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Research Article

A Cosmological Model with Varying G and Λ in General Relativity—Part III

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We are investigating Bianchi type-I cosmological model in perfect fluid. The cosmological model is obtained by assuming Λ proportional to $\ddot{R}/R$. We also observed some physical properties of the model and discussed them.

1. Introduction

Cosmology is the study of the largest-scale structures and dynamics of our universe, and it deals with subjects regarding their origin and evolution. Cosmology involves itself in studying the motions of the celestial bodies. At the present state of evolution, the universe is isotropic and homogeneous. The cosmological constant problem is very interesting. And the simplest way out of the problem is to consider a varying cosmological term. This can be done by considering different values for cosmological constant Λ.

As we are aware that the expansion of the universe is undergoing time acceleration Perlmutter et al. [1–3], Riess et al. [4, 5], Allen et al. [6], Peebles and Ratra [7], Padmanabhan [8], and Lima [9]. In the literature cosmological models with Λ proportional to scale factor have been studied by Chen and Wu [10], Pavn [11], Carvalho et al. [12], Lima and Maria [13], Lima and Trodden [14], Arbab and Abdel-Rahman [15], Cunha and Santos [16], and Carneiro and Lima [17]. A number of authors investigated Blanches models, using the approach that there is a link between variation of gravitational constant and cosmological constant (see [18–22]). A Lot of work has been done by Saha [23–26] in studying the anisotropic Bianchi type-I cosmological model in general relativity with varying G and Λ.

The cosmological constant is small because the universe is old. Models with dynamically decaying cosmological term representing the energy density of vacuum have been studied by Vishwakarma [27–29], Arbab [30], and Berman [31, 32]. In this paper, we study homogeneous Bianchi type-I space time with variable G and Λ containing matter in the form of a perfect fluid. We obtain solution of the Einstein field equations by assuming that cosmological term is proportional to $\ddot{R}/R$. (R is scale factor.)

2. The Metric and Field Equations

We consider the Bianchi type-I metric in the orthogonal form as follows:

$$ds^2 = -dt^2 + A^2(t)\,dx^2 + B^2(t)\,dy^2 + C^2(t)\,dz^2. \quad (1)$$

We assume that the cosmic matter is taken to be perfect fluid given by the energy-momentum tensor as the following:

$$T_{ij} = (\rho + p)\,v_i v_j + p g_{ij}, \quad (2)$$

where R_{ij} is Ricci tensor, and p and ρ are the isotropic pressure and energy density of the fluid. We take equation of state

$$p = w\rho, \quad 0 \le w \le 1. \quad (3)$$

v_i is the four velocity vector of the fluid satisfying

$$g_{ij} v^i v^j = -1. \quad (4)$$

Einstein's field equations with time dependent G and Λ are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G(t)\,T_{ij} + \Lambda(t)\,g_{ij}, \tag{5}$$

where

$$R_{ij} = \frac{\partial^2 \log\sqrt{-g}}{\partial x^i \partial x^j} - \Gamma^b_{ij}\frac{\partial}{\partial x^b}\log\sqrt{-g} + \Gamma^b_{ia}\Gamma^a_{bj} - \frac{\partial \Gamma^a_{ij}}{\partial x^a}. \tag{6}$$

For the metric (1) and energy-momentum tensor (2) in commoving system of coordinates, the field equation (5) yields

$$\begin{aligned}
\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} &= -8\pi G p + \Lambda,\\
\frac{\ddot{A}}{A} + \frac{\ddot{C}}{B} + \frac{\dot{A}\dot{C}}{AC} &= -8\pi G p + \Lambda,\\
\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} &= -8\pi G p + \Lambda,\\
\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} &= 8\pi G\rho + \Lambda.
\end{aligned} \tag{7}$$

In view of vanishing of the divergence of Einstein tensor, we have

$$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{8}$$

The usual energy conservation equation of general relativity quantities is

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \tag{9}$$

Equation (8) together with (9) puts G and Λ in some sort of coupled field given by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \tag{10}$$

implying that Λ is a constant whenever G is constant. Using (3) in (9) and then integrating, we get $k > 0$; in particular we are assuming $w = 0$.

Consider

$$\rho = \frac{k}{R^3}. \tag{11}$$

We define R as the average scale factor of Bianchi type-I universe.

Consider

$$R = (ABC)^{1/3}. \tag{12}$$

The Hubble parameter H, volume expansion θ, shear σ, and deceleration parameter q are given by

$$\begin{gathered}
\theta = 3H = \frac{3\dot{R}}{R}, \qquad \sigma = \frac{k}{\sqrt{3}R^3}, \quad k > 0 \text{ (constant)}\\
q = -1 - \frac{\dot{H}}{H^2} = -\frac{R\ddot{R}}{\dot{R}^2}.
\end{gathered} \tag{13}$$

Einstein's field equations (7) can be also written in terms of Hubble parameter H, shear σ, and deceleration parameter q as

$$H^2(2q - 1) - \sigma^2 = 8\pi G p - \Lambda, \tag{14}$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda. \tag{15}$$

On integrating (7), we obtain

$$\begin{aligned}
\frac{\dot{A}}{A} - \frac{\dot{B}}{B} &= \frac{k_1}{R^3},\\
\frac{\dot{B}}{B} - \frac{\dot{C}}{C} &= \frac{k_2}{R^3},
\end{aligned} \tag{16}$$

where k_1 and k_2 are constants of integration. From (15), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \tag{17}$$

implying that $\Lambda \geq 0$

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, \qquad 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}. \tag{18}$$

Thus, the presence of positive Λ lowers the upper limit of anisotropy whereas a negative Λ contributes to the anisotropy.

