Research Article

Compact and Wideband Parallel-Strip 180° Hybrid Coupler with Arbitrary Power Division Ratios

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This paper presents a class of wideband 180° hybrid (rat race) couplers implemented by parallel-strip line. By replacing the 270° arm of a conventional 180° hybrid coupler by a 90° arm with phase inverter, the bandwidth of the coupler is greatly enhanced and the total circuit size is reduced by almost half. Simple design formulas relating the characteristic impedance of the arms and power division ratio are derived. To demonstrate the concept, four couplers with different power division ratios of 1, 2, 4, and 8 were designed, fabricated, and tested. S-parameters of the coupler are simulated and measured with good agreement. All working prototypes operate more than 112% impedance bandwidth with more than 25 dB port-to-port isolation and less than 5° absolute phase imbalance. The proposed 180° hybrid couplers can be employed as a wideband in-phase/differential power divider/combiner, which are essential for many RF and microwave subsystem designs.

1. Introduction

Nowadays the demands for compactness, light weight, and broadband have a great impact on design issues of a high performance RF and microwave front-ends at both component and system levels. Power dividers and combiners play critical roles in RF and microwave circuit designs as they are the fundamental and indispensable components in the wireless communications systems. The Wilkinson power divider, 90° hybrid coupler, and 180° hybrid coupler have unique features in both dividing and combining RF signals with port-to-port isolation in 0°, 90°, and 180° phase difference, respectively [1]. For applications such as push-pull amplifiers, balanced mixers, frequency multipliers, and antenna arrays, the 180° hybrid coupler is preferred. It is because not only the load impedance has a small impact on the isolation and input/output impedance matching of the 180° hybrid coupler, but also its operation bandwidth is wider than that of the 90° hybrid coupler [2]. However, circuit area of the 180° hybrid coupler is larger compared with both the Wilkinson power divider and the 90° hybrid coupler. In addition, demands from many widebands sparked the exploration of bandwidth enhancement techniques of various RF and microwave components, and the 180° hybrid coupler is no exception.

Parallel-strip line [3, 4] belongs to the family of balanced transmission lines. It is a simple structure of a piece of dielectric substrate sandwiched by two strip conductors. Signals flowing on the upper and lower strip conductors are always equal in magnitude but 180° out of phase. This nature was applied to realize a 180° Wilkinson power divider/combiner for push-pull amplifier with wideband second-harmonic suppression [3]. In addition, the symmetry of the parallel-strip line implies that the “ground” and “signal” lines can be swapped freely in the circuit design. The parallel-strip phase reversal swap can easily be realized by “intercrossing” the upper and lower strip conductors by a pair of metal vias. The phase reversal swap, which is a simple passive microwave component, forms a compact realization of 180° phase shift which was employed to realize a power divider/combiner concept with enhanced isolation bandwidth [4].

A parallel-strip 180° hybrid coupler was proposed in [5] with a performance similar to the conventional design. In this letter, a compact wideband 180° hybrid coupler, implemented by parallel-strip line, is achieved by replacing the 270° arm of the conventional 180° hybrid coupler with a section of 90°...
parallel-strip line and the phase reversal swap. In addition to size reduction, the coupler's bandwidth is dramatically enhanced. A wideband parallel-strip 90° hybrid coupler was proposed in [6]. However, the size of the proposed 90° hybrid coupler is almost double that of the 180° hybrid coupler reported in [4]. Similar to [4], a swap or phase inverter is employed to enhance bandwidth. Unlike the 180° hybrid coupler, large magnitude imbalance is obtained over the working frequency band. In this work, we not only design a 180° hybrid coupler with equal power division, but also extend the concept to that with arbitrary power division. Design formula is derived based on even- and odd-mode analysis. Four couplers with different power divisions are designed, fabricated, and measured.

2. Analysis

A schematic diagram of a 180° hybrid coupler with a phase inverter is shown in Figure 1. It consists of 4 arms with quarter-wavelength long. By changing the characteristic impedance of each arm, different power divisions can be achieved. Equations relating characteristic impedances and power division ratio are derived in this section.

