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Research Article

Water-Filling Solution for Distributed Estimation of Correlated Data in WSN MIMO System

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We consider the distributed estimation of a random vector signal in a power constraint wireless sensor network (WSN) that follows a multiple-input and multiple-output (MIMO) coherent multiple access channel model. We design linear coding matrices based on linear minimum mean-square error (LMMSE) fusion rule that accommodates spatial correlated data. We obtain a closed-form solution that follows a water-filling strategy. We also derive a lower bound to this model. Simulation results show that when the data is more correlated, the distortion in terms of mean-square error (MSE) degrades. By taking into account the effects of correlation, observation, and channel matrices, the proposed method performs better than equal power method.

1. Introduction

The wireless sensor network (WSN) is a potential technology in many application areas including environmental monitoring, health, security and surveillance, and robotic exploration [1]. WSNs have many interested issues, one of them is distributed estimation. In the distributed estimation scenario, sensors observe phenomena from the target(s) and transmit to a fusion center (FC). Received signal at FC is estimated using an estimation technique.

Distributed estimation by considering power consumption has attracted much attention in [2–7]. Because the sensors deployed in a certain region are difficult to change the batteries, the low power consumption is important to guarantee a lifetime of the sensors. The distributed estimation is also applied on the orthogonal multiple access channel (MAC) model [4, 6, 8] and the coherent MAC model [2, 5] that considers single-input single-output (SISO). To save energy, multiple-input and multiple-output (MIMO) system has been analyzed by involving signaling overhead [9], in which the MIMO system can offer substantial energy savings in WSN. Cooperative MIMO with data aggregation also has been investigated in [10, 11].

In practical scenario, if the targets or the sensors are close to each other, the data will be potentially correlated. Such a problem has been investigated in [3, 6, 12, 13]. In this paper, we consider multiple targets that are spatially distributed. The targets are observed by multiple sensors that apply an analog forwarding scheme. This scheme will multiply the observed data with a designed coding matrix in each sensor, which results in encoded messages. The encoded messages from the sensors are transmitted to the FC over a coherent MAC. The channel follows MIMO model that has multiple antenna at transmitter and at receiver. At the FC, the received signals are estimated using linear minimum mean-square error (LMMSE) rule. To perform the estimation, we calculate a distortion in terms of mean-square error (MSE). We also consider the separation of the estimation under two conditions, that is, distortion due to noisy observation and distortion due to channel noise. Both of the distortions should be minimized by designing coding matrices under total power constraint. To derive the coding matrix, we use singular value decomposition (SVD) technique. We show that the equations can be formulated as a convex optimization problem. We derive a closed-form solution that can be solved using water-filling algorithm. The designed coding matrices

will be sent back to the sensors and will optimize transmit power of them. Under the above scenario, the proposed method will be compared to the equal power method. The equal power method allocates equal transmit power to each sensor. Compared to the existing literature, the contribution of this work lies in the following aspects: we consider MIMO model based on the water-filling algorithm and a spatial correlated data as an extension of [2, 5]. We obtain a closed-form solution for the optimization problem and give the coding matrices for each sensor. We also derive a lower bound of the MSE.

The rest of the paper is organized as follows. We review related work in Section 2. Section 3 describes the network setup under consideration. In Section 4, we formulate the linear coding matrix based on total power constraint and correlated sources. Section 5 presents some numerical simulation examples and the conclusion is drawn in Section 6.

Throughout this paper, we use the following notations. A lower case letter denotes a scalar, a boldface lower case letter denotes a vector, and a boldface upper case denotes matrix. The superscripts, $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^{-1}$, and $\mathbf{A}^{-H}$ denote the transpose, the Hermitian, the inverse, and the inverse Hermitian of matrix $\mathbf{A}$, respectively. The operator diag$(\cdot)$, manipulates the diagonal elements of matrix or a column vector into diagonal matrix and tr$(\cdot)$ is the trace of a matrix.

2. Related Work

In [2], they proposed a closed-form solution of coding matrices design in distributed estimation that follows coherent multiple access channel model. They exploited antenna diversity to improve received signal in low transmit power. However, they did not take into account the target correlation. Fang and Li studied power allocation problem in correlated sensor observation and provided lower bound of the objective function for single-input single-output (SISO) model [3]. A best linear unbiased estimator (BLUE) was used for estimating a target in multiple wireless sensor network [4], where the system performance was analyzed by the concept of estimation outage. The power allocation strategy was also employed in the multiple-input single-output (MISO) system.

In [5], Guo et al. investigated joint estimation of random vector and minimized the gap to the performance benchmark using water-filling strategy. They revealed that extra transmissions beyond the dimension of the target do not improve the estimation performance. An orthogonal multiple access channel (MAC) model for distributed estimation was studied in [6, 8]. To save energy, multiple-input and multiple-output (MIMO) system has been analyzed by involving signaling overhead [9], in which the MIMO system can offer substantial energy savings in WSN. Cooperative MIMO with data aggregation has also been investigated in [10, 11]. Compared to the existing literature, the contribution of this work lies in the following aspects: we considered MIMO model based on the water-filling algorithm and a spatial correlated data as an extension of [2, 5]. We obtained a closed-form solution for the optimization problem and gave the coding matrices for each sensor.

