4987	0.5712	0.6303	0.6788
	11	1.0	0.0059	0.0751	0.1971	0.3221	0.4307	0.5199	0.5919	0.6501	0.6974
	7	1.5	0.0047	0.0942	0.2439	0.3841	0.4975	0.5859	0.6446	0.7083	0.7509
	5	2.0	0.0042	0.1328	0.3177	0.4694	0.5819	0.6644	0.7257	0.7721	0.8079
	4	2.5	0.0029	0.1730	0.3843	0.5395	0.6469	0.7222	0.7764	0.8164	0.8467
	3	3.0	0.0054	0.2983	0.5323	0.6726	0.7597	0.8166	0.8557	0.8836	0.9041

From Table 4, the experimenter could get the values of $d_{0.5}$ for different choices of c and $t/t_{0.5}^0$ in order to assert that the producer's risk is less than 0.05. In this example, the value of $d_{0.5}$ should be 7.9120 for $c = 1$, $t_{0.5}/t_{0.5}^0 = 2.0$, and $p^* = 0.75$. This means that the product can have a life of 7.9120 times the required 50th percentile life time in order that under the above acceptance sampling plan the product is accepted with probability at least 0.75.

4.2. Example 2. The second data refers to the data obtained from Aarset [21]. It represents the lifetimes of 50 devices in hours. The data set was reported in hours as 0.1, 0.2, 1, 1, 1, 1, 1, 2, 3, 6, 7, 11, 12, 18, 18, 18, 18, 18, 21, 32, 36, 40, 45, 46, 47, 50, 55, 60, 63, 63, 67, 67, 67, 67, 72, 75, 79, 82, 82, 83, 84, 84, 84, 85, 85, 85, 85, 86, and 86. On the similar lines of Example 1, for this data, the goodness of fit for 50 observations showed that $R = 0.9175$.

TABLE 3: OC values of sampling plans.

$c = 5$		$t/t_{0.5}^0$									
p^*	n	$\delta_{0.01}$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0.75	137	0.1	0.2484	0.6314	0.8348	0.9234	0.9623	0.9802	0.9891	0.9936	0.9961
	68	0.2	0.2493	0.6407	0.8427	0.9284	0.9652	0.9820	0.9901	0.9943	0.9966
	34	0.4	0.2408	0.6506	0.8533	0.9354	0.9694	0.9845	0.9916	0.9952	0.9972
	23	0.6	0.2228	0.6527	0.8596	0.9400	0.9723	0.9862	0.9926	0.9959	0.9976
	17	0.8	0.2359	0.6888	0.8833	0.9527	0.9790	0.9898	0.9947	0.9971	0.9983
	14	1.0	0.2120	0.6898	0.8883	0.9561	0.9809	0.9909	0.9953	0.9974	0.9985
	10	1.5	0.1711	0.7193	0.9118	0.9685	0.9871	0.9942	0.9971	0.9985	0.9991
	8	2.0	0.1556	0.7826	0.9428	0.9816	0.9930	0.9970	0.9986	0.9993	0.9996
	7	2.5	0.1297	0.8341	0.9625	0.9889	0.9960	0.9983	0.9992	0.9996	0.9998
	6	3.0	0.2323	0.9326	0.9880	0.9969	0.9989	0.9996	0.9998	0.9999	1.0000
0.90	171	0.1	0.0008	0.0143	0.0541	0.1154	0.1868	0.2595	0.3288	0.3925	0.4497
	85	0.2	0.0007	0.0137	0.0536	0.1156	0.1879	0.2615	0.3314	0.3955	0.4531
	42	0.4	0.0005	0.0129	0.0534	0.1173	0.1918	0.2673	0.3387	0.4038	0.4619
	28	0.6	0.0004	0.0117	0.0519	0.1169	0.1930	0.2701	0.3428	0.4087	0.4675
	21	0.8	0.0002	0.0108	0.0516	0.1185	0.1970	0.2760	0.3501	0.4170	0.4763
	17	1.0	0.0001	0.0096	0.0501	0.1182	0.1983	0.2789	0.3542	0.4219	0.4818
	11	1.5	0.0001	0.0120	0.0654	0.1499	0.2431	0.3321	0.4117	0.4810	0.5404
	9	2.0	0.0000	0.0093	0.0619	0.1492	0.2459	0.3377	0.4193	0.4898	0.5499
	8	2.5	0.0000	0.0078	0.0604	0.1503	0.2499	0.3439	0.4268	0.4980	0.5584
	7	3.0	0.0000	0.0109	0.0779	0.1823	0.2907	0.3885	0.4722	0.5422	0.6006
0.95	193	0.1	0.0003	0.0071	0.0328	0.0787	0.1376	0.2019	0.2661	0.3273	0.3841
	96	0.2	0.0002	0.0068	0.0324	0.0787	0.1382	0.2031	0.2680	0.3298	0.3869
	47	0.4	0.0002	0.0067	0.0335	0.0824	0.1449	0.2124	0.2793	0.3425	0.4006
	31	0.6	0.0001	0.0063	0.0339	0.0849	0.1499	0.2197	0.2884	0.3529	0.4117
	23	0.8	0.0001	0.0062	0.0352	0.0892	0.1575	0.2301	0.3008	0.3666	0.4262
	18	1.0	0.0001	0.0068	0.0395	0.0991	0.1728	0.2495	0.3228	0.3901	0.4503
	12	1.5	0.0000	0.0070	0.0462	0.1166	0.2004	0.2845	0.3624	0.4320	0.4930
	10	2.0	0.0000	0.0046	0.0399	0.1092	0.1938	0.2794	0.3590	0.4299	0.4920
	8	2.5	0.0000	0.0078	0.0604	0.1503	0.2499	0.3439	0.4268	0.4980	0.5584
	7	3.0	0.0000	0.0109	0.0779	0.1823	0.2907	0.3885	0.4722	0.5422	0.6006
0.99	240	0.1	0.0000	0.0015	0.0110	0.0339	0.0700	0.1153	0.1657	0.2177	0.2691
	119	0.2	0.0000	0.0015	0.0109	0.0343	0.0710	0.1172	0.1683	0.2211	0.2731
	58	0.4	0.0000	0.0015	0.0117	0.0369	0.0764	0.1254	0.1792	0.2341	0.2877
	38	0.6	0.0000	0.0015	0.0122	0.0393	0.0813	0.1331	0.0189	0.0246	0.3012
	28	0.8	0.0000	0.0015	0.0132	0.0428	0.0883	0.1435	0.2026	0.2617	0.3183
	22	1.0	0.0000	0.0016	0.0149	0.0480	0.0978	0.1571	0.2195	0.2810	0.3392
	15	1.5	0.0000	0.0014	0.0159	0.0535	0.1096	0.1750	0.2424	0.3075	0.3681
	11	2.0	0.0000	0.0023	0.0256	0.0793	0.1518	0.2299	0.3057	0.3756	0.4383
	9	2.5	0.0000	0.0034	0.0365	0.1055	0.1912	0.2781	0.3588	0.4307	0.4937
	8	3.0	0.0000	0.0045	0.0457	0.1256	0.2197	0.3115	0.3945	0.4668	0.5289

