Hindawi Publishing Corporation
International Journal of Atmospheric Sciences
Volume 2013, Article ID 419694, 7 pages
http://dx.doi.org/10.1155/2013/419694

Research Article

Self-Organized Criticality: Emergent Complex Behavior in PM_{10} Pollution

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Received 14 February 2013; Revised 9 May 2013; Accepted 16 May 2013

Academic Editor: Prodromos Zanis

We analyze long-term time series of daily average PM_{10} concentrations in Chengdu city. Detrended fluctuation analysis of the time series shows long range correlation at one-year temporal scale. Spectral analysis of the time series indicates $1/f$ noise behavior. The probability distribution functions of PM_{10} concentrations fluctuation have a scale-invariant structure. Why do the complex structures of PM_{10} concentrations evolution exhibit scale-invariant? We consider that these complex dynamical characteristics can be recognized as the footprint of self-organized criticality (SOC). Based on the theory of self-organized criticality, a simplified sandpile model for PM_{10} pollution with a nondimensional formalism is put forward. Our model can give a good prediction of scale-invariant in PM_{10} evolution. A qualitative explanation of the complex dynamics observed in PM_{10} evolution is suggested. The work supports the proposal that PM_{10} evolution acts as a SOC process on calm weather. New theory suggests one way to understand the origin of complex dynamical characteristics in PM_{10} pollution.

1. Introduction

The adverse effects of PM_{10} have been recognized in environmental sciences. Besides the reduction of visibility, the direct impact on human health via inhalation is an important issue [1]. It will be very useful to develop accurate PM_{10} concentrations forecasting methods, which can help to put forward effective warning strategies to reduce impacts on public health during episodes or poor air quality [2]. In recent years, some PM_{10} concentrations forecasting methods have been developed [3]. These methods mainly come from two approaches. One is to establish accurate atmospheric model based on meteorologic, physical, and chemical process. The other is to find inherent correlations based on the statistical analysis of the collected data. However, there are still some pending problems with predicting PM_{10} concentrations. PM_{10} evolution is highly complex events involving human factors as well as meteorologic, topographic, physical, and chemical conditions. Interrelationships between these processes and PM_{10} concentrations are complex and nonlinear. The circumstances that determine high PM_{10} concentrations are uncertain sometimes [4]. So the microscopic physical and chemical mechanisms that drive PM_{10} temporal evolutions are not well understood. However, even if such microcosmic dynamical mechanisms have been illuminated, it is likely that the system would be highly nonlinear without any simple way to predict emergent behavior [5].

As asked by Nagel [6], "is there anything we can say about these systems from first principles without knowing about the 'microscopic' details of the problem?" It is obvious that the complexity theory and related methods are needed when confronted with these complex and nonlinear problem. So many researchers have investigated the nonlinear dynamics of air pollutants evolution with different aspects [7–9]. The relational studies have observed that air pollutants concentrations exhibited long range correlation, scale-invariant, and multifractal behavior. These progresses enhance our fundamental knowledge on the complex structure of PM_{10} concentrations. Thus, one has to be very careful to employ the standard statistical methods in air pollution prediction (as, e.g., ARIMA and ARIMAX models). If one wants to make robust forecasting, the first important thing to do is to

identify the structure of the process. If it is approximately linear, then a linear method (e.g., an autoregressive one) can be helpful. On the contrary, if it is nonlinear, completely different types should be used. In this case, predicting models can be validated by using the information on the persistence characteristics of air pollution time series.

However, why do the complex structures of PM_{10} concentrations evolution exhibit scale-invariant?

As an introduction to the concept, self-organized criticality (SOC) has been proposed by Bak et al. [10] to provide a framework of modeling such phenomena as persistent behavior, $1/f$ noise, and scale-invariant, which are widespread in nature. The Bak-Tang-Wiesenfeld (BTW) sandpile model is a classical numerical model in SOC theory. This concept is well illustrated with a model of sandpile. One considers a grain is dropped into a sandpile randomly and slowly. As we add new grains, the pile grows more and more until the pile reaches a critical slope in a statistically stationary state. At some critical point, the addition of other grains may cause either small avalanches or trigger a very large avalanche. Some phenomena, such as power-law temporal correlations and scale-invariant, emerged from SOC state in a dissipative system. The issue of determining whether evolution of PM_{10} is governed by SOC is a difficult one. There are no unequivocal determining criteria. One approach is to compare characteristic measures of air pollution process to those obtained from a known SOC system [11, 12]. We consider that PM_{10} pollution is a complex dynamical system, which will automatically adjust itself to a critical state characterized by power-law correlations in both space and time. This state is "critical" in the sense of an equilibrium critical point where there is no characteristic length or time scale that controls the behaviour of the system. So PM_{10} pollution can be analogous to a variety of nonequilibrium relaxation processes in nature such as earthquakes and avalanches.