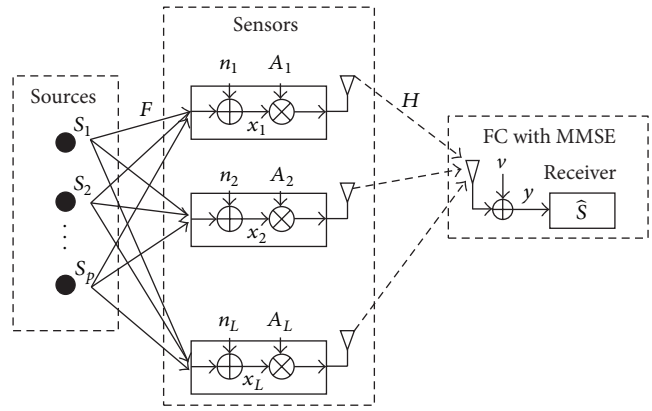

Figure 1: Distributed estimation over a wireless sensor network.

3. Problem Formulation

Assume that there are L sensors for estimating p random source signals, written in a vector form $\mathbf{s} = [s_1, \ldots, s_p]^T \in \mathbb{C}^p$, as shown in Figure 1. The sensors observe the sources through a sensing matrix $\mathbf{F} \in \mathbb{C}^{k\times p}$ and each sensor, lth, has k measurements given by

$$\mathbf{x}_l = \mathbf{F}_l\mathbf{s} + \mathbf{n}_l, \quad 1 \le l \le L, \tag{1}$$

where $\mathbf{n}_l \in \mathbb{C}^k$ is the additive noise. We assume that the targets are close to each other. They become potentially correlated. Therefore, the sensing matrix, $\mathbf{F}$, can be modeled as $\mathbf{F} = \mathbf{F}_o\mathbf{r}^{1/2}$, where $\mathbf{F}_o$ is a matrix with size $k \times p$ that has i.i.d zero mean circularly symmetric complex Gaussian (ZMCSCG) entries with unit variance and $\mathbf{r}$ is the $p\times p$ spatial correlation matrix. If we have L sensors, (1) can be written as

$$\mathbf{x} = \mathbf{F}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_L^T]^T \in \mathbb{C}^{Lk}$, $\mathbf{F} = [\mathbf{F}_1^T, \ldots, \mathbf{F}_L^T]^T \in \mathbb{C}^{Lk\times p}$, and $\mathbf{n} = [\mathbf{n}_1^T, \ldots, \mathbf{n}_L^T]^T \in \mathbb{C}^{Lk}$ with $Lk = \sum_{l=1}^{L} k_l$. The sensor observations are encoded using a linear coding matrix $\mathbf{A}_l \in \mathbb{C}^{N\times k}$, where N is the number of encoded messages transmitted from the lth sensor. The message vector is transmitted through channel matrix $\mathbf{H}_l \in \mathbb{C}^{N\times N}$ using N different frequencies. The received signal at FC can be written as

$$\begin{aligned}\mathbf{y} &= \sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}_l\left(\mathbf{F}_l\mathbf{s} + \mathbf{n}_l\right) + \mathbf{v}\\ &= \mathbf{B}\left(\mathbf{F}\mathbf{s} + \mathbf{n}\right) + \mathbf{v},\end{aligned} \tag{3}$$

where $\mathbf{B} = [\mathbf{B}_l, \ldots, \mathbf{B}_L] \in \mathbb{C}^{N\times Lk}$ with $\mathbf{B}_l = \mathbf{H}_l\mathbf{A}_l \in \mathbb{C}^{N\times k}$ and $\mathbf{v} \in \mathbb{C}^N$ is additive Gaussian noise. FC employs LMMSE estimator to estimate parameter $\mathbf{s}$ based on the received signal $\mathbf{y}$ in (3) [14]

$$\hat{\mathbf{s}} = E\left[\mathbf{s}\mathbf{y}^H\right]\left(E\left[\mathbf{y}\mathbf{y}^H\right]\right)^{-1}\mathbf{y}, \tag{4}$$

and the total distortion in terms of MSE can be expressed as

$$D_t = \operatorname{tr}\left(E\left[(\mathbf{s}-\hat{\mathbf{s}})(\mathbf{s}-\hat{\mathbf{s}})^H\right]\right). \tag{5}$$

We can view the total distortion as a summation of noise observation distortion, D_o, and channel noise distortion, D_c, as follows:

$$\begin{aligned} D_t &= E\left[\left(\bar{\bar{\mathbf{s}}}-\mathbf{s}\right)^2\right] \\ &= E\left[\left(\bar{\bar{\mathbf{s}}}-\bar{\mathbf{s}}+\bar{\mathbf{s}}-\mathbf{s}\right)^2\right] \\ &= \underbrace{E\left[\left(\bar{\bar{\mathbf{s}}}-\bar{\mathbf{s}}\right)^2\right]}_{:=D_c} + \underbrace{E\left[(\bar{\mathbf{s}}-\mathbf{s})^2\right]}_{:=D_o} + \underbrace{2E\left[\left(\bar{\bar{\mathbf{s}}}-\bar{\mathbf{s}}\right)(\bar{\mathbf{s}}-\mathbf{s})\right]}_{:=D_m}, \end{aligned} \tag{6}$$