Suppose that we would like to establish the unknown 50th percentile life time for the devices to be at least 10 hours and the life test would be ended at 15 hrs which should have led to the ratio of $t_{0.5}/t_{0.5}^0 = 1.5$. Thus with $c = 5$, $p^* = 0.99$, the experimenter may take, from Table 1, the sample size n which must be at least 15. Thus the sampling plan to the given data is given by $(n, c, t/t_{0.5}^0) = (15, 5, 1.5)$. Since there are 13 items with a failure time less than or equal to 15 hours in the given sample of 50 observations, the lot is accepted as the

result indicates that the 50th percentile lifetime $t_{0.5}$ is at least 10 hours with a confidence level of $p^* = 0.99$.

5. Summary and Conclusions

This paper provides the minimum sample size required to decide upon accepting/rejecting a lot based on its specified 50th percentile (median) when the data follows half normal distribution. Assuming that the size of the given sample

TABLE 4: Minimum ratio for the acceptability of a lot with producer's risk 0.05.

p^*	c	0.1	0.2	0.4	0.6	0.8	1.0	1.5	2.0	2.5	3.0
0.75	0	27.2995	27.3264	25.2820	25.3354	25.3887	31.7358	31.8683	42.4911	53.1138	32.2612
0.75	1	7.5211	7.4707	7.0638	6.9551	6.8415	7.0279	5.9340	7.9120	5.9356	7.1227
0.75	2	4.6982	4.6579	4.5740	4.4857	4.3925	4.1606	4.2240	4.2572	5.3215	4.2191
0.75	3	3.6690	3.6354	3.4860	3.5693	3.4898	3.5650	3.5335	3.0410	3.8013	3.1989
0.75	4	3.1398	3.1106	3.0490	2.9280	3.0224	2.9465	2.7276	3.0537	3.0610	2.6281
0.75	5	2.7959	2.7492	2.6938	2.6752	2.5684	2.5816	2.5929	2.5693	2.6240	2.3656
0.75	6	2.5649	2.5411	2.4575	2.4019	2.4074	2.3411	2.2305	2.2542	2.3369	2.8043
0.75	7	2.3985	2.3630	2.3429	2.2910	2.2887	2.1705	2.1968	2.3400	2.1320	2.5584
0.75	8	2.2725	2.2520	2.2076	2.1353	2.1038	2.1610	1.9808	2.1336	1.9782	2.3739
0.75	9	2.1635	2.1442	2.1022	2.0753	2.0432	2.0464	1.9775	1.9748	2.1740	2.2298
0.75	10	2.0847	2.0577	2.0176	1.9725	1.9931	1.9550	1.9712	1.8485	2.0464	2.1137
0.90	0	44.0826	44.1095	42.0653	44.2167	42.1723	42.2257	31.8683	42.4911	53.1138	63.7366
0.90	1	10.7007	10.4994	10.0945	9.6860	9.2735	10.0728	8.2474	7.9120	9.8900	7.1227
0.90	2	6.4092	6.2376	6.1553	6.0698	5.9809	5.4907	5.2368	5.6320	5.3215	6.3857
0.90	3	4.8111	4.7384	4.5906	4.5183	4.4422	4.3623	4.1426	3.8877	3.8013	4.5615
0.90	4	3.9865	3.9303	3.8153	3.7515	3.6839	3.5015	3.5805	3.6368	3.0610	3.6731
0.90	5	3.4961	3.4497	3.3544	3.2966	3.2348	3.2105	2.9167	3.0187	3.2117	3.1488
0.90	6	3.1546	3.0983	3.0488	2.9954	2.9380	2.8428	2.7494	2.6193	2.8177	2.8043
0.90	7	2.9122	2.8770	2.7767	2.6995	2.7270	2.5857	2.6252	2.6377	2.5401	2.5584
0.90	8	2.7309	2.6877	2.6215	2.5512	2.4762	2.5130	2.3481	2.3904	2.3338	2.3739
