Research Article

Power Consideration in a Piezoelectric Generator

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A piezoelectric generator converts mechanical energy into electricity and is used in energy harvesting devices. In this paper, synchronisation conditions in regard to the excitation vibration are studied. We show that a phase shift of ninety degrees between the vibration excitation and the bender's displacement provides the maximum power from the mechanical excitation. However, the piezoelectric material is prone to power losses; hence the bender's displacement amplitude is optimised in order to increase the amount of power which is converted into electricity. In the paper, we use active energy harvesting to control the power flow, and all the results are achieved at a frequency of 200 Hz which is well below the generator's resonant frequency.

1. Introduction

A piezoelectric generator (PEG) [1] can be used to extract energy from ambient vibrations. For that purpose, a proof mass is firmly attached to one end of a bender, while the other end is fixed onto a vibrating case [2]. The voltage supplied to the piezoelectric material produces internal stresses, which create active damping [3]. The power which brakes the movement of the proof mass is not dissipated into heat but is converted into electrical power and fed back to an electrical load.

For a given design of the PEG, the harvested power depends on the operating conditions. First, the resonant behaviour of the bender and its proof mass lead to a strong dependency of harvested power on vibrations [2]. Moreover, the impedance of the PEG and of the load should match [4], and, then, there exists an optimal electric load to be connected to the PEG connection [5, 6].

Optimal energy harvesting can be achieved using active solutions. For example, [7] proposes to use a full *H-bridge* in order to accurately control the voltage supplied to the PEG. For an excitation close to the resonant frequency of the PEG, semiactive (or semipassive) solutions can also be used. The *SSHI* technique, for example, uses a switched inductor to reverse the voltage across the piezoelectric generator and

synchronises the voltage on maxima and minima of the displacement [8]. The *SECE* technique [9] uses a flyback topology to extract the charges for the piezoelectric generator at each maximum of the voltage. In this way, energy scavenging is obtained at any load value and can be used to charge up a battery, in a wireless communication application [10], for example.

In this work, we are using active energy harvesting, because we control the instantaneous voltage across the piezoelectric material. However, this voltage is not synchronised to a current but directly to the vibration which is measured by an accelerometer. The paper is organised as follows. First, we present a model of the piezoelectric generator, and we calculate the mechanical power losses inside the device while it is being bent. Then we present experimental results of the conversion of mechanical power into electrical power at an excitation frequency which is *less than* one-half of the generator's resonance frequency.

2. Modelling

2.1. Modelling of a Piezoelectric Generator (PEG). We consider a piezoelectric bender, excited at one end by sinusoidal

FIGURE 1: The studied system.

vibration $z_c(t)$ of amplitude A and frequency ω , with a proof mass M attached to the other end, as described in Figure 1.

The model of the device is established for the actuator mode and is derived from equations proposed by [11]. In this modelling, the mass of the piezoelectric material is not taken into account, and the model is valid for frequencies below the first vibration mode of the bender and its mass. We name v the voltage across the piezoelectric material; if the piezoelectric bender is supposed to exhibit a linear behaviour, with a mechanical stiffness K_s and an internal damping D_s , the equation of the displacement w of the bender's tip is given by

$$M\ddot{w} + D_s\dot{w} + K_s w = f_p - f_{acc}, \quad (1)$$

where f_p is an internal piezoelectric force and f_{acc} is the equivalent force due to the structure's oscillations; f_{acc} is given by

$$f_{acc} = M\ddot{z}_c = -MA\omega^2 \sin(\omega t). \quad (2)$$

The piezoelectric conversion does not take into account any nonlinearity; therefore we write

$$f_p = Nv, \quad i_m = N\dot{w}, \quad (3)$$

where i_m is a motional equivalent current and N is called the piezoelectric force factor. Finally, the electrical behaviour is modelled using (4), where i is the current supplied to the bender and C_b is the equivalent blocked capacitance of the piezoelectric generator

$$i - i_m = C_b \frac{dv}{dt}. \quad (4)$$

This model can be represented using the energetic macroscopic representation because this representation tool is suitable not only to deduce by inversion control laws [12] but also to obtain the power flowing into the system. For the purpose of a better understanding of the representation, [13] introduces two variables f_s and f given by

$$f_s = K_s w + D_s \dot{w}, \quad (5)$$

$$f = f_p + f_s. \quad (6)$$

FIGURE 2: Response of $w(t)$ to voltage step: comparison with modelling.