where D_m characterizes the mutual term. The $\bar{\bar{\mathbf{s}}}$ and $\bar{\mathbf{s}}$ are estimated parameters due to channel noise and due to observation noise, respectively. Based on *Cauchy-Schwarz* inequality, we can have the upper bound distortion D_t as follows:

$$\begin{aligned} D_t &\le E\left[\left(\bar{\bar{\mathbf{s}}}-\bar{\mathbf{s}}\right)^2\right] + E\left[(\bar{\mathbf{s}}-\mathbf{s})^2\right] + 2\sqrt{E\left[\left(\bar{\bar{\mathbf{s}}}-\bar{\mathbf{s}}\right)^2(\bar{\mathbf{s}}-\mathbf{s})^2\right]} \\ &= \left(\sqrt{D_c}+\sqrt{D_o}\right)^2 \le 2\left(D_c+D_o\right) = 2\widetilde{D}_t. \end{aligned} \tag{7}$$

To obtain (7), we have used $(E[xy])^2 \le E[x^2]E[y^2]$ in the first inequality and $(\sum_{k=1}^{K} G_k)^2 \le K\sum_{k=1}^{K} G_k^2$ in the second inequality. Then, we have

$$\widetilde{D}_t = D_c + D_o. \tag{8}$$

We can calculate distortion due to noisy observation, D_o, by assuming that channel noise is free, $\mathbf{v} = 0$. Then, the received signal can be written as

$$\mathbf{y} = \sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}_l\mathbf{x}_l = \mathbf{B}(\mathbf{F}\mathbf{s}+\mathbf{n}). \tag{9}$$

Based on (9), we can derive D_o as

$$D_o = \operatorname{tr}\left(\mathbf{R}_s - \mathbf{R}_{sx}\mathbf{B}^H\left[\mathbf{B}\mathbf{R}_{xo}\mathbf{B}^H\right]^{-1}\mathbf{B}\mathbf{R}_{sx}^H\right), \tag{10}$$

where $\mathbf{R}_{xo} = \sigma_s^2\mathbf{F}\mathbf{r}\mathbf{F}^H + \mathbf{R}_{nl}$ with $\mathbf{r}$ being spatial correlation matrix. We assume covariance matrices of the targets, $\mathbf{R}_s = \sigma_s^2\mathbf{I}_p$, noise observation, $\mathbf{R}_{nl} = \sigma_n^2\mathbf{I}_k$, and channel noise, $\mathbf{R}_v = \sigma_v^2\mathbf{I}_N$. Introducing $\mathbf{R}_{sx} = \mathbf{r}^{1/2}\mathbf{R}_s\mathbf{F}^H$ is covariance between $\mathbf{s}$ and $\mathbf{x}$. Moreover, we can calculate distortion due to channel noise, D_c, by assuming that observation noise is free, $\mathbf{n}_l = 0$. Then, the received signal can be written as

$$\mathbf{y} = \sum_{l=1}^{L}\mathbf{H}_l\mathbf{A}_l\mathbf{x}_l + \mathbf{v} = \mathbf{B}(\mathbf{F}\mathbf{s}) + \mathbf{v}. \tag{11}$$

Based on (11), we can derive D_c as

$$D_c = \operatorname{tr}\left(\mathbf{R}_s - \mathbf{R}_{sx}\mathbf{B}^H\left[\mathbf{B}\mathbf{R}_{xc}\mathbf{B}^H + \mathbf{R}_v\right]^{-1}\mathbf{B}\mathbf{R}_{sx}^H\right), \tag{12}$$

where $\mathbf{R}_{xc} = \sigma_s^2\mathbf{F}\mathbf{r}\mathbf{F}^H$.

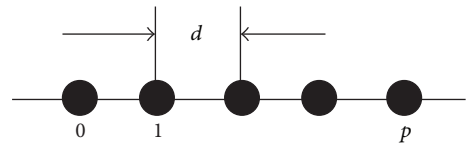

Figure 2: The targets are placed in line with spacing distance, d.

3.1. Correlated Sources. We assume that data observed by sensors are spatially correlated. The targets are placed in line with spacing distance, d, as shown in Figure 2 [13]. The model of correlation is as follows:

$$\begin{aligned} [\mathbf{r}]_{i,j} &= \rho^{|d_{ij}|}; \quad i,j \in 1,\ldots,p \\ &= \begin{bmatrix} \rho^{|d_{11}|} & \cdots & \rho^{|d_{1j}|} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \rho^{|d_{i1}|} & \cdots & \rho^{|d_{ij}|} \end{bmatrix}, \end{aligned} \tag{13}$$

where d_{ij} is a distance between the ith and the jth targets and ρ is the spatial correlation coefficient. In general, $\mathbf{r}$ is Hermitian and positive definite, so we can write $\mathbf{r} = \mathbf{r}^{1/2}\mathbf{r}^{1/2}$.

