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Research Article

Transverse Spin Structure Function $g_2(x, Q^2)$

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The transverse spin-dependent nucleon structure function $g_2(x, Q^2)$ at large Q^2 is computed within the Wandzura-Wilczek relation, using NLO g_1 fit. Also we investigate the twist-3 contribution of $g_2(x, Q^2)$ via calculating d_2. It turns out that although the twist-3 part of $g_2(x, Q^2)$ can be negligible for DIS processes, it has a significant value in resonance region.

1. Introduction

Over the past 30 years, significant progress has been made in understanding the spin structure of the nucleon through measurements using polarized deep inelastic lepton scattering (DIS). Most of these experiments were focused on precise measurements of the spin structure function g_1. In the naive quark parton model (QPM), g_1 is directly related to contributions of the individual quark flavors to the overall spin of the nucleon. Sum rules based on this simple model have provided fertile ground for understanding the origin of the nucleon spin in terms of quark degrees of freedom. In addition, next-to-leading-order (NLO) analyses of the world g_1 data (see, e.g., [1–5]) have provided indirect information about the role of gluons in the nucleon's spin. Polarized DIS also provides information about a second spin structure function, g_2, which is identically zero in the naive QPM [6]. Interest in g_2 arises because, unlike g_1, contributions from certain nonperturbative QCD processes such as quark-gluon correlations enter at the same order in Q^2 as asymptotically free contributions [7].

2. Theoretical Background

2.1. The g_2 Structure Function. The nucleon's second polarized structure function $g_2(x, Q^2)$ had never been measured till 1990, and there had been a few theoretical studies of it. If we define $q_f(x)$ and $\overline{q}_f(x)$ as the expectation value for the number of quarks and antiquarks of flavor f in the hadron whose momentum fraction lies in the interval $[x; x+dx]$, then in the parton model it can be shown that

$$F_1(x) = \frac{1}{2}\sum_f e_f^2 \left[q_f(x) + \overline{q}_f(x)\right],$$
$$g_1(x) = \frac{1}{2}\sum_f e_f^2 \left[q_f(x) - \overline{q}_f(x)\right], \tag{1}$$

where the quark charge e_f enters due to the fact that the cross section is proportional to the squared charge of the target. The Callan-Gross [8] relation shows that F_2 can be defined entirely in terms of F_1, but there is no such simple physical interpretation of g_2. This spin-dependent structure function is determined by the x-dependence of the quarks transverse momenta and the off-shellness, both of which are unknown in the parton model [9]. Ignoring quark mass effect of order $\mathcal{O}(m_q/\Lambda_{\mathrm{QCD}})$, g_2 can be separated into leading twist and higher-twist components as follows:

$$g_2\left(x, Q^2\right) = g_2^{WW}\left(x, Q^2\right) + \overline{g}_2\left(x, Q^2\right), \tag{2}$$

where

$$g_2\left(x, Q^2\right) = -\int_x^1 \frac{\partial}{\partial y}\left[\frac{m_q}{M} h_T\left(y, Q^2\right) + \zeta\left(y, Q^2\right)\right]\frac{dy}{y}. \tag{3}$$

To twist-3, there are three contributions to g_2 [10]:

g_2^{ww}: the leading twist-2 term, which depends only on g_1,

h_T: arises from the quark transverse polarization distribution. Also twist-2, this term, is suppressed by the smallness of the quark mass,

ζ: the twist-3 part which arises from quark-gluon interactions,

Using the fact that g_1 and g_2 contain the same twist-2 operator, Wandzura-Wilczek [11] derived the following expression for the asymptotically free contribution to g_2, in terms of g_1:

$$g_2^{ww}\left(x,Q^2\right) = -g_1\left(x,Q^2\right) + \int_x^1 \frac{g_1\left(x,Q^2\right)}{x}\,dx. \quad (4)$$

Also this relation can be extracted directly from twist-2 operators of OPE.