TABLE 1: Parameters of the modelling for a CMPB01 from Noliac.

M	D_s	K_s	N
1.2 g	0.17 Nm/s	10 kN/m	0.012 N/V

Thus (1) can be revised into:

$$\dot{w} = \frac{1}{M} \int (f - f_{acc}) dt. \quad (7)$$

The representation in Figure 3 depicts the relationship between each of the variables of the system, which are represented by arrows; on each side of each block, we find two variables: one input and one output. By multiplying an input by an output, we obtain the power flowing in the system. For example, $P_1 = f_{acc} \times \dot{w}$ is the power provided to the environment; because the modelling is established in an actuator mode, P_1 should be negative in an energy harvesting operation. In the same way, $P_2 = v \times i_m = Nv \times \dot{w}$ is the mechanical power which is converted to electrical. The power appearing in Figure 3 will be used later in the paper.

Each block is a nutshell, inside which we can find an equation. The orange rectangles encircled by a red line correspond to (7) (block M), (5) (block K_s), and (4) (block C_b). It is important to notice that these blocks also represent energy storage in the system. For example, the block K_s stores elastic energy, while the block C_b stores electrostatic energy.

The orange circle shows the electrical to mechanical conversion stage inside the piezoelectric material, as presented by (3): on the left side of the circle, the energy is electrical; on the other side, the energy is mechanical. The two aggregated rectangles depict the internal coupling inside the piezoelectric bender, producing the spring-mass system. Finally, the environment is represented by two green ellipses (blocks SE and SM) at the edge of the representation.

We compare these modelling results to actual measurements in Figure 2. For this purpose, we used a bender which is a CMPB01 from Noliac [14], and we applied a step variation of v . Hence, for these trials, the piezoelectric generator actually is an actuator.

The parameters used for the simulation are given in Table 1.

FIGURE 3: Energetic macroscopic representation of the system.

Simulation and modelling are similar, validating the approach. In the next section of the paper, we focus our attention on the mechanical power which can be harvested from the mechanical excitation.

2.2. Harvested Power from Vibrations. The instantaneous mechanical power harvested at the proof mass p_1 is provided by the mechanical source SM and is given by

$$p_1 = f_{acc} \times \dot{w}. \quad (8)$$

By convention, p_1 should be < 0 in the energy harvesting mode. If we introduce γ , the acceleration of the rigid structure, as $\gamma = \ddot{z}_c$, then (8) becomes

$$p_1 = M\gamma \times \dot{w}. \quad (9)$$

If we consider monochromatic excitations, we make the assumption that both the acceleration of the rigid structure and the displacement w of the bender's tip are sinusoidal functions of time. We then state

$$\begin{aligned} \gamma &= \Gamma \sin(\omega t), \\ w &= W \sin(\omega t - \varphi), \end{aligned} \quad (10)$$

where Γ is the amplitude of the vibration's acceleration, W is the amplitude of the displacement, and φ is the phase shift which exists between $w(t)$ and $\gamma(t)$. Under these considerations, the average power harvested at the proof mass is equal to

$$P_1 = -\frac{1}{2}MT\omega W \sin(\varphi). \quad (11)$$

In Figure 4, we show the power which is provided by the system to its environment, at a given vibration level (Γ), frequency ω , and displacement (W).

This figure illustrates that P_1 depends on the phase shift between the vibration acceleration and the tip's displacement; P_1 can be

- (i) positive if $\sin(\varphi) < 0$, and the displacement of the mass generated a power need;
- (ii) equal to 0 if $\varphi = \pi$, and the displacement of the mass does not produce power;
- (iii) negative if $\sin(\varphi) > 0$, when energy harvesting occurs.