3.2. Objective Function. According to the total distortion, we need to minimize $\widetilde{D}_t$ under a total power constraint, P. The total transmit power for the L sensors is defined as

$$\sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{A}_l\mathbf{R}_{xl}\mathbf{A}_l^H\right) \le P, \tag{14}$$

where $\mathbf{R}_{xl} = \sigma_s^2\mathbf{F}_l\mathbf{r}\mathbf{F}_l^H + \sigma_n^2\mathbf{I}_k$.

Furthermore, based on (8) and (14), we have an objective function as follows:

$$\begin{aligned} &\min \quad \widetilde{D}_t \\ &\text{subject to} \quad \sum_{l=1}^{L}\operatorname{tr}\left(\mathbf{A}_l\mathbf{R}_{xl}\mathbf{A}_l^H\right) \le P. \end{aligned} \tag{15}$$

4. Proposed Approach

4.1. Proposed Method. In this section, we aim to solve the objective function in (15). First, we need to minimize it through SVD technique. Second, we formulate a convex optimization function by considering total power constraint. Afterwards, we further show that the optimal solution can be obtained by a water filling algorithm.

Let us introduce a lemma (cf. [15]).

Lemma 1. *For any two positive semidefinite matrices* $\mathbf{X}$ *and* $\mathbf{Y}$ *with size n, it holds that*

$$\operatorname{tr}(\mathbf{X}\mathbf{Y}) \le \sum_{i=1}^{n}\alpha_i\beta_i, \tag{16}$$

where α_i *and* β_i *are the ith eigenvalues of* $\mathbf{X}$ *and* $\mathbf{Y}$*, respectively, in an increasing order.*

Since $\mathbf{R}_{xo}$ and $\mathbf{R}_{xc}$ are positive definite, they can be written as $\mathbf{R}_{xo} = \mathbf{R}_{xo}^{1/2}\mathbf{R}_{xo}^{1/2}$ and $\mathbf{R}_{xc} = \mathbf{R}_{xc}^{1/2}\mathbf{R}_{xc}^{1/2}$, respectively. By performing SVD, we can write them as

$$\mathbf{R}_{xo}^{-1/2}\mathbf{F}\mathbf{F}^H\mathbf{R}_{xo}^{-1/2} = \mathbf{U}_f\Sigma\mathbf{f}\mathbf{U}_f^H, \tag{17}$$

$$\mathbf{R}_{xc}^{-1/2}\mathbf{F}\mathbf{F}^H\mathbf{R}_{xc}^{-1/2} = \mathbf{U}_g\Sigma\mathbf{g}\mathbf{U}_g^H, \tag{18}$$

where $\Sigma\mathbf{f} = \mathrm{diag}(f_1,\ldots,f_p)$ with $f_1 \geq f_2 \geq \cdots \geq f_p > 0$ and $\Sigma\mathbf{g} = \mathrm{diag}(g_1,\ldots,g_p)$ with $g_1 \geq g_2 \geq \cdots \geq g_p > 0$. $\mathbf{U}_f \in \mathbb{C}^{Lk\times Lk}$ and $\mathbf{U}_g \in \mathbb{C}^{Lk\times Lk}$ are unitary, respectively.

Let $\mathbf{B}\mathbf{R}_{xo}^{1/2}$ and $\mathbf{B}\mathbf{R}_{xc}^{1/2}$ be performed as SVD as follows:

$$\mathbf{T} = \mathbf{B}\mathbf{R}_{xo}^{1/2} = \mathbf{U}_t\Sigma\sqrt{\mathbf{t}}\mathbf{V}_t^H \tag{19}$$

$$\mathbf{W} = \mathbf{B}\mathbf{R}_{xc}^{1/2} = \mathbf{U}_w\Sigma\sqrt{\mathbf{w}}\mathbf{V}_w^H, \tag{20}$$

where $\mathbf{U}_t \in \mathbb{C}^{N\times N}$ and $\mathbf{U}_w \in \mathbb{C}^{N\times N}$ are unitary, respectively. Matrix $\Sigma\sqrt{\mathbf{t}} = \mathrm{diag}(\sqrt{t_1},\ldots,\sqrt{t_N})$ with $t_1 \geq \cdots \geq t_N > 0$, $\Sigma\sqrt{\mathbf{w}} = \mathrm{diag}(\sqrt{w_1},\ldots,\sqrt{w_N})$ with $w_1 \geq \cdots \geq w_N > 0$, $\mathbf{V}_t \in \mathbb{C}^{Lk\times N}$, and $\mathbf{V}_w \in \mathbb{C}^{Lk\times N}$ have orthonormal columns.