By applying odd- and even-mode analysis, the coupler is divided by half in which the line of symmetry is terminated either by open circuit or by short circuit. Figure 2 shows

\[
(S_{e}) = \begin{pmatrix}
\frac{Z_A/Z_0 - Z_0/Z_A - j(2Z_A/Z_B)^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\frac{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\end{pmatrix}
\]

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the odd- and even-mode half structures of the coupler. The characteristic impedances of the series transmission line and the two shunt (open- and short-circuit) stubs are \(Z_A\) and \(Z_B\), and their electrical lengths are 90° and 45° at the center frequency, respectively. The two half structures are exactly the same except that the port assignments are reversed. Therefore, the odd- and even-mode S-parameters can be related by

\[
S_{11e} = S_{22o},
\]

\[
S_{22e} = S_{11o},
\]

\[
S_{12e} = S_{21o} = S_{34o},
\]

where \((S_{e}) = \begin{pmatrix} S_{11e} & S_{12e} \\ S_{21e} & S_{22e} \end{pmatrix}\) and \((S_{o}) = \begin{pmatrix} S_{11o} & S_{21o} \\ S_{34o} & S_{22o} \end{pmatrix}\) are the even- and odd-mode S-matrices, respectively. To simplify the analysis, we only consider the parameters at center frequency. The overall ABCD-matrix of the even-mode half structure is the product of the ABCD-matrix representing the shunt short open-circuit stub, series transmission line, and shunt short-circuit stub given by

\[
\begin{pmatrix}
1 & 0 & \cos 90° & jZ_A \sin 90°
\end{pmatrix}
\begin{pmatrix}
Z_A & jZ_A \sin 90° & \cos 90° & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0
\end{pmatrix}
\begin{pmatrix}
1 & jZ_B \tan 45° & 1
\end{pmatrix},
\]

and the overall matrix is expressed by

\[
\begin{pmatrix}
\frac{Z_A}{Z_B} & jZ_A/Z_B
\end{pmatrix}
\begin{pmatrix}
j \left( \frac{1}{Z_A} + \frac{Z_A}{Z_B} \right) - \frac{Z_A}{Z_B}
\end{pmatrix}.
\]

The even-mode S-matrix can be conversed from the ABCD-parameter given by

\[
\begin{pmatrix}
\frac{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\frac{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\end{pmatrix}.
\]

Using (1), the odd-mode S-matrix can be expressed by

\[
\begin{pmatrix}
\frac{Z_A/Z_0 - Z_0/Z_A - j(2Z_A/Z_B)^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\frac{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}{2Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}
\end{pmatrix}.
\]
Once both even- and odd-mode S-matrixes are determined, the S-parameters, \( S_{11}, S_{21}, S_{31}, \) and \( S_{41} \), of the entire coupler can be determined by the following well-known formulas:

\[
S_{11} = \frac{S_{11e} + S_{11o}}{2} = \frac{Z_A/Z_0 - Z_0/Z_A - Z_0Z_A/Z_B^2}{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2},
\]

\[
S_{21} = \frac{S_{11e} - S_{11o}}{2} = \frac{-j(2Z_A/Z_B)}{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2},
\]

\[
S_{31} = \frac{S_{21e} - S_{21o}}{2} = 0,
\]

\[
S_{41} = \frac{S_{21e} + S_{21o}}{2} = \frac{2}{Z_A/Z_0 + Z_0/Z_A + Z_0Z_A/Z_B^2}.
\]

\( S_{31} \) is always zero resulting in port 3 always being isolated from port 1. If \( S_{31} = 0 \) is assumed, then it indicates that perfect impedance matching is achieved at the center frequency. A straightforward calculation derives

\[
\frac{Z_A}{Z_0} = \frac{Z_0 + Z_0Z_A}{Z_A + Z_0Z_A}. \tag{7}
\]

Hence, the expressions of \( S_{21} \) and \( S_{41} \) can be reduced to

\[
S_{21} = -\frac{Z_0}{Z_B}, \quad S_{41} = \frac{Z_0}{Z_A}. \tag{8}
\]

