Hindawi Publishing Corporation
ISRN High Energy Physics
Volume 2013, Article ID 364852, 5 pages
http://dx.doi.org/10.1155/2013/364852

Research Article

Super-Liouville Theory from Superstring Theory in the Presence of Gauge Worldsheet Fields

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Received 7 January 2013; Accepted 28 January 2013

Academic Editors: C. A. d. S. Pires, F.-H. Liu, and L. Marek-Crnjac

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We obtain in this work the super-Liouville theory induced from the action of the superstring theory in the presence of gauge worldsheet fields; such construction is based on a special ansatz that gives a special value of B-fields $B = e^{\varphi} + (i/2)\overline{\psi}\psi$. We discuss the integrability of super-Liouville theory through the Lax formulation, and we establish also the superfields formulation of super-Liouville equations.

1. Introduction

Gauge superstring theories have been studied in different points of view [1, 2]. They are obtained by introducing the worldsheet abelian gauge fields in superstring action [3–5]. The action of the gauged superstring allows us to build two worldsheet fields from elements of the gauge field. They appear as the coordinates of space and time.

The super-Liouville field theory (SLFT) is a generalization supersymmetric of classical bosonic Liouville theory, which is known to be theory of matter-induced gravity in two dimensions. Similarly SLFT describes 2D supergravity, induced by supersymmetric matter [6].

Our goal in this paper is to find the super-Liouville fields equations from gauged superstring theory and study the superfields formulation and integrability of such equations.

This paper is organized as follows. In Section 2, we introduce some elements of the action of the superstring in the presence of worldsheet gauge fields and its symmetries. In Section 3, we present in detail the method to find the Liouville equations from gauged superstring action. In Section 4, we give the superfield formulation. In Section 5, the integrability of Liouville superstring theory is discussed. Section 6 is for concluding remarks.

2. Superstring Theory Coupled to Gauge Theory

In this section we recall some basic elements of superstring theory in the presence of gauge superfields, so that we have the following.

2.1. Bosonic Action. To obtain the action with gauge field, we use the superfields in the worldsheet superspace. The bosonic action is given by

$$S_1 = -\int d^2\sigma \left(\frac{1}{4\pi\alpha'} \partial_a X^\mu \partial^a X_\mu + \frac{1}{4g^2} F_{ab} F^{ab} \right), \qquad (1)$$

where g is the gauge coupling constant, $F_{ab} = \partial_a A_b - \partial_b A_a$ are the fields strength associated with the worldsheet gauge fields, $A_a(\sigma, \tau)$, $\{\sigma, \tau\}$ are the worldsheet coordinates, and X^μ are the string coordinates. This action has the gauge symmetry. Therefore, we have the condition

$$\partial_a A^a = 0. \qquad (2)$$

2.2. Supersymmetric Action. To get the supersymmetric action the bosonic fields X^μ and A^a should be replaced by the superfields

$$Y^\mu\left(\sigma,\tau;\ \theta^1,\theta^2\right) = X^\mu(\sigma,\tau) + \overline{\theta}\,\psi^\mu(\sigma,\tau) + \frac{1}{2}\overline{\theta}\theta B^\mu(\sigma,\tau),$$
$$\mathrm{A}^a\left(\sigma,\tau;\ \theta^1,\theta^2\right) = A^a(\sigma,\tau) + \overline{\theta}\rho^a\chi(\sigma,\tau) + \frac{1}{2}\overline{\theta}\theta W^a(\sigma,\tau), \tag{3}$$

where $\psi^\mu = (\psi^\mu_\alpha)$ are dynamical fields on the worldsheets, B^μ and W^a are auxiliary fields, the Majorana spinor χ is the superpartner of A_a, and the Grassmannian coordinates θ^1 and θ^2 form a Majorana spinor $\theta = \left(\begin{smallmatrix}\theta^1\\ \theta^2\end{smallmatrix}\right)$.

We introduce also the following superspace covariant derivative:

$$\mathscr{D}^a = k\,\varepsilon^{ab}\rho_b D,$$
$$D = \frac{\partial}{\partial\overline{\theta}} - i\rho^a\theta\partial_a, \tag{4}$$

where $\varepsilon^{01} = -\varepsilon^{10} = 1$ and k is a constant which is finding such that $\overline{\mathscr{D}}^a Y_\mu \mathscr{D}_a Y^\mu = \overline{D}Y_\mu DY^\mu$; this gives $k \in \{\pm 1/\sqrt{2}, \pm i/\sqrt{2}\}$. Reformulating the action by introducing (3) and (4) we obtain the following supersymmetric action:

$$S = \int d^2\sigma d^2\theta\left(\frac{i}{8\pi\alpha'}\overline{\mathscr{D}}^a Y_\mu \mathscr{D}_a Y^\mu + \frac{i}{4g^2}\overline{\mathscr{F}}_{ab}\mathscr{F}^{ab}\right). \tag{5}$$