Based on (17) and (19), the distortion due to observation noise, D_o, can be reformed as

$$\begin{aligned} D_o &= \mathrm{tr}\Big(\sigma_s^2\mathbf{I}_p - \sigma_s^4\mathbf{R}_{xo}^{-1/2}\mathbf{F}\mathbf{F}^H\mathbf{R}_{xo}^{-1/2}\mathbf{B}^H\mathbf{R}_{xo}^{1/2} \\ &\quad\times\left[\mathbf{B}\mathbf{R}_{xo}^{1/2}\mathbf{B}^H\mathbf{R}_{xo}^{1/2}\right]^{-1}\mathbf{B}\mathbf{R}_{xo}^{1/2}\Big) \\ &= \mathrm{tr}\left(\sigma_s^2\mathbf{I}_p - \sigma_s^4\mathbf{U}_f\sum\mathbf{f}\mathbf{U}_f^H\mathbf{T}^H\left[\mathbf{T}\mathbf{T}^H\right]^{-1}\mathbf{T}\right). \end{aligned} \tag{21}$$

By using Lemma 1, we have

$$\begin{aligned} D_o &\leq p\sigma_s^2 - \sigma_s^4\sum_{i=1}^{p}\frac{f_ib_i}{b_i} \\ &\leq p\sigma_s^2 - \sigma_s^4\sum_{i=1}^{p}f_i. \end{aligned} \tag{22}$$

According to (18) and (20), the distortion due to channel noise, D_c, can be reformed as

$$\begin{aligned} D_c &= \mathrm{tr}\Big(\sigma_s^2\mathbf{I}_p - \sigma_s^4\mathbf{R}_{xc}^{-1/2}\mathbf{F}\mathbf{F}^H\mathbf{R}_{xc}^{-1/2}\mathbf{B}^H\mathbf{R}_{xc}^{1/2} \\ &\quad\times\left[\mathbf{B}\mathbf{R}_{xc}^{1/2}\mathbf{B}^H\mathbf{R}_{xc}^{1/2} + \mathbf{R}_v\right]^{-1}\mathbf{B}\mathbf{R}_{xc}^{1/2}\Big) \\ &= \mathrm{tr}\Big(\sigma_s^2\mathbf{I}_p - \sigma_s^4\mathbf{U}_g\sum\mathbf{g}\mathbf{U}_g^H\mathbf{W}^H \\ &\quad\times\left[\mathbf{W}\mathbf{W}^H + \sigma_v^2\mathbf{I}_N\right]^{-1}\mathbf{W}\Big). \end{aligned} \tag{23}$$

By using Lemma 1, we have

$$D_c \leq p\sigma_s^2 - \sigma_s^4\sum_{i=1}^{p}\frac{g_iw_i}{w_i+\sigma_v^2}. \tag{24}$$

We have $\widetilde{D}_t$ in terms of SVD as follows:

$$\begin{aligned} \widetilde{D}_t &\leq D_o + D_c \\ &\leq 2p\sigma_s^2 - \sigma_2^4\left(\sum_{i=1}^{p}f_i + \frac{g_iw_i}{w_i+\sigma_v^2}\right). \end{aligned} \tag{25}$$

For simplifying analysis, we set $\mathbf{V}_w^H\widehat{\mathbf{U}}_g = \left[\mathbf{I}_N \;\; 0\right]$, where $\widehat{\mathbf{U}}_g = \mathbf{U}_g(:,1:N)$. This is intended to keep D_c diagonal. When the number of transmitters is set minimum, $N = p$, we have

$$\mathbf{V}_w = \widehat{\mathbf{U}}_g; \quad \text{where } \widehat{\mathbf{U}}_g = \mathbf{U}_g(:,1:p), \tag{26}$$

where $\widehat{\mathbf{U}}_g$ is taken from the first p columns of matrix $\mathbf{U}_g$. Then, from (26), we can express (20) as

$$\begin{aligned} \mathbf{B}\mathbf{R}_{xc}^{1/2} &= \mathbf{U}_w\sum\sqrt{\mathbf{w}}\widehat{\mathbf{U}}_g^H \\ \mathbf{B} &= \mathbf{U}_w\sum\sqrt{\mathbf{w}}\underbrace{\widehat{\mathbf{U}}_g^H\mathbf{R}_{xc}^{-1/2}}_{\mathbf{U}_R}, \end{aligned} \tag{27}$$

where $\mathbf{U}_w \in \mathbb{C}^{N\times N}$ is unitary in (27); thus, we have $\mathbf{B} = \Sigma\sqrt{\mathbf{w}}\mathbf{U}_R$. Because of $\mathbf{B} = [\mathbf{B}_1,\ldots,\mathbf{B}_l]$, we can express $\mathbf{B}_l$ as

$$\mathbf{B}_l = \Sigma\sqrt{\mathbf{w}}\mathbf{U}_{Rl}, \tag{28}$$

where $\mathbf{U}_R = [\mathbf{U}_{R1},\ldots,\mathbf{U}_{RL}]$ with $\mathbf{U}_{Rl} \in \mathbb{C}^{p\times k}$.

Let us reform the total power constraint in terms of SVD. We have $\mathbf{A}_l = \mathbf{H}_l^{-1}\mathbf{B}_l$. We can rewrite (14) by taking into account (28) as follows:

$$\begin{aligned} &\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{H}_l^{-1}\mathbf{B}_l\mathbf{R}_{xl}\mathbf{B}_l^H\mathbf{H}_l^{-H}\right) \leq P \\ &\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{H}_l^{-1}\Sigma\sqrt{\mathbf{w}}\mathbf{U}_{Rl}\mathbf{R}_{xl}\Sigma\sqrt{\mathbf{w}}\mathbf{U}_{Rl}^H\mathbf{H}_l^{-H}\right) \leq P \\ &\sum_{l=1}^{L}\mathrm{tr}\left(\mathbf{H}_l^{-1}\mathbf{U}_{Rl}\mathbf{R}_{xl}\mathbf{U}_{Rl}^H\mathbf{H}_l^{-H}\Sigma\mathbf{w}\right) \leq P \\ &\sum_{i=1}^{p}n_iw_i \leq P, \end{aligned} \tag{29}$$