The power division ratio is defined as \( k = \left| S_{21}/S_{41} \right|^2 \), and the expression relating \( k, Z_A, \) and \( Z_B \) is obtained as follows:

\[
Z_A = \sqrt{k}Z_B. \tag{9}
\]

By solving (7) and (9), the design equations for \( Z_A \) and \( Z_B \) as function of \( k \) are given by

\[
Z_A = \sqrt{1+k}Z_0, \quad Z_B = \sqrt{1+1/k}Z_0. \tag{10}
\]

### 3. Bandwidth Consideration

The 180° hybrid coupler with a phase inverter has wider bandwidth than that without phase inverter. The frequency response of the input impedance or the input reflection coefficient at port 1 should be considered. By considering port 1 as the input port, the signal will be divided into two paths and will be transmitted to port 2 and port 4. The ports 2 and 4 should be considered as \( Z_0 \) resistors to ground in this case. With an assumption of ideal phase inverter, the port 3 is virtually grounded because the phase inverter introduces an additional 180° phase difference at port 3. Therefore, the hybrid coupler becomes a single-port network as shown in Figure 3. According to the design equation (10), the four transmission line have 90° electrical length at the centre frequency \( f_0 \). Electrical length, input impedance, and input reflection coefficient are the functions of frequency \( f \). The input impedance at port 1 \( Z_{in} \) is given by

\[
\frac{1}{Z_{in}} = \left[ \frac{Z_A}{Z_0} \frac{1}{1 + jZ_B \tan \theta} - \frac{jZ_A \tan \theta}{Z_A + j(1/Z_0 + 1/Z_B \tan \theta)^{-1} \tan \theta} \right]^{-1}
\]

\[
+ \left[ \frac{Z_B}{Z_0} \frac{1}{1 + jZ_A \tan \theta} - \frac{jZ_B \tan \theta}{Z_B + j(1/Z_0 + 1/jZ_A \tan \theta)^{-1} \tan \theta} \right]^{-1}, \tag{11}
\]

where \( \theta = (f/f_0)90° \). To simplify the analysis, 180° hybrid coupler with equal power division \( k = 1 \) is considered. Using (10), the two characteristic impedances are \( Z_A = Z_B = \sqrt{2}Z_0 \), and the input impedance becomes

\[
Z_{in} = \frac{Z_0}{\sqrt{2} + j \left( 1/\sqrt{2} \right) (\tan \theta - \cot \theta)}. \tag{12}
\]

We consider the magnitude of the input reflection coefficient given by

\[
\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{j \cot \theta}{2 \sqrt{2} + j (2 \tan \theta - \cot \theta)}. \tag{13}
\]

The impedance bandwidth is defined as the frequency range with reflection coefficient less than \(-10 \text{ dB}\); hence, we have

\[
|\Gamma_{in}| = \frac{1}{2 \tan^2 \theta + 1} \leq -10 \text{ dB}. \tag{14}
\]
Equation (14) represents the frequency response of the input reflection coefficient or $S_i$, where $i = 1, 2, 3,$ or 4. We also built a 180° hybrid coupler with equal power division using microwave circuit simulator, Agilent’s ADS. Equation (14) is verified by comparing the simulation as shown in Figure 4. The required frequency range is determined as

$$\frac{1}{90^\circ} \left( 180^\circ - \tan^{-1} \left( \frac{\sqrt{10} - 1}{2} \right) \right) \leq f \leq \frac{1}{90^\circ} \left( 180^\circ + \tan^{-1} \left( \frac{\sqrt{10} - 1}{2} \right) \right).$$

(15)

For the 180° hybrid coupler with phase inverter, the theoretical percentage bandwidth with reflection coefficient less than $-10$ dB is given by $(2/90^\circ) \tan^{-1} \left( \sqrt[4]{10} - 1 \right)/2 \sim 102\%$.

### 4. Parallel-Strip Phase Reversal Swap

The phase inverter is achieved by interchanging the two signal lines in the balance transmission line so that the signal is said to be "reversed"; therefore, it provides 180° phase shift without requiring a delay line, which is often used in microstrip implementation. Phase inverter is possible in coplanar strip [7] and coplanar waveguide [8] implementations; however, stubs are required for the phase reversal section, and transitions between different transmission lines (for the port...
Figure 5: 3D view of the parallel-strip phase reversal swap (conductors only).

