Research Article

Transverse Spin Structure Function $g_2(x, Q^2)$

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The transverse spin-dependent nucleon structure function $g_2(x, Q^2)$ at large $Q^2$ is computed within the Wandzura-Wilczek relation, using NLO $g_1$ fit. Also we investigate the twist-3 contribution of $g_2(x, Q^2)$ via calculating $d_2$. It turns out that although the twist-3 part of $g_2(x, Q^2)$ can be negligible for DIS processes, it has a significant value in resonance region.

1. Introduction

Over the past 30 years, significant progress has been made in understanding the spin structure of the nucleon through measurements using polarized deep inelastic lepton scattering (DIS). Most of these experiments were focused on precise measurements of the spin structure function $g_1$. In the naive quark parton model (QPM), $g_1$ is directly related to contributions of the individual quark flavors to the overall spin of the nucleon. Sum rules based on this simple model have provided fertile ground for understanding the origin of the nucleon spin in terms of quark degrees of freedom. In addition, next-to-leading-order (NLO) analyses of the world $g_1$ data (see, e.g., [1–5]) have provided indirect information about the role of gluons in the nucleon's spin. Polarized DIS also provides information about a second spin structure function, $g_2$, which is identically zero in the naive QPM [6]. Interest in $g_2$ arises because, unlike $g_1$, contributions from certain nonperturbative QCD processes such as quark-gluon correlations enter at the same order in $Q^2$ as asymptotically free contributions [7].

2. Theoretical Background

2.1. The $g_2$ Structure Function. The nucleon’s second polarized structure function $g_2(x, Q^2)$ had never been measured till 1990, and there had been a few theoretical studies of it. If we define $q_f(x)$ and $\bar{q}_f(x)$ as the expectation value for the number of quarks and antiquarks of flavor $f$ in the hadron whose momentum fraction lies in the interval $[x; x+dx]$, then in the parton model it can be shown that

$$F_1(x) = \frac{1}{2} \sum_f e_f^2 \left[ q_f(x) + \bar{q}_f(x) \right],$$

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \left[ q_f(x) - \bar{q}_f(x) \right],$$

(1)

where the quark charge $e_f$ enters due to the fact that the cross section is proportional to the squared charge of the target. The Callan-Gross [8] relation shows that $F_2$ can be defined entirely in terms of $F_1$, but there is no such simple physical interpretation of $g_2$. This spin-dependent structure function is determined by the $x$-dependence of the quarks transverse momenta and the off-shellness, both of which are unknown in the parton model [9]. Ignoring quark mass effect of order $O(m_q/\Lambda_{QCD})$, $g_2$ can be separated into leading twist and higher-twist components as follows:

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + g_2^T(x, Q^2),$$

(2)

where

$$g_2(x, Q^2) = - \int_1^x \frac{dy}{y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y}. \quad (3)$$

To twist-3, there are three contributions to $g_2$ [10]:

- $g_2^{ww}$: the leading twist-2 term, which depends only on $g_1$,
Using the fact that $g_1$ and $g_2$ contain the same twist-2 operator, Wandzura-Wilczek [11] derived the following expression for the asymptotically free contribution to $g_2$, in terms of $g_1$:

$$g_{2w}^{w}(x,Q^2) = -g_1(x,Q^2) + \int_{x}^{1} \frac{g_1(x',Q^2)}{x'} \ dx'$$  (4)

Also this relation can be extracted directly from twist-2 operators of OPE.

2.2. Operator Product Expansion. An appropriate formalism to study $g_2$ beyond the simple QPM is the operator product expansion (OPE) [13, 14] which is the model-independent approach based directly on QCD. The OPE allows us to write the hadronic matrix element in deep inelastic scattering (DIS) in terms of a series of normalized operators of increasing twist. The leading contribution is twist-2, with higher-twist terms suppressed by powers of $1/Q^2$. Keeping only terms up to twist-3, the moments of $g_1$ and $g_2$ at fixed $Q^2$ can be related to the twist-2 and twist-3 reduced matrix elements, $a_n$ and $d_n$ [14],

$$\int_{0}^{1} x^n g_1(x,Q^2) \ dx = \frac{a_n}{2}, \ n = 0, 2, 4, \ldots,$$
$$\int_{0}^{1} x^n g_2(x,Q^2) \ dx = \frac{1}{2n+1} (d_n - a_n), \ n = 2, 4, \ldots$$  (5)

In the expression above, $d_n$ directly appears in the equation for $g_2$ allowing us to study the higher-twist structure of the nucleon at leading order. One obtains

$$\int_{0}^{1} x^n \left[ g_1(x,Q^2) + \frac{n+1}{n} g_2(x,Q^2) \right] \ dx$$
$$= \frac{1}{2} d_n (Q^2), \ n = 2, 4, 6, \ldots.$$  (6)

Hence, for $n = 2$,

$$d_2 = 2 \int_{0}^{1} x^2 \left[ g_1(x,Q^2) + \frac{3}{2} g_2(x,Q^2) \right] \ dx.$$  (7)

At large $Q^2$, the $d_n$ matrix element is related to the color polarization, which describes how the color electric and magnetic fields respond to the nucleon spin. At lower momentum transfer, $d_n(Q^2)$ provides a mean to study the transition from perturbative to nonperturbative behavior and to quantify higher-twist effects. And although $d_n$ is a higher-twist OPE object, the definition holds for all $Q^2$.  