where $\sum_{l=1}^{L}(\mathbf{H}_l^{-1}\mathbf{U}_{Rl}\mathbf{R}_{xl}\mathbf{U}_{Rl}^H\mathbf{H}_l^{-H})$ have diagonal entries n_i, $1 \leq i \leq p$. We also have $\mathbf{R}_{xl} = \sigma_s^2\mathbf{F}_l\mathbf{r}\mathbf{F}_l^H + \sigma_n^2\mathbf{I}_k$.

From (25) and (29), we can express the objective function in terms of SVD as follows:

$$\begin{aligned} \max \quad & -\left(\sum_{i=1}^{p}f_i + \frac{g_iw_i}{w_i+\sigma_v^2}\right) \\ \text{subject to} \quad & \sum_{i=1}^{p}n_iw_i \leq P, \\ & w_i \geq 0, \quad i = 1,\ldots,p. \end{aligned} \tag{30}$$

It is a convex problem since the objective function is a linear combination of convex functions and the constraints are

linear. To solve (30), it can be written as the Lagrangian equation as

$$L(\mu, w_i) = -\left(\sum_{i=1}^{p} f_i + \frac{g_i w_i}{w_i + \sigma_v^2}\right) + \mu\left(\sum_{i=1}^{p} n_i w_i - P\right) - \sum_{i=1}^{p} u_i w_i. \tag{31}$$

We can obtain the global optimum by solving the KKT conditions [16]:

$$\begin{aligned} \frac{g_i \sigma_v^2}{(w_i + \sigma_v^2)^2} &= \mu n_i - u_i, \quad i = 1, \ldots, p \\ \mu\left(\sum_{i=1}^{p} n_i w_i - P\right) &= 0 \\ u_i w_i &= 0, \quad i = 1, \ldots, p \\ \mu &\geq 0. \end{aligned} \tag{32}$$

The solution of the above can be expressed as follows: for $i = 1, \ldots, L$

$$w_i = \begin{cases} \left(\sqrt{\dfrac{g_i}{\mu n_i \sigma_v^2}} - 1\right)\sigma_v^2, & \dfrac{g_i}{n_i \sigma_v^2} \geq \mu, \\ 0, & \dfrac{g_i}{n_i \sigma_v^2} < \mu. \end{cases} \tag{33}$$

The parameter μ satisfies

$$\sum_{i=1}^{L}\left(\sqrt{\frac{g_i}{\mu n_i \sigma_v^2}} - 1\right)^{+} \sigma_v^2 n_i = P, \tag{34}$$

where $(x)^+ = \max(0, x)$. We solve the parameters w_i and μ using water-filling algorithm [5]:

input: $\mathbf{g}_i = (g_1, \ldots, g_p)$; $\mathbf{n}_i = (n_1, \ldots, n_p)$; σ_v^2; P

output: μ; m_i, $i = 1, \ldots, p$

(1) Reorder the sequence $\mathbf{t}_i = \mathbf{g}_i / \mathbf{n}_i \sigma_v^2$, in the increasing order, and set $m = 1$

(2) **do** (

$\mu \leftarrow t_m$

$\tau \leftarrow \sum_{i=m}^{p} \sqrt{t_i} \sigma_v^2 n_i / P + \sum_{i=m}^{p} \sigma_v^2 n_i$

$m = m + 1$)

while ($\mu < \tau$ and $m \leq p$)

(3) $\mu \leftarrow \tau$

$\omega_i \leftarrow \sigma_v^2 (\sqrt{g_i / \mu n_i \sigma_v^2} - 1)^{+}$

After we obtain $\Sigma\sqrt{\mathbf{w}} = \mathrm{diag}(\sqrt{w_1}, \ldots, \sqrt{w_p})$, we can write coding matrix $\mathbf{A}_l$ as

$$\mathbf{A}_l = \mathbf{H}_l^{-1} \Sigma \sqrt{\mathbf{w}} \mathbf{U}_{Rl}. \tag{35}$$

We also define a lower bound from (25); as $P \to \infty$, we have $w_i \to \infty$ as

$$D_{\text{low}} = 2p\sigma_s^2 - \sigma_s^4\left(\sum_{i=1}^{p} f_i + g_i\right). \tag{36}$$

4.2. Equal Power Method. Under equal power strategy, the transmit power for all sensors is set to be equal as follows:

$$P_l = \frac{P}{L}, \quad 1 \leq l \leq L, \tag{37}$$

where P_l is the transmit power for the lth sensor. Then, we have the lth coding matrix, $\mathbf{A}_l = \lambda_l [\mathbf{I}_N \;\; 0]$, where $\lambda_l = \sqrt{P/[L\,\mathrm{tr}(\mathbf{R}_{xl}(1:N, 1:N))]}$, so that $\mathrm{tr}(\mathbf{A}_l \mathbf{R}_{xl} \mathbf{A}_l^H) = P_l$, $1 \leq l \leq L$. The $\mathbf{R}_{xl}(1:N, 1:N)$ denotes the first N rows and columns of $\mathbf{R}_{xl}$ and $[\mathbf{I}_N \;\; 0]$ is a $N \times k$ matrix with its diagonal entries equal to 1 and other entries equal to 0.