Equation (17) can also be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c}, \tag{19}$$

where $\rho_c = 3H^2/8\pi G$ is the critical density and $\rho_v = \Lambda/8\pi G$ is the vacuum density.

From (14) and (15), we get

$$\frac{d\theta}{dt} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2. \tag{20}$$

Thus, the universe will be in decelerating phase for negative Λ, and for positive Λ the universe will slow down the rate of decrease, showing that the rate of volume expansion decreases during time evolution, and presence of positive Λ slows down the rate of this decrease whereas a negative Λ would promote it.

3. Solution of the Field Equations

The system of (7) and (10) supplies only five equations in seven unknown parameters ($A, B, C, \rho, p, \Lambda$, and G). Two extra equations are needed to solve the system completely. For this purpose, we take cosmological term to be proportional to $\ddot{R}/R$. [33, 34]. That is,

$$\Lambda = a\frac{\ddot{R}}{R}. \tag{21}$$

Using (11) and (21) in (10) we get

$$G = \frac{ae^{3t_0/(a-3)}}{2k\pi} \times \left(\frac{a-1}{(a-3)(a+3)}\right) \times \frac{1}{t^{(3+a)/(3-a)}}. \quad (22)$$

From (14), (15), and (21), where t_0 is a constant of integration, we get

$$R = \frac{e^{t_0/(a-3)}}{t^{(1-a)/(a-3)}}. \quad (23)$$

By using (23) in (16) and the metric (1), we get

$$\begin{aligned} ds^2 = {} & R^2\left[m_1^2 \exp\left\{\left[\frac{2k_1+k_2}{3}\right] \times 2M\right\}\right] dx^2 \\ & + R^2\left[m_2^2 \exp\left\{\left[\frac{k_2-k_1}{3}\right] \times 2M\right\}\right] dy^2 \\ & + R^2\left[m_3^2 \exp\left\{\left[\frac{-2k_2-k_1}{3}\right] \times 2M\right\}\right] dz^2, \end{aligned} \quad (24)$$

where $M = ((3-a)/2ae^{3t_0/(a-3)}) \times (1/t^{2a/(a-3)})$ and m_1, m_2, and m_3 are constants.

For the model (24), the spatial V, density ρ, gravitational constant G, and cosmological constant Λ are

$$\begin{aligned} V = R^3 &= \left[e^{t_0/(a-3)}\right]^3 \times \left[t^{(a-1)/(a-3)}\right]^3, \\ \rho &= \frac{k}{\left[e^{t_0/(a-3)}\right]^3} \times \left[t^{(1-a)/(a-3)}\right]^3, \\ \Lambda &= 2a\left[\frac{a-1}{(a-3)^2}\right]\frac{1}{t^2}. \end{aligned} \quad (25)$$

Expansion scalar θ and shear σ are

$$\theta = \left[\frac{a-1}{a-3}\right]\frac{3}{t}, \quad (26)$$

$$\sigma = \frac{k}{e^{3t_0/(a-3)}\sqrt{3}} \times \frac{1}{t^{3(a-1)/(a-3)}}, \quad (27)$$

$$q = \frac{-2}{a-1}, \quad (28)$$

$$\Omega = \frac{\rho}{\rho_c} = \frac{2ae^{3t_0/(a-3)}}{k} \times \left[\frac{a-1}{(a-3)^2}\right] \times \frac{1}{t^{-(a+3)/(a-3)}}, \quad (29)$$

$$\frac{\sigma}{\theta} = \frac{k}{3\sqrt{3}} \times \frac{1}{e^{3t_0/(a-3)}} \times \left[\frac{a-3}{a-1}\right]\frac{1}{t^{2a/(a-3)}}. \quad (30)$$

4. Observations and Conclusion

(1) We observe that the spatial volume $V \to 0$ at $t = 0$ and expansion scalar θ is infinite, which shows that universe starts evolving with zero volume at $t = 0$ with an infinite rate of expansion. Hence, the model has a point type singularity at initial epoch.

(2) Initially at $t = 0$, the energy density "ρ," pressures "p," shear σ, and cosmological term Λ tend all to be infinite.

(3) As t increases the spatial volume increases, but the expansion rate decreases. Thus, the rate of expansion slows down with increase in time and tends to be zero.

(4) As $t \to \infty$, the spatial volume V becomes infinitely large. All parameters θ, ρ, p, $\Omega = \rho/\rho_c$, σ, and $\Lambda \to 0$ asymptotically but G is decreasing. Therefore, at large value of t the model gives empty universe. The cosmic scenario starts from a big bang at $t = 0$ and continues until $t = \infty$.

(5) From (28), we observed that when $a < 0$ the model is decelerating as q is positive, and the model is accelerating when $a > 1$.

(6) The ratio $\sigma/\theta \to 0$ as $t \to \infty$. So the model approaches isotropy for a large value of t.

The possibility of G increasing with time, at least in some stages of the development of the universe, has been investigated by Abdel-Rahman [18], Chow [35], Levitt [36], and Milne [37]. $\Lambda\alpha(1/T^2)$ include Berman [38], Berman and Som [39], Berman et al. [40], and Bertolami [41, 42]. This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. A decreasing functional form permits Λ to be large in the early universe.

In summary, we have investigated the Bianchi type-I cosmological model with variable G and Λ in presence of perfect fluid where the cosmological term is proportional to $a(\ddot{R}/R)$. (R is scale factor) as suggested by Silveira and Waga [43, 44] and others. Initially the model has a point type singularity, gravitational constant $G(t)$ is decreasing, and cosmological constant Λ is infinite at this time, when time increases Λ decreases. The model approaches isotropy for a large value of "t," the model is quasi-isotropic that is, $\sigma/\theta = 0$.

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