In this work, at first, long range correlation, $1/f$ noise, and scale-invariant of PM_{10} concentrations measured at Chengdu are examined by detrended fluctuation analysis, power spectrum, and cumulative magnitude-frequency distribution, respectively. Then, in the SOC framework, we put forward the self-organized evolution theory of PM_{10} pollution under calm condition. The SOC mechanism of PM_{10} evolution is discussed. At last, a modified sandpile model of SOC, which describes the essential mechanisms of self-organized evolution of PM_{10}, is put forwarded to simulate the scaling characteristic of PM_{10} concentrations and to illuminate the origin of scale-invariant in PM_{10} pollution. This work provides new insight and approaches to research the nonlinear dynamics and emergent complex behavior in PM_{10} evolution.

2. Materials and Methods

2.1. Data. Chengdu city is located in western Sichuan Basin of China. Sichuan Basin covers 260,000 km^2, generally at low altitudes of about 500 m. These lower lying areas are surrounded by mountains and a plateau higher than 4 km. The unique geographical environment directly affects meteorological condition of pollutant diffusion and increases the frequency of calm wind at Chengdu. At Chengdu city, the average annual frequency of calm wind, namely, wind speed being 0~0.2 m/s, is 46%. So the local pollution sources play a more significant role in air quality of Chengdu [13].

There are eight automatic monitoring stations at Chengdu city. The daily average concentrations of pollutants are made at each station. These concentrations are further averaged over the stations to provide the daily average values of pollutant to represent the daily average air quality of Chengdu city. In this work, the examined data set is daily average PM_{10} concentrations data from 2001 to 2011 in Chengdu city, as shown in Figure 1. The length of data set is 4018. These data are provided by the Sichuan Environment Monitoring Center. These data are characterized by many large fluctuations with no obvious correlation that is difficult to interpret (see Figure 1).

2.2. Detrended Fluctuation Analysis. Detrended fluctuation analysis (DFA), which was proposed by Peng et al. [14], can be used to demonstrate the scaling behavior associated with SOC systems. It can avoid spurious detection of correlations that are artifacts of nonstationary, which often affects the time series data. It is that, for sample size n, the root mean square fluctuation of this integrated and detrended time series $F(n)$ behaves as a power-law function of n within the scaling region, data present scaling

$$F(n) \propto n^H. \tag{1}$$

A DFA exponent $H = 0.5$ indicates a wholly stochastic process lacking correlation; $0.5 < H \leq 1$ indicates persistent long range power-law correlations; for $0 < H \leq 0.5$, power-law anticorrelations are present such that large values are more likely to be followed by small values and vice versa. Although a DFA exponent between 0.5 and 1.0 does not absolutely prove the presence of SOC, it is a strong (in a statistical sense) indication of the presence of SOC.

2.3. Spectral Analysis. The power spectrums have been used in investigating the $1/f$ noise behavior of some time series. If a time series spectrum obeys a power-law form

$$S(f) \propto f^{-\beta}, \tag{2}$$

where f is the frequency, it indicates the absence of a characteristic time scale, that is, a scaling behavior. Thus, fluctuations at all scales are related to each other and a fractal behavior may be assumed. Malamud and Turcotte [15] have shown theoretically that one should have $\beta = 2H - 1$. However, in practice this relation is only weakly satisfied.