5. Simulation Results

We present simulation results to illustrate the estimation performance of the previous section. We use P to denote the total transmit power constraint across the network. In all simulations, the random vectors $\mathbf{s}$, $\mathbf{n}_l$, and $\mathbf{v}$ are complex Gaussian with zero mean and unit variance. Note that power P is taken relative to the channel noise power. Since the channel noise has unitary variance, thus we label the total transmit power in unit of dB. The channel matrix $\mathbf{H}_l$ is also Gaussian random variable with zero mean and unit variance. We set the number of encoding matrix, $N = 5$, equal to that of sources, $p = 5$. This is because distortion performance does not degrade when $N > p$ [17]. Assuming that targets are close to each other so that we set distance among them, $d = 1$. The channels between sensors and the fusion center are chosen to be independent and the average LMMSE is calculated over 500 times. Therefore, we calculate the total distortion, $\widetilde{D}_t$, instead of D_t due to mathematical tractability.

Figure 3 plots distortion in terms of average MSE performance comparison between the proposed method, the lower bound, and the equal power method, in which we take $L = 10$, $k = 8$, and the correlation coefficient, $\rho = 0$. From Figure 3, we can see that the proposed method performs better than the equal power method. The proposed method converges to the lower bound as P increases. This is because the proposed method allocates power by taking into account the effects of observation and channel noise, but the equal power method does not.

Figure 4 shows distortion for uncorrelated, $\rho = 0$, and correlated, $\rho = 0.9$, data. We can see that the distortion of both methods becomes worse as the data is being correlated. If the data is more correlated, sensors provide redundant information about the targets. It becomes difficult to estimate the targets. Once $\rho = 0.9$, the distortion of the proposed method remains constant for $P < 25$ dB. Moreover, the equal power method performs better than the proposed method particularly for P between 10 and 25 dB because the proposed method does not have enough power to accomplish the threshold of water filling. However, the distortion of the

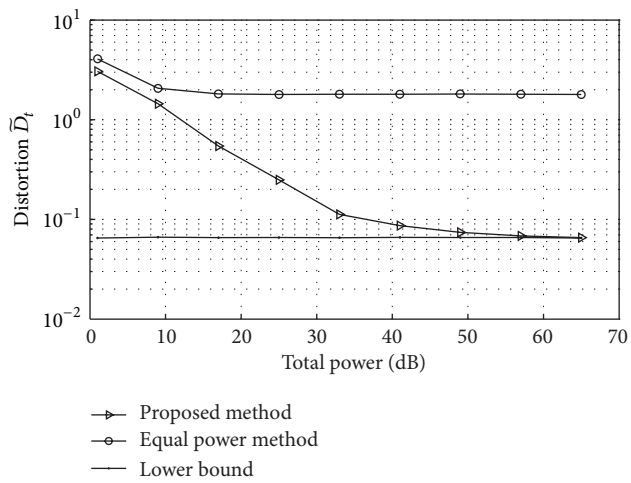

Figure 3: Distortion comparison between the proposed method, equal power method, and the lower bound with parameters $\rho = 0$, $L = 10$, $k = 8$, $N = 5$, $p = 5$.

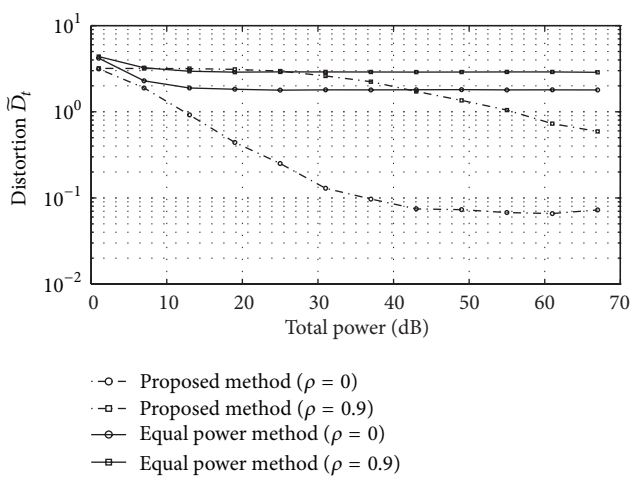

Figure 4: Distortion comparison between different correlation coefficients with parameters $\rho = 0, 0.9$, $L = 10$, $k = 8$, $N = 5$, $p = 5$.

proposed method becomes smaller than that of the equal power method for $P > 25$ dB, because it allocates power by taking into account the effects of the correlation, observation, and channel noise.

The distortion of the proposed method and the lower bound with different power levels ($P = 5$, 10, and 20 dB), and spatial correlation coefficient, $\rho = 0.7$, are shown in Figure 5. Both distortions become smaller as the number of measurements, k, increases. This is because increasing the number of k leads to an increase of measurement power. In Figure 5, we can also see that the gap of the distortion between the proposed method and the lower bound becomes larger. This is because the proposed method has a power constraint compared with the lower bound that does not have a power constraint.