2.2. Operator Product Expansion. An appropriate formalism to study g_2 beyond the simple QPM is the operator product expansion (OPE) [13, 14] which is the model-independent approach based directly on QCD. The OPE allows us to write the hadronic matrix element in deep inelastic scattering (DIS) in terms of a series of normalized operators of increasing twist. The leading contribution is twist-2, with higher-twist terms suppressed by powers of $1/Q^2$. Keeping only terms up to twist-3, the moments of g_1 and g_2 at fixed Q^2 can be related to the twist-2 and twist-3 reduced matrix elements, a_n and d_n [14],

$$\int_0^1 x^n g_1\left(x,Q^2\right)dx = \frac{a_n}{2}, \quad n = 0,2,4,\ldots,$$
$$\int_0^1 x^n g_2\left(x,Q^2\right)dx = \frac{1}{2}\frac{n}{n+1}\left(d_n - a_n\right), \quad n = 2,4\ldots. \quad (5)$$

In the expression above, d_n directly appears in the equation for $g_2$allowing us to study the higher-twist structure of the nucleon at leading order. One obtains

$$\int_0^1 x^n\left[g_1\left(x,Q^2\right) + \frac{n+1}{n}\,g_2\left(x,Q^2\right)\right]dx = \frac{1}{2}d_n\left(Q^2\right), \quad n = 2,4,6\ldots. \quad (6)$$

Hence, for $n = 2$,

$$d_2 = 2\int_0^1 x^2\left[g_1\left(x,Q^2\right) + \frac{3}{2}g_2\left(x,Q^2\right)\right]dx. \quad (7)$$

At large Q^2, the d_n matrix element is related to the color polarization, which describes how the color electric and magnetic fields respond to the nucleon spin. At lower momentum transfer, $d_n(Q^2)$ provides a mean to study the transition from perturbative to nonperturbative behavior and to quantify higher-twist effects. And although d_n is a higher-twist OPE object, the definition holds for all Q^2.

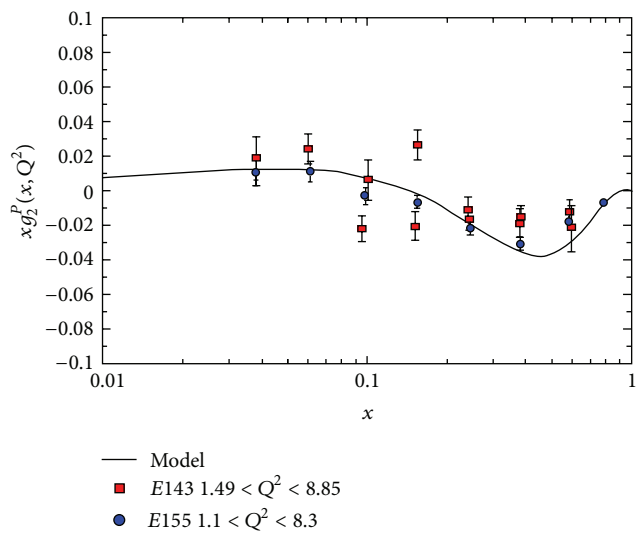

Figure 1: The polarized structure function xg_2^p as a function of Q^2 in intervals of x.

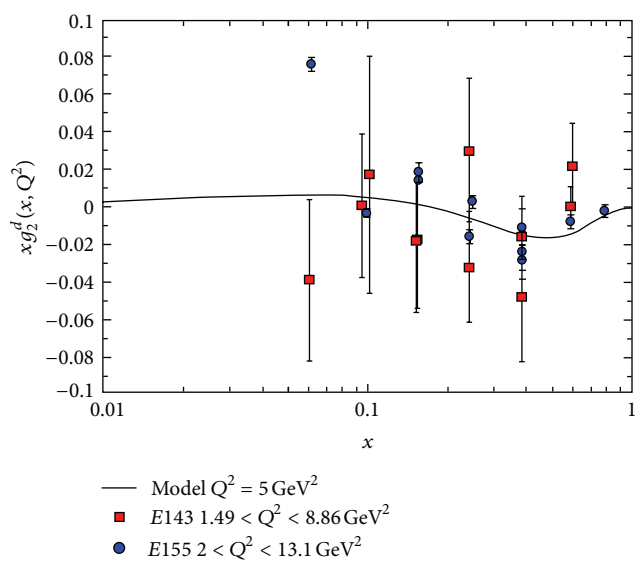

Figure 2: The polarized structure function xg_2^d as a function of Q^2 in intervals of x.