3. Formalism

We summarize the key features of our QCD analysis here. Knowing the fact that world data for $g_1$ cover a broad range in $x$ and $Q^2$ with relatively high precision motivates us to extract polarized parton distributions from NLO fits to the data. In our calculation, the polarized parton densities are parameterized at a starting scale $Q_0^2 = 4$ GeV$^2$ and are evolved to higher factorization scales using a numerical solution of the polarized NLO DGLAP evolutions [15–17]. We take the QCD coupling as a free parameter which is fitted to the data. The QCD DGLAP equations are solved in the Mellin space.

The centerpiece of our approach is the Jacobi polynomial expansion; this method was developed and applied to a
This relates the \( g_1(x, Q^2) \) structure function to their moments \([4]\).

In this analysis we use \( N_{\text{max}} = 9 \), \( \alpha = 3.0 \), and \( \beta = 0.5 \). The world data are at relatively large \( Q^2 \), where higher-twist effects should be negligible; therefore the evolved parton distributions allow one to calculate the twist-2 spin-dependent structure functions \( g_1 \) and \( g_2^{WW} \) in most kinematic regions accessible today. Due to the fact that our calculations are performed in DIS region where transverse spin is a suppressed leading twist, we can neglect the twist-3 part of \( g_2 \), \( \zeta(y, Q^2) \). Also, ignoring the masses of three light quarks is another fact that allows us to have an estimate of \( g_2 \), by these approximations, \( g_2 \approx 0 \) then \( g_2^{WW} \) (see (2), (3)). By the last assumption, we can see that the contribution of quark transverse distribution, \( h_T \), is omitted too. Precise measurements of \( g_2 \) at specific values of \( x \) and \( Q^2 \) can be compared to \( g_2^{WW} \), providing a unique opportunity to cleanly isolate higher-twist contributions.

### 4. Results

We have performed a QCD analysis of the inclusive polarized DIS data at NLO and extracted the spin structure function \( g_1(x, Q^2) \). We have used an expansion in the Jacobi polynomials to facilitate the analysis. The Jacobi polynomials help us to produce \( g_1(x, Q^2) \) in an analytical (not numerical) form.

For the measured region \( 0.03 < x < 0.8 \), proton and deuteron spin structure functions \( g_2^{WW}(x, Q^2) \) are calculated using our parametrization on \( g_1(x, Q^2) \) and the possible contributions of higher-twist effects to \( g_2^{p,d} \) are quantified.

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**Figure 3:** \( xg_2 \) for the proton and deuteron as a function of \( Q^2 \) for selected value of \( x \). Data are for E155 [12] experiment. The error bars shown are the statistical and systematic ones added in quadrature. The curves show \( xg_2^{WW} \) based on model (solid) and the BB model (dash-dot) [2].

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\[ xg_1^{N_{\text{max}}} = x^\beta (1 - x)^\alpha \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha, \beta}(x) \sum_{j=0}^{n} \left( \alpha, \beta \right) [xg_1, j + 2]. \]
Table I: Comparison of model results with proton and deuteron data.

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<td>Lattice QCD [27]</td>
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<td>(-22 \pm 6 \times 10^{-3})</td>
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using twist-3 reduced matrix element \(d_2\) at fixed \(Q^2\). By using (7) and substitution \(g_2^{WW}\) for \(g_2\), as discussed, we neglect the contribution from the region \(0 < x < 0.03\) because of the \(x^2\) suppression factor. For \(0.8 < x < 1\), we assume that both \(g_1\) and \(g_2\) behave as \((1 - x)^3\). We obtained the value of \(d_p^{(2)} = 1.3 \times 10^{-2}\) and \(d_d^{(2)} = -8.8 \times 10^{-3}\). The computational processes were performed at \(Q^2 = 5\) GeV\(^2\). The consistency of these values with zero is another reasonable proof for our assumptions in the most common kinematic region. Comparisons of our results for the third moments of \(g_2\) with recent data and other model predictions are listed in Table I.

Figure 3 shows \(xg_2\) for the proton and deuteron as a function of \(Q^2\) for selected value of \(x\). Data are for E155 experiment. The error bars shown are the statistical and systematic ones added in quadrature. The curves show \(xg_2^{WW}\) based on our model (solid) and the BB model (dash-dot), both using NLO \(g_1\) fit, which are in good agreement with the data. \(xg_2\) from E143 (square) [20] and E155 (circle) [12] experiments together with our twist-2 \(g_2\) at \(Q^2 = 5\) GeV\(^2\) for \(xg_2^p\) and \(xg_2^d\) are presented in Figures 1 and 2.

Disclosure

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References