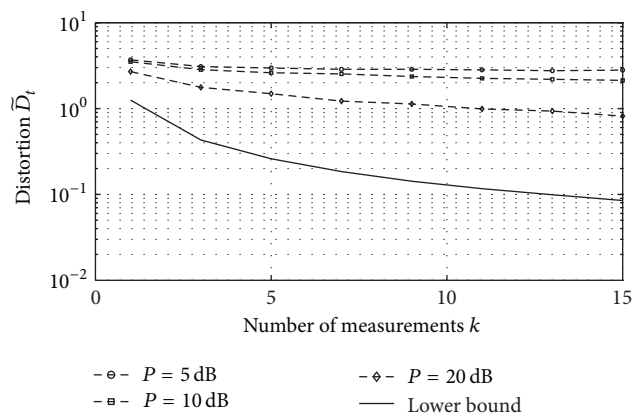

Figure 5: Distortion versus k with parameters $\rho = 0.7$, $L = 10$, $N = 5$, $p = 5$, $P = 5$, 10, 20 dB.

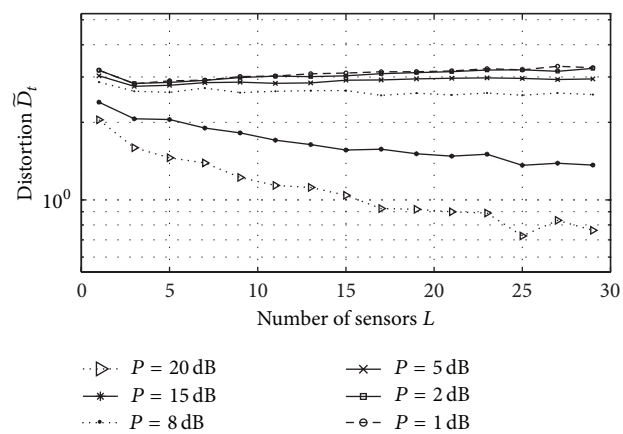

Figure 6: Distortion versus L with parameters $\rho = 0.7$, $k = 8$, $N = 5$, $p = 5$, $P = 1, 2, 5, 8, 15, 20$ dB.

From Figure 6, we simulate the proposed method with different power levels ($P = 1, 2, 5, 8, 15, 20$ dB) and correlation coefficient, $\rho = 0.7$. The distortions of the proposed method become smaller as the number of sensors, L, increases. However, they rise slightly after reaching the optimum number of sensors. For example, the total power constraint is $P = 1$ dB; the distortion rises when the number of sensors is $L > 3$. This is owing to the fact that the power allocated to each sensor becomes smaller as the number of sensors increases.

In Figure 7, we show the simulation results of the proposed method with an ideal sensor condition and an ideal channel condition. The simulation with ideal sensor condition is taken by assuming that the sensor observations have no distortion. To accommodate the assumption, we set the distortion due to observation noise, $D_o = 0$, in (8). Then, the simulation with ideal channel condition is taken by assuming that the fusion center has a perfect channel knowledge between the sensors and the fusion center. In a similar way to the ideal sensor condition, we set the distortion due to channel noise, $D_c = 0$, in (8). We use a complex Gaussian random variable with zero mean and unit variance for observation noise, $\mathbf{n}$, and noisy channel, $\mathbf{v}$, respectively. The distortion

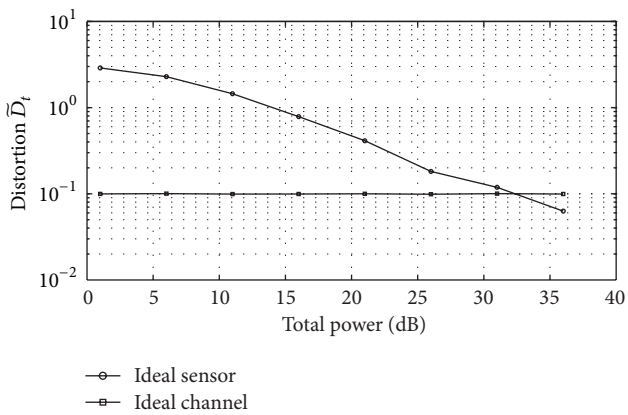

Figure 7: Distortion comparison of the proposed method with ideal sensor and ideal channel condition with parameters $\rho = 0.5$, $k = 8$, $N = 5$, $p = 5$.

with ideal sensor becomes smaller as P increases, but the distortion with ideal channel still remains constant. It means that the water-filling method is more effective to combat the noisy channel rather than the observation noise.

6. Conclusion

We studied distributed estimation of a random vector in MIMO sensor network by considering total power constraint and spatial correlated data. For the spatial correlated data, we obtained a water-filling-based closed-form solution that follows water-filling strategy. We also derived lower bound of distortion to this system. From the simulation results, we showed that the distortion increases as the data becomes more correlated, because the sensors become difficult to estimate the targets. Moreover, we showed that the noisy channel was more harmful than the observation noise. By taking into account the effects of correlation, observation, and channel noise, the proposed method has a better distortion performance than the equal power method.

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