The superfield strength $\mathscr{F}_{ab}$ is defined as follows:

$$\mathscr{F}_{ab} = \mathscr{D}_a\mathscr{A}_b - \mathscr{D}_b\mathscr{A}_a. \tag{6}$$

After making integration over the Grassmannian coordinates θ^1 and θ^2, this action takes the form

$$S = \int d^2\sigma\left(-\frac{1}{4\pi\alpha'}\left(\partial_a X^\mu\partial^a X_\mu - i\,\overline{\psi}^\mu\rho^a\partial_a\psi_\mu - B^\mu B_\mu\right)\right.$$
$$\left.-\frac{1}{2g^2}\left(\partial_a A^b\partial^a A_b - W_a W^a\right)\right). \tag{7}$$

As we see, the gaugino field χ from the two-dimensional action disappeared.

2.3. Extracoordinates in Gauged Superstring Action. In the action (7) the kinetic terms of the fields X^μ and A^b have the same feature. In other words, A^0 and A^1 have the roles of the time and space coordinates. Let $\{X^a\} = \{A^0, A^1\}$ denote the coordinates of this (1 + 1) dimensional spacetime. Thus, we have the field redefinition

$$X^a = \frac{\sqrt{2\pi\alpha'}}{g}A^a,$$
$$B^a = \frac{\sqrt{2\pi\alpha'}}{g}W^a. \tag{8}$$

According to these definitions, the action (7) can be written as

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\partial_a X^M\partial^a X_M - i\overline{\psi}^\mu\rho^a\partial_a\psi_\mu - B^M B_M\right), \tag{9}$$

where $M \in \{\mu, a\}$, and we will use the convention $a \in \{0,1\}$ and $\mu \in \{0', 1', \ldots, 9'\}$,

$$\left\{X^M\right\} = \left\{X^\mu\right\} \cup \left\{X^a\right\}. \tag{10}$$

Since both X^a and σ^a carry the worldsheet index, the partial derivative ∂_a always shows derivative with respect to σ^a. The bosonic part of this action apparently describes a 12-dimensional spacetime with the signature 10 + 2 and the coordinates. However, in the superstring theory the dimension of the spacetime is always 9 + 1. Therefore, *they are called the extradimensions or the fictitious coordinates* [6].

The fermionic term of the action (9) also can be written with the 12-dimensional indices. For this, the Majorana spinor ψ^a is defined by

$$\psi^0 = \frac{\sqrt{2\pi\alpha'}}{g}\begin{pmatrix}\chi_2\\ \chi_1\end{pmatrix}, \qquad \psi^1 = \frac{\sqrt{2\pi\alpha'}}{g}\begin{pmatrix}\chi_2\\ -\chi_1\end{pmatrix}. \tag{11}$$

The spinors ψ^0 and ψ^1 satisfy the identities

$$\overline{\psi}^b\rho^a\partial_a\psi_b = 0,$$
$$\overline{\psi}^a\psi_a = \frac{4\pi\alpha'}{g^2}\overline{\chi}\chi, \tag{12}$$

where $\chi = \left(\begin{smallmatrix}\chi_1\\ \chi_2\end{smallmatrix}\right)$. Introducing the identity (12) in the action (9) leads to the covariant form of this action

$$I = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\partial_a X^M\partial^a X_M - i\overline{\psi}^M\rho^a\partial_a\psi_M - B^M B_M\right). \tag{13}$$

The metric of the extended manifold is

$$\eta_{MN} = \mathrm{diag}\left(\eta_{\mu\nu}, \eta_{ab}\right), \tag{14}$$

where $\eta_{\mu\nu}$ belongs to the 9 + 1 actual spacetime and η_{ab} to the fictitious coordinates.

The equations of motion, extracted from the action (13), are

$$\partial_a\partial^a X^M = 0,$$
$$\rho^a\partial_a\psi^M = 0, \tag{15}$$
$$B^M = 0.$$

In addition, we should also consider the gauge condition

$$\partial_a X^a = 0. \tag{16}$$

This condition and the equation of motion of X^a can be written as

$$X^a = \varepsilon^{ab}\partial_b\phi,$$
$$\partial_a\partial^a\phi = c, \tag{17}$$

respectively. The constant c is independent of σ and τ.