2.4. Cumulative Magnitude-Frequency Distribution. In some natural phenomena, cumulative magnitude-frequency distributions exhibit power-law scaling. It is regarded as the typical "critical" dynamical behavior of SOC systems [16]. A power-law applied to a cumulative distribution has the relation

$$N(c > c_0) \propto \int_{C_0}^{\infty} c^{-\tau} dc \propto c^{-(\tau+1)}, \tag{3}$$

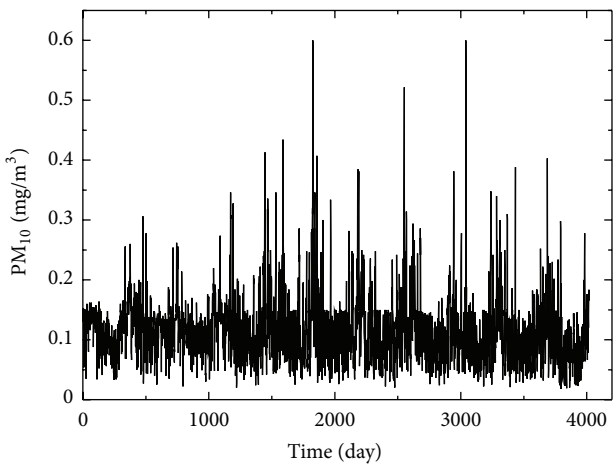

Figure 1: The daily average PM_{10} concentrations data from 2001 to 2011 in Chengdu city. The data lengths all are 4018 (days).

where N is the cumulative number of events with size greater than the magnitude (c) and τ is the scaling exponent.

2.5. Simplified Sandpile Model for PM_{10} Pollution. In the study, a general sandpile model for PM_{10} pollution has been established with a nondimensional formalism.

The model is defined on a square lattice of size $L \times L$ with open boundaries in 2D. The boundaries of the lattice are open. So sands are allowed to leave the system through the boundaries. A given amount of PM_{10} pollutants $h(i, j)$ is associated with each site (i, j).

Driving mechanism: at a given time, as a consequence of pollution source emission at some site (i, j), amount of PM_{10} pollutants changes as follows:

$$h(i, j) = h(i, j) + \Delta h. \tag{4}$$

Redistribution and relaxation mechanism: when the amounts of PM_{10} pollutants at some site (i, j) reach some threshold magnitude h_c, the site becomes unstable or critical and it relaxes by a toppling. The redistribution and relaxation rule is

$$\begin{aligned} h(i, j) &\longrightarrow \frac{\Delta h}{5}, \\ h(i \pm 1, j) &\longrightarrow h(i \pm 1, j) + \left[h(i, j) + \frac{4}{5}\Delta h\right] \times 0.24, \\ h(i, j \pm 1) &\longrightarrow h(i, j \pm 1) + \left[h(i, j) + \frac{4}{5}\Delta h\right] \times 0.24. \end{aligned} \tag{5}$$

This rule will be circulating running in accordance with the above method until a new stable configuration is reached, namely, all $h(i, j) < h_c$. Avalanche size (s) is measured as the total number of toppling during an avalanche.

These rules represent the movements and transformation process of PM_{10} under calm condition. When PM_{10} pollutants diffuse, the site of pollution source will reserve partial pollutants, which are set to one-fifth of its original value. At the same time, owing to precipitation, adsorption, and chemical action, some PM_{10} pollutants will be lost during transportation and diffusion. In our model, we presume that 4% of PM_{10} pollutants will be lost when they topple to the four adjacent neighbor sites. So the model is local and nonconservative.

Temporal degradation mechanism: PM_{10} pollutants will decay with time owing to self-purification of atmospheric environment. We simplify this process and presume that degradation of PM_{10} follows the first level of decaying kinetics. So when a new stable configuration is reached after each relaxation rule, PM_{10} pollutants at all sites will decay to e^{-k} of the original level as follows:

$$h(i, j) \longrightarrow h(i, j) \times e^{-k}. \tag{6}$$

After all lattice sites are stable, another grain of sand is added.

If PM_{10} evolution is an example of a SOC process, the avalanche size distribution will follow power-law distribution. In a nondimensional formalism, we select $\Delta h = 1$ and $h_c = 4$ referring to the classical BTW sandpile model.

3. Results and Discussion

3.1. Long Range Correlation as Detected by DFA Method. Figure 2 shows the DFA for daily average PM_{10} concentrations data of Chengdu from 2001 to 2011. The observed $F(n) \propto n^H$ relationship shows obviously two different period regimes, with a critical time scale (n_c) of about one year. Fitting by the least square method the $F(n) \sim n$ plot, for $n < n_c$, $H_1 = 0.829$, while $n > n_c$, $H_2 = 0.493$. For one-year periods ($n < n_c$), it indicates high persistence. For example, there is a tendency for increase in PM_{10} concentration to be followed by another increased tendency in PM_{10} concentration at a different time in a power-law fashion. This suggests that the correlations between the fluctuations in PM_{10} concentrations do not obey the classical Markov-type stochastic behavior (exponential decrease with time) but display more slowly decaying correlations. Over longer time periods, $n > n_c$, H_2 is close to 0.5 and indicates that the fluctuations in PM_{10} concentrations behave like the stochastic process at a large temporal scale. This phenomenon perhaps reflects an influence of the annual climate cycle.