Figure 6: (a) Lumped-circuit model of the parallel-strip phase reversal swap. (b) Comparison of simulated S-parameters between EM simulation and lumped-circuit model. (c) Phase difference of the transmission lines with phase inverter.

Table 1: Required values of $Z_A$ and $Z_B$ for different power division ratios.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$Z_A$ (Ω)</th>
<th>$Z_B$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70.71</td>
<td>70.71</td>
</tr>
<tr>
<td>2</td>
<td>86.60</td>
<td>61.24</td>
</tr>
<tr>
<td>4</td>
<td>111.8</td>
<td>55.90</td>
</tr>
<tr>
<td>8</td>
<td>141.4</td>
<td>53.03</td>
</tr>
</tbody>
</table>

and the phase reversal swap) complicate the entire circuit design. Fortunately, a phase inverter can be realized easily by the parallel-strip line. Figure 5 shows the 3D view of the parallel-strip phase inverter. The upper and lower strip lines are interconnected by two vertical metallic vias. Within the whole simulation band (0–4 GHz), less than 0.5 dB additional insertion loss is introduced and 180° phase shift is provided with less than 2° phase error caused by the inclusion of the vias.

The lump-circuit model of the parallel-strip phase reversal swap is illustrated in Figure 6(a). The parasitic capacitance ($C_S$) is used to model the edge couplings between strips residing on different layers. The parasitic capacitance ($C_C$) is used to model the total effect due to edge couplings.
between strips on the same layers and coupling between the metal vias. The parasitic inductance ($L_V$) and resistance ($R_V$) are introduced by the vertical conductors in via holes and soldering. The parasitic components can be extracted from electromagnetic simulations to complete the lump-circuit model.

Commercial electromagnetic simulator, Ansoft HFSS, was employed to determine the optimal dimensions of the phase reversal swap. The substrate has a dielectric constant of 2.65 and thickness of 1.5 mm. The gap widths on the top and bottom conductors are 0.2 mm, the line width is 3.4 mm, and the diameter of metallic vias is 1.1 mm with 70.71 $\Omega$ terminating ports. Deembedding of the parameters was performed by utilizing the microwave circuit simulator, Agilent’s ADS. Values of parasitic elements are $L_V = 2.181$ nH, $C_S = 0.2939$ pF, $C_C = 0.3878$ pF, and...
\( R_V = 0.2624 \, \Omega \). With these parameters, the magnitude of the \( S \)-parameters of the phase inverter versus frequency is shown in Figure 6(b). The phase difference of the section of parallel-strip lines with and without phase inverter is shown in Figure 6(c). It is obvious that the lumped-circuit model provides accurate performance prediction of the phase reversal swap.

### 5. Measured Results

Four 180° hybrid couplers with four different power division ratios \( k \) and the same centre frequency of 2 GHz were designed to verify the concept. All couplers were prototyped on a printed circuit board with a dielectric constant of 2.65 and substrate thickness of 1.5 mm. Figure 7 shows the layout...
Figure 11: Simulated and measured $S$-parameters of the coupler with $k = 4$. (a) Frequency responses of magnitude. (b) Frequency responses of phase differences of the coupler’s outputs.

Table 2: Physical dimensions of the couplers with different power division ratios.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$w_a$</th>
<th>$l_a$</th>
<th>$w_b$</th>
<th>$l_b$</th>
<th>$w$</th>
<th>$s$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.4 mm</td>
<td>24.9 mm</td>
<td>3.4 mm</td>
<td>24.9 mm</td>
<td>5.3 mm</td>
<td>0.2 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>2</td>
<td>2.5 mm</td>
<td>25.1 mm</td>
<td>4.1 mm</td>
<td>24.7 mm</td>
<td>5.3 mm</td>
<td>0.2 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>4</td>
<td>1.7 mm</td>
<td>25.5 mm</td>
<td>4.6 mm</td>
<td>24.9 mm</td>
<td>5.3 mm</td>
<td>0.2 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td>8</td>
<td>1.1 mm</td>
<td>25.9 mm</td>
<td>4.9 mm</td>
<td>24.5 mm</td>
<td>5.3 mm</td>
<td>0.2 mm</td>
<td>1.1 mm</td>
</tr>
</tbody>
</table>