0.90	9	2.5801	2.5413	2.5003	2.4353	2.3657	2.3516	2.2950	2.2008	2.1740	2.2298
0.90	10	2.4598	2.4243	2.3678	2.3422	2.2768	2.2240	2.2503	2.2469	2.3107	2.1137
0.95	0	57.7180	56.6969	54.6527	50.5105	50.5640	52.7154	47.6037	42.4911	53.1138	63.7366
0.95	1	13.1233	12.9222	12.5182	12.4144	11.7015	11.5919	10.5419	10.9966	9.8900	11.8680
0.95	2	7.5278	7.4222	7.2090	6.8610	6.5094	6.8151	6.2408	5.6320	5.3215	6.3857
0.95	3	5.5593	5.4474	5.2215	5.2290	5.0756	4.7596	4.7466	4.7113	4.8596	4.5615
0.95	4	4.5600	4.4766	4.3623	4.2449	4.1238	4.0540	4.0012	3.6368	3.8171	3.6731
0.95	5	3.9491	3.9029	3.7669	3.6688	3.5670	3.4192	3.2373	3.4572	3.2117	3.1488
0.95	6	3.5477	3.4916	3.3770	3.2917	3.2025	3.1754	3.0050	2.9739	2.8177	2.8043
0.95	7	3.2501	3.2015	3.1018	3.0256	2.9455	2.8609	2.8366	2.6377	2.9250	3.0482
0.95	8	3.0289	2.9859	2.8972	2.8279	2.7544	2.7464	2.7084	2.6410	2.6670	2.8005
0.95	9	2.8479	2.7993	2.7389	2.6749	2.6067	2.5540	2.4517	2.4208	2.4685	2.6088
0.95	10	2.7041	2.6687	2.6127	2.5529	2.4889	2.4024	2.3882	2.2469	2.3107	2.4557
0.99	0	88.1383	86.0673	84.0232	81.9790	75.7389	73.6945	63.3385	63.4716	53.1138	63.7366
0.99	1	18.2712	17.7675	17.3645	16.9596	16.5526	16.1433	15.1091	14.0558	13.7457	11.8680
0.99	2	10.0941	9.9229	9.5790	9.2329	8.6206	8.7969	8.2360	8.3211	7.0400	8.4480
0.99	3	7.2133	7.1017	6.7979	6.6493	6.3403	6.3445	5.9462	5.5234	5.8892	5.8316
0.99	4	5.7617	5.6786	5.4559	5.2305	5.2214	4.8799	4.8365	4.7740	4.5460	4.5805
0.99	5	4.9170	4.8504	4.6741	4.5363	4.3954	4.2512	4.1877	3.8890	3.7734	3.8540
0.99	6	4.3502	4.2779	4.16472	3.9820	3.8621	3.8379	3.5116	3.6659	3.2741	3.3812
0.99	7	3.9395	3.8776	3.7517	3.6770	3.5993	3.4087	3.2557	3.2162	3.2971	3.0482
0.99	8	3.6363	3.5820	3.4483	3.3804	3.3092	3.2113	3.0646	2.8874	2.9880	3.2004
0.99	9	3.4034	3.3549	3.2557	3.1533	3.0872	3.0576	2.9161	2.8494	2.7510	2.9622
0.99	10	3.2100	3.1574	3.0673	2.9738	2.9118	2.8460	2.7972	2.6279	2.5624	2.7713

TABLE 5: OC values of sampling plans for $n = 3$, $c = 1$, and $\delta_0 = 2.0$.

$t_{0.5}/t_{0.5}^0$	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
OC	0.0832	0.4276	0.6317	0.7464	0.8157	0.8602	0.8905	0.9120	0.9272

is minimum, we can use the tables in this paper to pick up acceptance number c and the upper bound in order to accept/reject the lot for a given probability of acceptance, say p^* . This paper also provides the sensitivity of the sampling plans in terms of the OC. We advise the industrial practitioner and the experimenter to adopt this plan in order to save the cost and time of the experiment. This plan can be further studied for many other distributions and various quality and reliability characteristics as future research.

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