3.2. $1/f$ Noise as Detected by Spectral Analysis. On the power spectrum plot shown in Figure 3, we can see that the power spectral density obeys two different power laws in the high-frequency and low-frequency regimes. We have power-law fits $\beta_1 = 1.021$ for 1 year$^{-1} < f < 1$ day^{-1} and $\beta_2 = 0.583$ for $f < 1$ year^{-1}. In shorter period, spectral analysis shows that the fluctuations in PM_{10} concentrations are characterized by $1/f$ noise and self-affine type fractal behaviors, which are similar to the results of DFA methods. For β_1 and β_2 values, we note that they are higher than the ones estimated by the relation $\beta = 2H - 1$. It shows that this relation is only weakly satisfied.

3.3. The Magnitude-Frequency Distribution of PM_{10} Concentrations. Figure 4 shows the number (N) of PM_{10} events from 2001 to 2011, with size greater than some PM_{10} concentration (c) on a double logarithmic plot. Fitting by the least

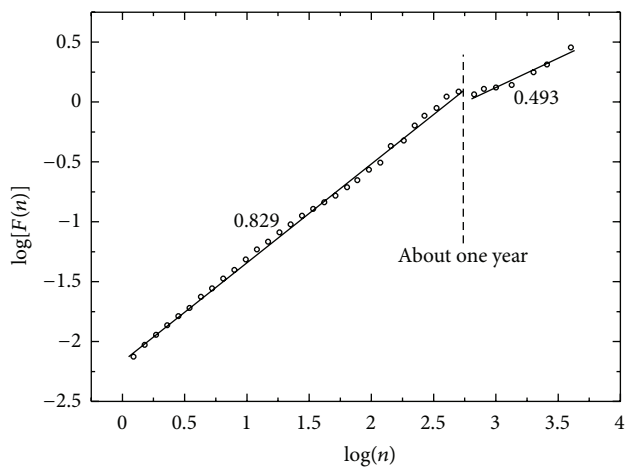

Figure 2: DFA of the study data. The dots are values of $\log[F(n)]$ against the corresponding $\log[n]$. The solid lines are power law $F(n) \propto n^H$, with $H_1 = 0.829$ and $H_2 = 0.493$, respectively. The vertical line indicates that n_c is about one year.

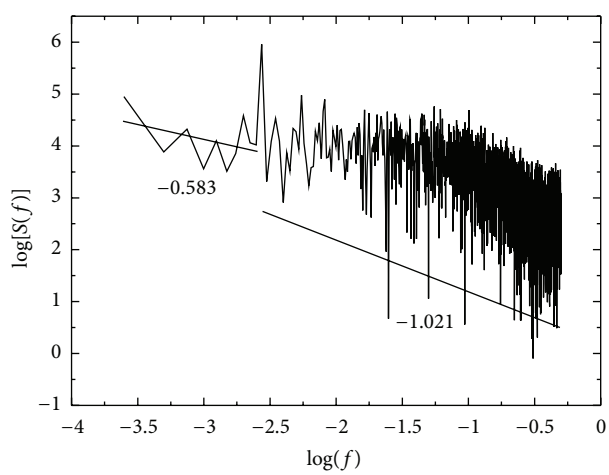

Figure 3: The power spectrum plot for the study data. The black lines are power law $S(f) \propto f^{-\beta}$, with $\beta_1 = 1.021$ and $\beta_2 = 0.583$ corresponding to high-frequency and low-frequency regimes, respectively.

square method, the scaling exponent (τ) is 3.58 according to (1). The scale invariance region starts from 0.128 mg/m^3 and ends at 0.518 mg/m^3. In the PM_{10} concentrations regions, a typical scale of pollution events does not exist. There is inherent dynamical connection among the fluctuations in PM_{10} concentrations. In smaller PM_{10} concentrations regions, the power-law breaks down obviously. We consider that low monitoring frequency of PM_{10} concentrations results in the low-size tail of the frequency distribution. A similar phenomenon in rainfall has been reported by Peters and Christensen [17].