2.4. Symmetries of the Model

2.4.1. Worldsheet Supersymmetry. Using the superfield (3) we obtain the supersymmetry transformations of A^a and χ as in the following:

$$\delta A^a = \bar{\epsilon}\rho^a\chi, \qquad \delta W^a = -i\,\bar{\epsilon}\rho^b\rho^a\partial_b\chi,$$
$$\delta\chi = -\frac{i}{4}\rho^{ab}F_{ab}\epsilon - \frac{1}{2}\rho_a W^a\epsilon, \tag{18}$$

where $\rho^{ab} = (1/2)[\rho^a, \rho^b]$. The supersymmetry parameter ε is an anticommuting infinitesimal constant spinor. In terms of the fields $\{X^a, \psi^a, B^a\}$ these transformations take the form

$$\delta X^a = i\varepsilon^{ab}\,\bar{\epsilon}\psi_b,$$
$$\delta B^a = \varepsilon^{ab}\,\bar{\epsilon}\rho^c\partial_c\psi_b, \tag{19}$$
$$\delta\psi^a = -\frac{1}{2}\left(\rho^a\varepsilon^{bc}\partial_b X_c - i\varepsilon^{ab}\rho_b\,\rho_c B^c\right)\epsilon.$$

The transformations (19) form a closed algebra. The supercurrent associated with the supersymmetry transformations (19), accompanied by $\delta X^\mu = \delta\psi^\mu = 0$, is

$$k_a = \frac{i}{4}\rho^{a'}\rho_a\psi_{a'}\varepsilon^{bc}\partial_b X_c. \tag{20}$$

According to the identity $\rho^a\rho^{a'}\rho_a = 0$ there is $\rho^a k_a = 0$ then k_a is a conserved current $\partial^a k_a = 0$.

2.4.2. The Poincare Symmetry. The action (13), with $B^M = 0$, under the Poincare transformations

$$\delta X^M = a^M{}_N X^N + b^M, \qquad \delta\psi^M = a^M{}_N\psi^N, \tag{21}$$

is symmetric. The matrix a_{MN} is a constant antisymmetric, and b^M is a constant vector. The associated currents to these transformations are

$$P_a^M = \frac{1}{2\pi\alpha'}\partial_a X^M,$$
$$J_a^{MN} = \frac{1}{2\pi\alpha'}\left(X^M\partial_a X^N - X^N\partial_a X^M + i\overline{\psi}^M\rho_a\psi^N\right). \tag{22}$$

There are a conserved currents, that is, $\partial^a P_a^M = \partial^a J_a^{MN} = 0$.

3. Liouville Equations from Gauged Superstring Action

Let us consider the action (13) of the superstring in the presence of the worldsheet gauge fields

$$S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\left(\partial_a X^M\partial^a X_M - i\overline{\psi}^M\rho^a\partial_a\psi_M - B^M B_M\right), \tag{23}$$

where X^M, ψ^M and B^M are the fields of conformal weights 1, 3/2, and 2, respectively.

We assume that these fields can be processed as follows:

$$X^M = k^M\cdot\varphi,$$
$$\psi^M = k^M\cdot\psi, \tag{24}$$
$$B^M = k^M\cdot B,$$

where k^M is the Lorentz field of conformal weight 1 which does not depend on worldsheet variables, and the new fields φ, ψ, and B are, respectively, of conformal weights 0, 1/2 and 1.

By introducing the Ansatz (24), the action (23) becomes

$$S_1 = -\frac{k^2}{4\pi\alpha'}\int d^2\sigma\left(\partial_a\varphi\partial^a\varphi - i\,\overline{\psi}\rho^a\partial_a\psi - B\cdot B\right). \tag{25}$$

The equations of motion relative to the fields φ, ψ, $\overline{\psi}$, and B are given by

$$\partial_a\partial^a\varphi = 0,$$
$$\rho^a\partial_a\psi = 0,$$
$$\partial_a\overline{\psi}\rho^a = 0, \tag{26}$$
$$B = 0.$$

To obtain the Liouville equations we need to take the following value for the field B:

$$B = e^\varphi + \frac{i}{2}\overline{\psi}\,\psi. \tag{27}$$

In that case the action becomes

$$S_2 = -\frac{k^2}{4\pi\alpha'}\int d^2\sigma\left[\partial_a\varphi\partial^a\varphi - i\,\overline{\psi}\rho^a\partial_a\psi - \left(e^{2\varphi} + i\,\overline{\psi}\psi e^\varphi\right)\right], \tag{28}$$

where $(\overline{\psi}\psi)^2 = 0$.