FIGURE 4: Power provided to the system's environment as a function of φ .

Hence, we can conclude that energy harvesting operation should occur at $\varphi = \pi/2$. To achieve this condition at any vibration frequency, $w(t)$ has to be controlled. This can be achieved with an active energy harvesting circuit. However, the power which is harvested at the proof mass is not totally converted into electricity. In fact, the piezoelectric material is prone to mechanical power losses, modelled by the parameter D_s . The next section considers these power losses in the energy harvesting operation.

2.3. Power Losses in the Bender. The power losses which are considered in this paper are *hysteresis* power losses and are localised within the block K_s of Figure 3. The hysteretic behaviour can be observed in the generator mode, when supplying a sinusoidal voltage to the bender, as depicted in Figure 5 for the same bender used in Figure 2.

In [15], the authors calculate the power losses due to the mechanical hysteresis. The average value over a period of time is given by

$$P_H = \frac{1}{2}K_s\omega W^2 \tan(\phi), \quad (12)$$

where $\tan(\phi)$ is a parameter of the piezoelectric material.

In Figure 6, the mechanical power losses are compared to the analytical modelling of (12) at a frequency of 100 Hz. The value found for the hysteresis power losses is equal to $\tan(\phi) = 0.13$, which is consistent with the value from Noliac.

FIGURE 5: Hysteretic behaviour of the bender for several voltage amplitudes.

FIGURE 6: Hysteretic power losses as a function of W ; •: experimental measurements; —: modelling.

Hence, these power losses increase with the displacement amplitude of the bender's tip, and, to increase the amount of power which can be converted into electricity, we note that W should be as small as possible, for a given frequency. However, reducing W also reduces the amount of power which is extracted from the proof mass, resulting in an optimal operating point, as presented in the next section.

In addition to the mechanical hysteresis losses, dielectric losses and electrical to mechanical conversion losses also occur. However, in this paper, we do not take into account these power losses.

2.4. Optimal Control of the Bender's Speed at a Given Pulsation ω . To obtain larger converted power, it is essential to lower the mechanical power losses, leading to the conclusion that the displacement amplitude W should be as small as possible. Equation (11) infers an optimal value of $\varphi = \pi/2$.

With $\varphi = \pi/2$, if we increase the vibration speed—for example, by adjusting the voltage across the piezoelectric material—we can harvest more power from the rigid structure. However, at the same time, we also increase the mechanical power losses. Figure 7 depicts this situation. In this figure, we represent $-P_1$ as a function of W , for $\varphi = \pi/2$, as well as P_H . We can see that, for low vibration amplitude, $P_H < -P_1$. For this condition, we convert mechanical power into electricity.

The power which is converted into electricity is named P_2 and is expressed by

$$P_2 = P_1 + P_H. \quad (13)$$

FIGURE 7: P_H and $-P_1$ as functions of W .

FIGURE 8: P_2 as a function of φ for $W_1 > W_2 > W_3$; • is the optimal point.

which results in

$$P_2 = \frac{1}{2}W\omega(-M\Gamma \sin(\varphi) + K_s W \tan(\varphi)). \quad (14)$$

Figure 8 shows the effect of φ on energy harvesting. In this figure, we represent P_2 as a function of φ , for several values of W .

We see that, for these three values of W , the minimal power is obtained for $\varphi = \pi/2$. Moreover, we observe that, for $W = W_1$, P_2 is always positive. For that operating condition, $|P_1|$ is large but not large enough to compensate for the large power losses induced by the displacement of the bender's tip. For $W = W_3$, the power losses are the smallest, because W_3 is the smallest, and mechanical power can be converted into electrical power. But more power can be converted for $W = W_2$ even if for that point we have more power losses due to a higher displacement compared to the case $W = W_3$. Indeed, we also have more mechanical input power $-P_1$.