In order to investigate the robustness of the scale invariance in PM_{10} concentrations, the same analysis is performed at the different time intervals, shown in Figure 5 (from 2006 to 2011) and Figure 6 (from 2010 to 2011). These observed

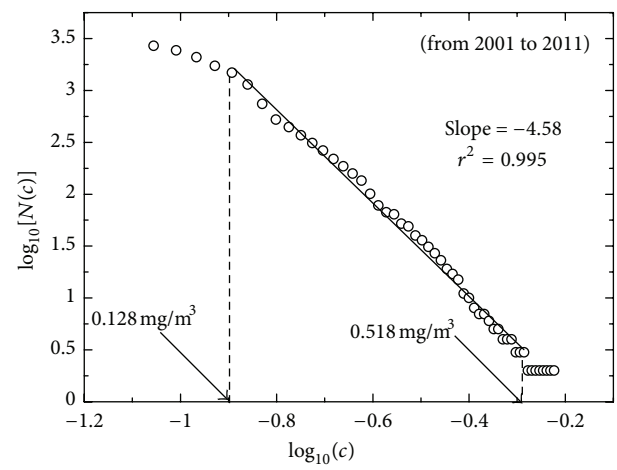

Figure 4: The number (N) of PM_{10} events from 2001 to 2011, with size greater than some PM_{10} concentration (c) on a double logarithmic plot.

statistical distributions all exhibit power laws and the scaling exponents are almost the same. The robustness to the time intervals is the critical behavior of SOC system.

3.4. Simulation Results of Sandpile Model. The SOC state is stationary in the sense that over long timescales, the average height $\langle h \rangle$ neither grows nor decays. The average height can be calculated according to the relation

$$\langle h \rangle = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} h(i, j). \tag{7}$$

The simulations are performed for 50×50 lattice sizes. The average height $\langle h \rangle$ is plotted against the number of avalanches up to 50000 in Figure 7 for $k = 3.5 \times 10^{-4}$. It can be seen that a constant average height is achieved and it remains constant over a large number of avalanches. The inset figure is a closeup on the average height curve, which looks like very low amplitude "ripples" propagating along the mean field. This phenomenon indicates that the SOC state is reached.

When reaching the nonequilibrium steady state, extensive data collection has been made for $L = 50$ in runs of 10^7 avalanches. The simulated result of avalanche size distribution is shown in Figure 8. We have found the value $\alpha = 3.57$ when $k = 3.5 \times 10^{-4}$ for the exponent of power-law relation $P(s > s_0) \propto s^{-(\alpha+1)}$, quite close to the exact value $\tau = 3.58$ in Figure 4. We emphasize that the parameter k is closely related to characteristics of PM_{10} pollutants in atmospheric environment. The emerged scale-invariant in the avalanches size distribution demands no external parameter tuning and should be seen as an evidence of SOC.

3.5. Possible Explanation of PM_{10} Evolution SOC. There are no unequivocal determining criteria to ascertain whether evolution of some natural phenomenon is governed by SOC. One accustomed approach is to compare characteristic

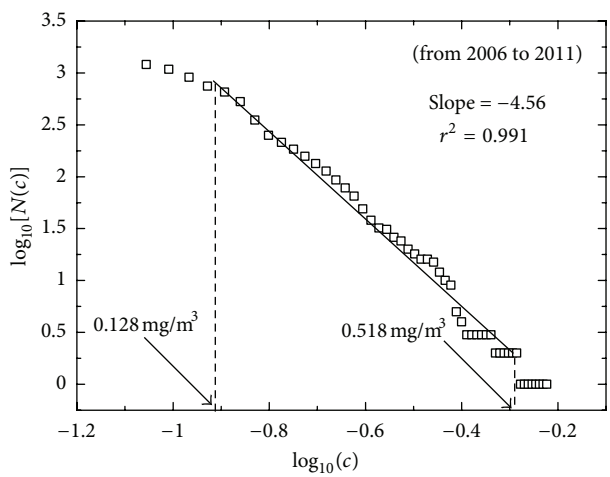

FIGURE 5: The number (N) of PM_{10} events from 2006 to 2011, with size greater than some PM_{10} concentration (c) on a double logarithmic plot.