3. Formalism

We summarize the key features of our QCD analysis here. Knowing the fact that world data for g_1 cover a broad range in x and Q^2 with relatively high precision motivates us to extract polarized parton distributions from NLO fits to the data. In our calculation, the polarized parton densities are parameterized at a starting scale $Q_0^2 = 4\,\mathrm{GeV}^2$and are evolved to higher factorization scales using a numerical solution of the polarized NLO DGLAP evolutions [15–17]. We take the QCD coupling as a free parameter which is fitted to the data. The QCD DGLAP equations are solved in the Mellin space.

The centerpiece of our approach is the Jacobi polynomial expansion; this method was developed and applied to a

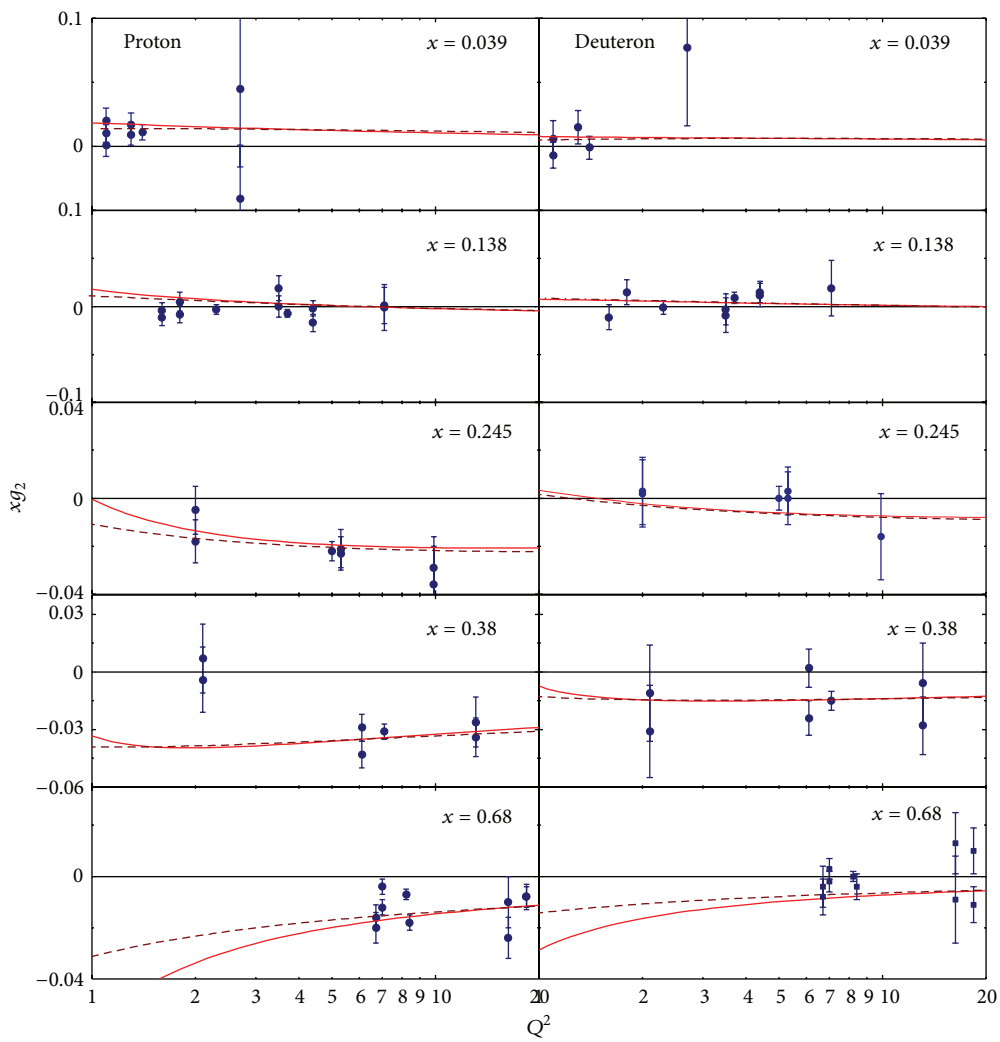

FIGURE 3: xg_2 for the proton and deuteron as a function of Q^2 for selected value of x. Data are for E155 [12] experiment. The error bars shown are the statistical and systematic ones added in quadrature. The curves show xg_2^{WW} based on model (solid) and the BB model (dash-dot) [2].

variety of different QCD analyses for unpolarized [18, 19] and polarized applications [4, 5]. In this approach, we expand the structure functions in terms of the Jacobi polynomials $\Theta_n^{\alpha,\beta}$

$$xg_1^{N_{\max}} = x^{\beta}(1-x)^{\alpha}\sum_{n=0}^{N_{\max}}\Theta_n^{\alpha,\beta}(x)\sum_{j=0}^{n}c_j^{(n)}(\alpha,\beta)\left[xg_1, j+2\right]. \quad (8)$$

This relates the $g_1(x,Q^2)$ structure function to their moments [4].