FIGURE 9: Bender system.

There exists then an optimal point, which minimizes the power P_2 . Equation (14) gives rise to

$$W_{\text{opt}} = \frac{MT}{2K_s \tan(\phi)}, \quad \varphi_{\text{opt}} = \frac{\pi}{2}. \quad (15)$$

Hence, we obtain two conditions to harvest a maximum energy from the mechanical excitation. First, the bender's movement should be synchronised to the excitation's acceleration. This implies that the acceleration should be measured, which is not often the case in energy scavenging application. However, there exist some cases where acceleration can be measured; for example, in [16], the authors propose to use the accelerometer embedded in a smartphone to synchronize the bender's deflection on the shaking produced by the walking motion of the user to harvest energy. A second condition leads to the optimal displacement amplitude for which a maximum amount of electrical power can be generated. This optimal displacement amplitude depends on the excitation's magnitude.

In the next section of the paper, we will try to confirm these optimal conditions through an experimental study.

3. Experimental Study

3.1. Presentation of the Experimental Test Bench. Figure 9 shows the experimental setup. The cantilever (CMBP01 from *Noliac*) is pinched by a plastic clamp, following the manufacturer's recommendations [14]. On the other end a magnet is glued. This magnet is used as the proof mass for the study. A Hall effect sensor (FHS40-P 600 from *LEM*) faces the magnet and thus gives a voltage which is a function of the bender end $w(t)$, achieving a position sensor.

In order to apply an acceleration force to the structure, the clamp is firmly attached to a shaker (*the modal shop inc.*, mod 2110E) through a wood frame, as presented in Figure 10, and the control scheme is depicted in Figure 11.

The experimental test bench consists of

- (i) an accelerometer (352C68 from *PCB Piezotronics*), a conditioner (DVC-8/4 from *Vibration World*), and a filter, to measure the structure's acceleration and thus estimate f_{acc} ;

FIGURE 10: Experimental setup.

FIGURE 11: Experimental model.

- (ii) a Digital Signal Processor (TMS32F4 from *STM*) which calculates the voltage v from the acceleration, according to an adjustable delay φ and amplitude;
- (iii) a PWM amplifier which supplies the sinusoidal voltage to the bender;
- (iv) a RS232 serial communication between a laptop computer and the DSP to input the value of φ .

A specific control of the bender's position $w(t)$ is needed and should require a position control, as shown in [16]; however, in our case, this could not be achieved since it has produced instabilities. In fact, the high bandwidth of the piezoelectric generator (~ 450 Hz) requires a high sampling frequency, which could not be handled by the DSP. During operation, the closed loop control was found unstable for the generator's eigenfrequency, leading to high frequency oscillations superimposed on the low frequency displacement produced by the shaker. Faster and more expensive controller

FIGURE 12: Hysteresis loop in the $f_e(w)$ plane for two conditions; measurements have been filtered.

should be used to accurately control $w(t)$. To remove these instabilities, we have chosen to keep an open loop: the voltage is controlled according to the acceleration of the structure.

However, the structure of the experimental test bench was found to be flexible. This results in an acceleration $\gamma(t)$ which differs from the acceleration of the shaker, because of the mechanical response of the structure. We have found that there exists a phase shift of 80 deg between the acceleration at the location of measurement and the case's acceleration. This correction angle was taken into account in the measurement of $\gamma(t)$.

In addition to these devices, an oscilloscope (Agilent, MSO6034A) measures $w(t)$, $v(t)$, and $\gamma(t)$ in order to estimate P_2 and measures the actual value of φ .

3.2. Estimation of P_2 . The bender chosen for the experimental study exhibits a large amount of power losses. However, we can estimate the mechanical power which is converted into electrical power, using $p_2 = Nv \times \dot{w}$. In fact, we are interested in P_2 , the average value calculated over a period of excitation. We should then calculate

$$P_2 = \frac{1}{T} \int_{(T)} Nv \times \dot{w} dt. \quad (16)$$

This calculation is possible if we measure the displacement speed directly. We could obtain $\dot{w}(t)$ by derivation of $w(t)$. However, $w(t)$ is a very noisy signal, and derivation gives bad results.