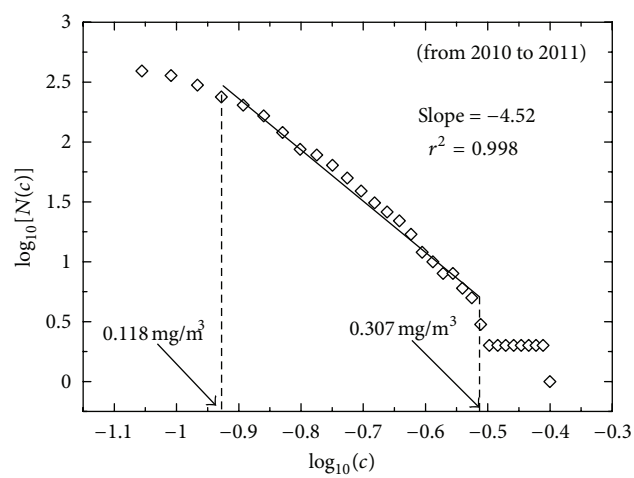

FIGURE 6: The number (N) of PM_{10} events from 2010 to 2011, with size greater than some PM_{10} concentration (We have found the value c) on a double logarithmic plot.

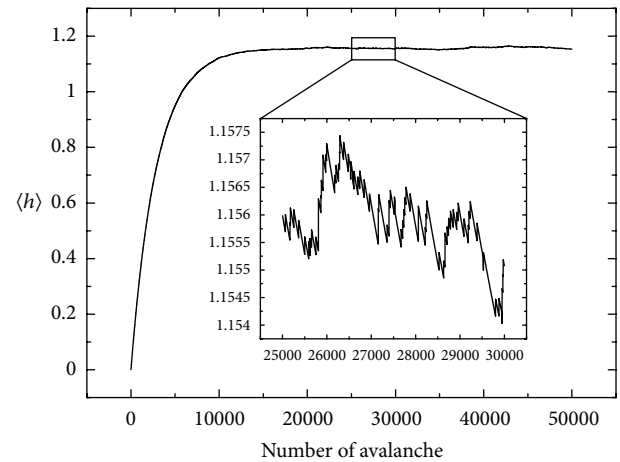

FIGURE 7: Plot of average height $\langle h \rangle$ against the number of avalanches. A closeup on the average height curve is shown in the inset.

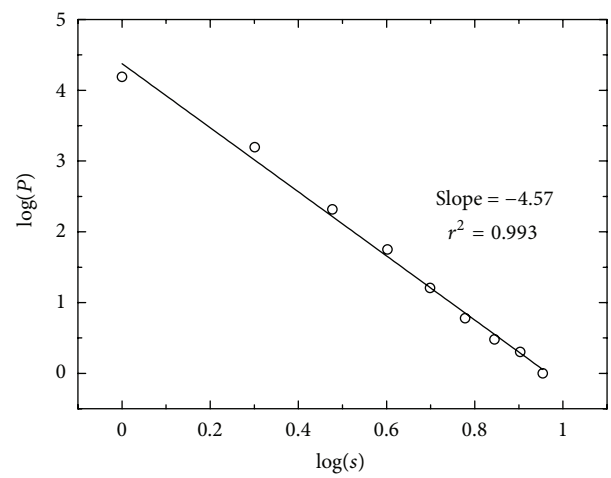

FIGURE 8: Avalanche size distribution for the PM_{10} pollution sand model when $k = 3.5 \times 10^{-4}$. It follows the power-law relation $P(s > s_0) \propto s^{-(\alpha+1)}$, with an exponent of $\alpha = 3.57$ (solid line), which is consistent with that of the size distribution of PM_{10} pollution events in Figure 4.

measures of some natural phenomenon to those obtained from a known SOC system.

To motivate comparisons between PM_{10} pollution and SOC sandpile, we firstly take a qualitative description of the complexity in PM_{10} pollution system which could give rise to SOC dynamics. PM_{10} pollution system contains many components such as pollutant sources, atmospheric pollutants components, solar radiation, wind speed, temperature, atmospheric self-purification, topographical feature, and other meteorologic factors. Each component has a certain influence on the average PM_{10} concentration each day. When all the components are considered together, they interact and correlate with each other on vastly different timescales. One group of comparatively fast radical chemical reactions relaxes on timescales of fractions of seconds up to hours, while another group the rather slow processes (e.g., the movement of PM_{10} pollutants) relaxes on timescales of days up to years or even longer. Thus, PM_{10} pollution system is a complex system composed of a series of interconnected components. These components have some complex pattern of influence on the daily average PM_{10} concentration, which result in the fluctuations of PM_{10} concentrations in long term.