In this analysis we use $N_{\max} = 9$, $\alpha = 3.0$, and $\beta = 0.5$. The world data are at relatively large Q^2, where higher-twist effects should be negligible; therefore the evolved parton distributions allow one to calculate the twist-2 spin-dependent structure functions g_1 and g_2^{WW}, in most kinematic regions accessible today. Due to the fact that our calculations are performed in DIS region where transverse spin is a suppressed leading twist, we can neglect the twist-3 part of g_2, $\zeta(y,Q^2)$. Also, ignoring the masses of three light quarks is another fact that allows us to have an estimate ofg_2, by these approximations, $\overline{g}_2 \approx 0$ then g_2^{WW} (see (2), (3)). By the last assumption, we can see that the contribution of quark transverse distribution, h_T, is omitted too. Precise measurements of g_2 at specific values of x and Q^2 can be compared to g_2^{WW}, providing a unique opportunity to cleanly isolate higher-twist contributions.

4. Results

We have performed a QCD analysis of the inclusive polarized DIS data at NLO and extracted the spin structure function $g_1(x,Q^2)$. We have used an expansion in the Jacobi polynomials to facilitate the analysis. The Jacobi polynomials help us to produce $g_1(x,Q^2)$ in an analytical (not numerical) form.

For the measured region $0.03 < x < 0.8$, proton and deuteron spin structure functions $g_2^{WW}(x,Q^2)$ are calculated using our parametrization on $g_1(x,Q^2)$ and the possible contributions of higher-twist effects to $g_2^{p,d}$ are quantified

Table 1: Comparison of model results with proton and deuteron data.

Paper	$d_p^{(2)}$	$d_d^{(2)}$
This paper	1.3×10^{-2}	-8.8×10^{-3}
c.m. bag model [21]	1.74×10^{-2}	6.79×10^{-3}
MIT bag model [22, 23]	1.0×10^{-2}	5.0×10^{-3}
QCD sum rule [24]	$-(0.6 \pm 0.3) \times 10^{-2}$	$-(17 \pm 5) \times 10^{-3}$
QCD sum rule [25]	$-(0.3 \pm 0.3) \times 10^{-2}$	$-(13 \pm 5) \times 10^{-3}$
Data [26]	$(0.32 \pm 0.17\ ) \times 10^{-2}$	$(7.9 \pm 4.8) \times 10^{-3}$
Lattice QCD [27]	$-(4.8 \pm 0.5) \times 10^{-2}$	$-(22 \pm 6) \times 10^{-3}$

using twist-3 reduced matrix element d_2 at fixed Q^2. By using (7) and substitution g_2^{WW} for g_2, as discussed, we neglect the contribution from the region $0 \leq x < 0.03$ because of the x^2 suppression factor. For $0.8 < x \leq 1$, we assume that both g_1 and g_2 behave as $(1-x)^3$. We obtained the value of $d_p^{(2)} = 1.3 \times 10^{-2}$ and $d_d^{(2)} = -8.8 \times 10^{-3}$. The computational processes were performed at $Q^2 = 5\,\mathrm{GeV}^2$. The consistency of these values with zero is another reasonable proof for our assumptions in the most common kinematic region. Comparisons of our results for the third moments of g_2 with recent data and other model predictions are listed in Table 1.

Figure 3 shows xg_2 for the proton and deuteron as a function of Q^2 for selected value of x. Data are for E155 experiment. The error bars shown are the statistical and systematic ones added in quadrature. The curves show xg_2^{WW} based on our model (solid) and the BB model (dash-dot), both using NLO g_1 fit, which are in good agreement with the data. xg_2 from E143 (square) [20] and E155 (circle) [12] experiments together with our twist-2 g_2^{WW} at $Q^2 = 5\,\mathrm{GeV}^2$ for xg_2^p and xg_2^d are presented in Figures 1 and 2.

Disclosure

This paper was published in PACS, number: 13.88.+e.

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