This last action (28) has the same form as that found in our paper [7]; by consequence we find the same equations of motion:

$$\partial_a\partial^a\varphi = e^{2\varphi} + \frac{i}{2}\overline{\psi}\psi e^\varphi,$$
$$\rho^a\partial_a\psi = \psi e^\varphi, \tag{29}$$
$$\partial_a\overline{\psi}\rho^a = -\overline{\psi}e^\varphi.$$

4. Superfield Analysis

In terms of a superfield formulation associated with an $N = 1$ supersymmetry we can set [8]

$$\Phi = \varphi + \theta\psi + \overline{\theta}\overline{\psi} + \theta\overline{\theta}B. \tag{30}$$

We can then easily show that the system of super-Liouville equations (29) can be expressed in terms of superderivative of the superfield Φ as follows:

$$D\overline{D}\Phi = e^\Phi, \tag{31}$$

where $D = \partial_\theta + \theta\partial$ and $\overline{D} = \partial_{\overline{\theta}} + \overline{\theta}\partial$. Indeed, straightforward computations lead to

$$D\overline{D}\Phi = -B - \overline{\theta}\,\overline{\partial}\overline{\psi} + \theta\partial\psi + \theta\overline{\theta}\partial\overline{\partial}\varphi. \tag{32}$$

By virtue of (30) and expanding the exponential of the superfield Φ, we find

$$e^{\Phi} = e^{\varphi}\left(1 - \overline{\theta}\psi + \theta\overline{\psi} + \theta\overline{\theta}\left(-e^{\varphi} + \overline{\psi}\psi\right)\right). \tag{33}$$

Identifying (31) with (32) one finds easily super-Liouville equations of motions (29).

Furthermore, using the complex transformations $z = \sigma + i\tau$ and $z = \sigma - i\tau$ one can easily rewrite the previous super-Liouville equations to become

$$\begin{aligned} \partial\overline{\partial}\varphi &= -e^{2\varphi} + \overline{\psi}\psi e^{\varphi}, \\ \partial\psi &= \overline{\psi}e^{\varphi}, \\ \overline{\partial}\overline{\psi} &= -\psi e^{\varphi} \end{aligned} \tag{34}$$

with $\partial_z = \partial$ and $\partial_{\overline{z}} = \overline{\partial}$. With the equation of motion (34) and the previous discussion, we write the superstring-Liouville action in terms of $N = 1$ superfield Φ (30) as follows:

$$S = \int d\theta\, d\overline{\theta}\, d^2\sigma \left(\frac{1}{2}D\Phi\overline{D}\Phi + \exp(\Phi)\right). \tag{35}$$

Forgetting about the fermionic fields, the super-Liouville equations are reduced simply to the Liouville equation

$$\partial\overline{\partial}\varphi = -e^{2\varphi}, \tag{36}$$

while the scalar superfield Φ is reduced to the scalar real-field φ. The associated Liouville action is

$$S = \frac{1}{2}\int d^2\sigma \left(\partial\varphi\overline{\partial}\varphi + \exp(2\varphi)\right), \tag{37}$$

whose single constant of motion is the stress energy momentum tensor T of weight 2 such that

$$T_2(z) = \partial^2\varphi - (\partial\varphi)^2, \qquad \overline{\partial}T_2(z) = 0. \tag{38}$$

5. Integrability of Liouville Superstring Theory

The super-Liouville Lax pair one way to introduce the integrability of the Liouville superstring theory, we are discussing here, is through the Lax pair formulation [9, 10]. A key step towards establishing this integrability is through an explicit determination of the Lax pair generators. The zero curvature condition is given by

$$DA_{\overline{\theta}} + \overline{D}A_\theta + \{A_\theta, A_{\overline{\theta}}\} = 0, \tag{39}$$

where the Lax pair $(A_\theta, A_{\overline{\theta}})$ is defined as functions of the osp(1 | 2) Lie superalgebra generators. One possible realization of this Lax pair is given by

$$\begin{aligned} A_\theta &= D\Phi h + f_+, \\ A_{\overline{\theta}} &= -2i\exp(\Phi)\, f_-. \end{aligned} \tag{40}$$

Indeed, by virtue of the zero curvature condition and the commutations relations of the osp(1 | 2) Lie superalgebra, we recover easily the super-Liouville equation of motion (34). Indeed

$$\begin{aligned} DA_{\overline{\theta}} &= -2iD\Phi\exp(\Phi)\, f_-, \\ \overline{D}A_\theta &= \overline{D}D\Phi h, \\ \{A_\theta, A_{\overline{\theta}}\} &= -2iD\Phi\exp(\Phi)\,[h, f_-] - 2i\exp(\Phi)\,\{f_+, f_-\} \\ &= 2iD\Phi\exp(\Phi)\, f_- - \exp(\Phi)\, h. \end{aligned} \tag{41}$$