We make an analogy between the sandpile and PM_{10} evolution. PM_{10} pollutants form mainly as a result of first and (or) secondary pollutants produced from the emission of air pollution sources. The driving force is the continuously PM_{10} pollutants emission in atmospheric environment, which serve as the grains continuously dropped on a pile. We can define the superposition of local PM_{10} pollutants concentration which represents the chain of forces in sandpile. When the amounts of microscopic condensed PM_{10} pollutants reach some threshold magnitude, the pollutants masses can be transported on microscopic scales by diffusion or

convection. They reach a new location, where the local PM_{10} pollutants concentration is lower and can be diluted. If the local PM_{10} pollutants concentration in the neighborhood is also high, the amount of condensed pollutants will increase. Once the system reaches some critical point, any small perturbation, in principle, can trigger a chain reaction like the avalanches in atmospheric system. The normal atmospheric environmental capability serves as the critical state. If the local PM_{10} pollutants concentration is higher than the same critical value, pollutants are assembled and precipitated in the atmosphere. Thus, the system will adapt itself by removing these dissidents to maintain the critical state just as the sandpile adapts itself by avalanching to reach its constant angle of repose. Therefore, we can define the fluctuations of PM_{10} concentrations as avalanches events in a SOC sandpile. At the critical state, long range correlation, $1/f$ noise, and scale-invariant of PM_{10} concentrations will emerge from the dissipative system. It is important to note that PM_{10} pollution system is "tuned" to a critical state solely by its own internal dynamics rather than external dynamics. The high correspondence of the simulated results to observations supports that PM_{10} evolution acts as a SOC process on calm weather. And SOC is a useful framework to explain the nonlinear evolution of PM_{10} concentrations.

The micromechanisms of PM_{10} concentrations evolution are very complex. Some microscopic physical and chemical mechanisms are still uncertain. For example, how do photochemical reaction rate change with first and (or) secondary pollutants? How do the components of pollutants affect mass transport and chemical reaction at gas and solid two phases? Based on this traditional "reductionism" science perspective, the origin of robust scale-invariant in PM_{10} concentrations evolution can be quite hard to understand. However, when we turn sight to "holism" science perspective, the satisfactory understanding is achieved to this problem. Considering the similarities between sandpile system and PM_{10} evolution, a simplified sandpile model for PM_{10} pollution with a nondimensional formalism is put forward. This model mechanism only includes the emission of PM_{10} pollutants, the movements and transformation of PM_{10}, and temporal degradation process of PM_{10}. The high correspondence of the results to observations indicates that the model provides an effective parameterization of the key physical process that governs PM_{10} concentrations evolution.

It is important to note that the power system organizes itself to an operating point near to, but not at, a critical value. This could make the system quite robust in different time intervals. One consequence is that the measured frequency of occurrence of small events can be used to estimate the frequency of occurrence of large events. For example, the recurrence interval for serious air pollution can be estimated from the frequency of smaller air pollution.

4. Conclusion

Based on DFA, power spectrum, and statistical analysis, we have identified long range correlation, $1/f$ noise, and scale-invariant of PM_{10} concentrations measured at Chengdu. These statistics seem consistent with avalanche sizes in a running sandpile known to be SOC. In order to explain the origin of scale-invariant in PM_{10} evolution, according to the characteristics of PM_{10} evolution on calm weather, a simplified sandpile model for PM_{10} pollution with a nondimensional formalism is put forward. The simulated result is consistent with the actual monitored data very well. The work supports the proposal that PM_{10} evolution acts as a SOC process on calm weather. Far from being equilibrium forms, PM_{10} pollutants will evolve into a nonequilibrium and critical state. At the critical state, long range correlation, $1/f$ noise, and scale-invariant of PM_{10} concentrations will emerge from the dissipative system. This insight will inspire new research into the macroeffect of air pollution processes and improvement of modeling of PM_{10} pollution.

Acknowledgments

The work is supported by the National Natural Science Foundation of China (no. 41105118) and Hunan Provincial Natural Science Foundation of China (13JJB012).

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