The four couplers were fabricated by the standard printed circuit fabrication process. The photograph of the fabricated coupler with power division ratio of $k = 4$ is shown in Figure 8. The three remaining fabricated couplers are similar to that shown in Figure 8. The circular core part is inside the red circle, where four parallel-strip line to microstrip line transitions are outside the red circle and they are required for measurement. The $S$-parameters were measured using 4-port network analyzer with port impedance of 50 Ω. Figures 9, 10, 11, and 12 show both simulated and measured performances of the four couplers. These performances include frequency responses of magnitudes of the $S$-parameters of the four couplers and frequency response of the phase differences of the couplers’ outputs. All fabricated couplers have similar core area of around 800 mm$^2$. Good agreements of the linear magnitudes of all $S$-parameters between the simulation and measurement are achieved. Since the magnitude of $S_{41}$ is at the order of $-30$ dB (order of the linear magnitude is at 0.001), small error in the linear magnitude introduces large error in that of the logarithmic magnitude.

All couplers are designed at the center frequency of 2 GHz. The proposed coupler works well at around center frequency. The proposed coupler does not work at DC and lower frequency range, since the swap introduces short circuit to all the ports. The proposed coupler also does not work at around second-harmonic frequency range, since the swap introduces 180° phase shift. Signals are cancelled at the thought and coupler ports. Table 3 summarizes the measured performances of the three couplers. The measured impedance
Figure 12: Simulated and measured $S$-parameters of the coupler with $k = 8$. (a) Frequency responses of magnitude. (b) Frequency responses of phase differences of the coupler’s outputs.

Table 3: Summarized performance of the four couplers.

<table>
<thead>
<tr>
<th>$k$</th>
<th>Impedance bandwidth</th>
<th>Average $k$</th>
<th>Port-to-port isolation</th>
<th>Phase error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120%</td>
<td>1.02</td>
<td>&gt;29 dB</td>
<td>&lt;5°</td>
</tr>
<tr>
<td>2</td>
<td>112%</td>
<td>1.85</td>
<td>&gt;27 dB</td>
<td>&lt;5°</td>
</tr>
<tr>
<td>4</td>
<td>119%</td>
<td>2.89</td>
<td>&gt;25 dB</td>
<td>&lt;7°</td>
</tr>
<tr>
<td>8</td>
<td>129%</td>
<td>7.55</td>
<td>&gt;28 dB</td>
<td>&lt;6°</td>
</tr>
</tbody>
</table>

Bandwidths of all four couplers are larger than the theoretical bandwidth, 102%, shown in Section 3 because of losses of circuit including transmission line, T-junction, and phase inverter.

6. Conclusion

A class of wideband and compact 180° parallel-strip hybrid coupler is presented. Unlike microstrip line or coplanar waveguide, the parallel-strip line is a balanced transmission line that consists of two metal strips at the top and bottom of the printed circuit board. The parallel-strip circuit fully unitizes the double layers of the printed circuit board to achieve higher performance than that of microstrip circuit, where microstrip circuit is totally grounded at the bottom layer. Parallel-strip phase reversal swap is unique, where swap is not realized by either microstrip line or coplanar waveguide. By employing the frequency-independent parallel-strip phase reversal swap in conjunction with the 90° section to replace the 270° section of the 180° hybrid coupler, the circuit area is reduced by half, and the overall bandwidth is enhanced. Design formula for the proposed coupler is provided. Based on the even- and odd-mode analysis. The required arm characteristic impedance of the coupler and its impedance bandwidth are determined by the analytical formula, which is derived in this paper for arbitrary coupling level. Four prototypes with different power divisions are designed, fabricated, and tested. Good agreement between the simulation and the measurement is achieved. It is well suited for compact, low-cost active parallel-strip circuit applications in RF and microwave integrated circuits.

References