Then with respect to the zero curvature condition $DA_{\overline{\theta}} + \overline{D}A_\theta + \{A_\theta, A_{\overline{\theta}}\} = 0$, and as suspected, were cover the super-Liouville equation of motion

$$\overline{D}D\Phi = \exp\Phi. \tag{42}$$

The $N = 1$ super-Liouville conserved current can be written as

$$G_{3/2}(z,\theta) = J_{3/2}(z) + \theta T_2(z), \qquad \overline{D}G_{3/2} = 0, \tag{43}$$

where $T_2(z)$ is the Virasoro conformal current of weight 2 and $J_{3/2}(z)$ is its supersymmetric partner of conformal spin 3/2. The explicit form of this $N = 1$ supercurrent in terms of the superfield Φ is given by

$$G_{3/2}(z,\theta) = D^2\Phi D\Phi - D^3\Phi, \qquad \overline{D}G_{3/2} = 0. \tag{44}$$

6. Conclusions

We have studied the superstring theory in the presence of worldsheet gauge fields, and we have extracted the associated super-Liouville theory by a special ansatz. This work contains some connections between the string and gauge fields on one hand and super-Liouville theory on the other hand.

References

[1] E. Witten, "Chern-Simons Gauge theory as a string theory," *Progress of Mathematics*, vol. 133, pp. 637–678, 1995, http://arxiv.org/abs/hep-th/9207094.

[2] G. Savvidy, "Gauge fields—strings duality and tensionless superstrings," *Acta Physica Polonica B*, vol. 34, no. 10, pp. 5063–5084, 2003.

[3] T. Fukuda and K. Hosomichi, "Super-Liouville theory with boundary," *Nuclear Physics B*, vol. 635, no. 1-2, pp. 215–254, 2002.

[4] J. Gomis and H. Suzuki, "Covariant currents in $N = 2$ super-Liouville theory," *Nuclear Physics B*, vol. 393, no. 1-2, pp. 126–148, 1993.

[5] M. Hssaini and M. B. Sedra, "Type IIB string backgrounds on parallelizable PP-waves and conformal Liouville theory," *The African Journal of Mathematical Physics*, vol. 1, pp. 91–97, 2004, http://128.84.158.119/abs/hep-th/0308187/.

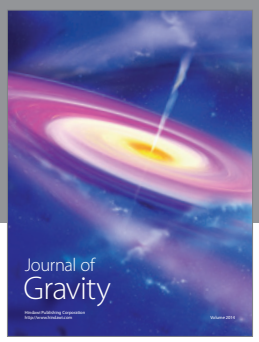
[6] D. Kamani, "Superstring in the presence of the worldsheet Gauge field," *International Review of Physics*, vol. 2, pp. 210–217, 2008, http://arxiv.org/abs/0710.3230/.

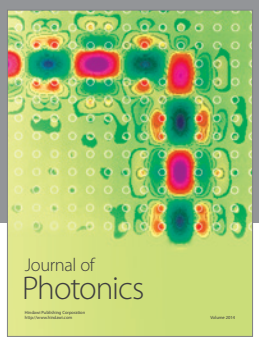
[7] H. Nishino and E. Sezgin, "Supersymmetric Yang-Mills equations in 10 + 2 dimensions," *Physics Letters B*, vol. 388, no. 3, pp. 569–576, 1996.

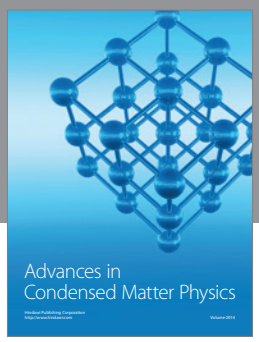
[8] K. Bilal, A. El Boukili, M. Nach, and M. B. Sedra, "Super string-Louville theory," *Advanced Studies in Theoretical Physics*, vol. 5, no. 2, pp. 91–96, 2011.

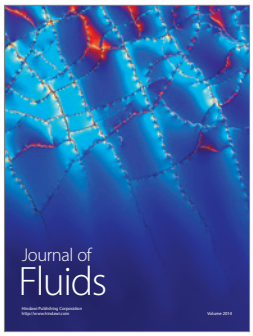
[9] A. Zamolodchikov and Al. Zamolodchikov, "Conformal bootstrap in Liouville field theory," *Nuclear Physics B*, vol. 477, no. 2, pp. 577–605, 1996.

[10] J. Teschner, "On the Liouville three point function," *Physics Letters B*, vol. 363, no. 1-2, pp. 65–70, 1995.









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