This is why P_2 is calculated from the hysteresis loop $f_e(w)$. For that purpose, (16) is revised into

$$P_2 = \frac{1}{T} \int_{\text{cycle}} f_e dw \quad (17)$$

which is the inner surface of the hysteresis $v(w)$ in one cycle of the excitation, divided by the excitation period T . In Figure 14, such hysteresis loops are presented for two conditions; it shows how the vibration conditions modify the hysteresis loop.

In Figure 12 two cycles are presented. The biggest one is operated in the clockwise direction and results in a positive power. The smallest one corresponds to lower value, because its cycle is thinner. Moreover, the cycle is operated counterclockwise and results in a negative power P_2 ; this last example represents the energy harvesting condition.

FIGURE 13: Power converted into electricity; $\blacklozenge$: $W = 11 \mu\text{m}$ $\blacksquare$: $W = 18 \mu\text{m}$ $\blacktriangle$: $W = 30 \mu\text{m}$ $\bullet$: $W = 47 \mu\text{m}$.

FIGURE 14: P_2 as a function of W at $\varphi = \pi/2$; $\blacktriangledown$: experimental measurements, —: modelling.

3.3. Power as a Function of φ . In this experiment, we shake the bender at constant frequency ($f = 200 \text{ Hz}$) and at constant vibration amplitude ($\Gamma = 3 \text{ g}$). We then control the voltage across the piezoelectric material v , in order to obtain a constant displacement amplitude, but phase is shifted by an angle φ compared to $\gamma(t)$.

We then draw in Figure 13 the power which is converted into electricity as a function of φ and for several displacement amplitudes W . Measurements are compared to the theory.

The results are found to be consistent with theory, Figures 8 and 13 are similar, and the effect of the phase shift is clearly demonstrated by this experimental study.

At $\varphi = \pi/2$, mechanical power can be converted into electricity for $W = 11 \mu\text{m}$. The amount of power is estimated to be equal to $P_2 = 132 \mu\text{W}$.

However, because the device does not operate at its mechanical resonant frequency, the voltage required to obtain $W = 11 \mu\text{m}$ is high: a peak-to-peak voltage of 15 V was needed. This value leads to dielectric power losses in the piezoelectric material, which are in the same order of P_2 . As a consequence, with our device, it was not possible to actually harvest energy: the amount which was converted into electricity was dissipated into dielectric losses.

3.4. Power as a Function of the Vibration Amplitude W . In this experimental study, we control the phase shift φ between W and γ in order to have $\varphi = \pi/2$. We measure then the power converted into electricity P_2 for $\Gamma = 3 \text{ g}$, $\omega = 2 \times \pi \times 200 \text{ Hz}$.

The results are presented in Figure 14 and compared to the analytical model.

We find that theory is confirmed by the experimental study. The optimal displacement calculated with the data of Table 1 and $\tan(\phi) = 0.13$ is equal to $W_{\text{opt}} = 13.5 \mu\text{m}$. This value is close to the experimental value found between $11 \mu\text{m}$ and $18 \mu\text{m}$.

4. Conclusion

In this paper, we considered the condition to harvest the maximum amount of energy from the mechanical excitation. A model of a cantilever piezoelectric generator is derived and gives rise to two conditions, in term of displacement amplitude of the bender's tip and in terms of synchronization of this movement to the excitation's acceleration of the structure. These conditions were verified through an experimental study. This study was achieved at 200 Hz, while the resonant frequency of the bender was 450 Hz.

Despite this result, we could not harvest electricity with our device. Indeed, when shifting away the PEG's resonant frequency, the voltage needed for the same displacement increases, and more power is lost into the dielectric power losses inside the piezoelectric material. This is why further work is needed to design a new device, which will harvest more mechanical power for the same dielectric losses. This can be achieved by increasing the proof mass, for example.

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