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Conference Paper

Dual Analysis of the Capacity of Spectrum Sharing Cognitive Radio with MRC under Nakagami-*m* Fading

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In this study, the maximum achievable information transmission rate of spectrum sharing cognitive radio with maximal ratio combining (MRC) antenna diversity technique is investigated when the channel between the secondary transmitter and the primary receiver and that between the secondary transmitter and the secondary receiver suffer Nakagami-*m* fading. With an assumption that both channels encounter Nakagami-*m* fading and the transmission of the secondary transmitter is subject to average interference power constraint, the approximated expressions for analyzing the effective capacity and the ergodic capacity of cognitive radio users with MRC are presented. The two capacity models are compared. In the case of the effective capacity, it is shown that different applications or users with different quality of service (QoS) requirements can be supported in cognitive radio, and when the delay QoS decreases, the effective capacity approaches the ergodic capacity.

1. Introduction

Communication through air medium practically relies on the radio portion of the electromagnetic spectrum with the main objective of allowing users on the move, anywhere, anytime to communicate with other users easily and efficiently. The major concern is how to accommodate this growth through efficient frequency spectrum usage. In response to this need, a renowned framework has been proposed to improve spectrum utilization by allowing the use of spectrum holes when and where they exist. This radio technology is known as cognitive radio (CR), which includes software defined radio (SDR), whose initial focus was on radio knowledge representation language (RKRL) [1]. With cognitive radio, spectrum utilization could therefore be improved by allowing secondary users to access the spectrum at the time and location when such band of frequencies is not being used by the primary user [2]. Conventionally, license restricts the use of specific spectrums to an authorized user only, and no other user (secondary) is allowed to transmit or receive on the same frequencies in order to prevent interference with the legitimate (primary) user's transmission. This restriction leads to underutilization of certain spectrum in a situation whereby the band of frequencies allocated to primary users is not being used during a certain period of time and in specific geographical area, which thus creates spectrum holes [2].

This spectrum sharing proposition has fostered researches on efficient sharing technique that ensures avoidance of harmful interference with the primary user (PU). Studies have been dedicated to analyzing the capacity of spectrum sharing cognitive radio when the secondary user's (SU) transmission is subject to interference power constraint under different types of fading. For example, the capacity of CR with maximal ratio combining (MRC) under asymmetric fading has been studied in [3], where the channel between the secondary user transmitter (SU_{tx}) and the PU receiver (PU_{rx}) is Nakagami fading while the channel path between the SU_{tx} and the secondary receiver (SU_{rx}) is Rayleigh fading. Also, the author in [4] investigated and analyzed the asymptotic performance of the ergodic capacity of CR system given a number of MRC receivers and fading is Rayleigh. In the case of effective capacity, the

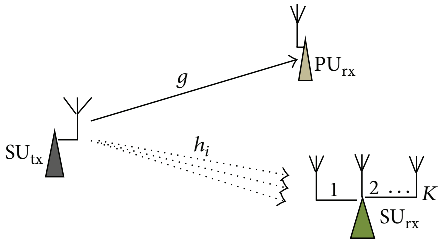

Figure 1: Cognitive radio system with MRC at the secondary receiver.

authors in [5] studied the maximum achievable information transmission rate of the SU subject to average interference power constraint in CR relay channels.

In this paper, the aim is to investigate both the ergodic capacity and effective capacity of spectrum sharing CR when the two channel paths, $SU_{tx} \rightarrow SU_{rx}$ and $SU_{tx} \rightarrow PU_{rx}$, both experience Nakagami-m fading and with an assumption that MRC diversity technique is implemented at the secondary receiver. The analysis of the capacity of CR system under Nakagami-m fading is very essential because Nakagami distribution is best fit for analyzing fading in mobile communication channel since it represents various channel fading conditions [6]. For instance, when the parameter $m = 1$ in Nakagami-m fading distribution, Rayleigh fading is a special case in Nakagami-m. Therefore, using Nakagami-m as the fading case, we can obtain both mathematical and numerical analyses for Nakagami and Rayleigh in one model.

The subsequent sections in this paper have been organized as follows. In Section 2, the assumed channel characteristics are presented in the channel and system models. The expression for analyzing the ergodic capacity of spectrum sharing CR with MRC under Nakagami-m fading is discussed and derived in Section 3, while under the same channel fading and assuming MRC at the SU_{tx}, Section 4 deals with the discussion on and the derivation of the effective capacity. The numerical results and analysis are presented in Section 5, while Section 6 concludes the paper.

2. Channel and System Models

In this section, the channel and system models used for this analysis are presented. Figure 1 [3] schematically represents a spectrum sharing system where the secondary receiver is implemented with MRC receiver diversity. In this channel model and analysis, we assume that both the primary channel, $SU_{tx} \rightarrow PU_{rx}$, and the secondary channel, $SU_{tx} \rightarrow SU_{rx}$, experience Nakagami-$m$ fading, and K-branch MRC antenna diversity is implemented at the CR receiver. The channel power gains for both $SU_{tx} \rightarrow PU_{rx}$ and $SU_{tx} \rightarrow SU_{rx}$ are g and h, respectively. The channel power gains are independent and identically distributed (i.i.d) and characterized by Gamma distributions as follows [3, 7, 8]:

$$P_h(h) = \frac{m^m h^{m-1}}{\Gamma(m)} e^{-mh}, \tag{1}$$

$$P_g(g) = \frac{{m_o}^{m_o} g^{m_o-1}}{\Gamma(m_o)} e^{-m_o g}. \tag{2}$$

Without losing generality, we assume that the average channel power gain of each channel is one. With MRC K antenna elements, $K \in (1, 2, 3, \ldots, k)$ at the secondary receiver, the Gamma distribution of the channel gain, h, of the secondary channel becomes [9]

$$P_{h\mathrm{MRC}}(h) = \frac{m^{km} h^{km-1}}{\Gamma(km)} e^{-mh}. \tag{3}$$

For perfect combining with MRC, the channel side information (CSI) must be known at both the transmitter and the receiver such that the combining technique has the perfect knowledge of the branch amplitudes and phases in order to achieve perfect combining. With this perfect combining, MRC diversity provides the optimal diversity that improves the maximum achievable capacity as opposed to other diversity techniques [9–11]. Hence, for perfect knowledge of the channel, we assume that the perfect CSI is available to both the transmitter and the receiver.

3. Ergodic Capacity of Spectrum Sharing Cognitive Radio with MRC over Nakagami-*m* Fading

The ergodic capacity of CR channel has been studied in [3, 4] under asymmetric fading and Rayleigh fading, respectively, when the secondary receiver has MRC diversity technique implemented. The authors in [3] studied the capacity gains resulted from MRC diversity technique in CR system subject to average interference power constraint. Also, the capacity of a single or noncognitive communication channel with MRC under Rayleigh fading has been studied in [9]. In this analysis, we study a distinct case of the ergodic capacity of a CR channel with MRC under Nakagami-m fading as opposed to the asymmetric case in [3], the Rayleigh case in [4], and single communication channel case in [9].

In this section, the ergodic capacity of a CR system is studied when both the primary and the secondary channels experience Nakagami-m fading, and the secondary receiver uses MRC diversity combining. Also, the transmission of the SU_{tx} is subject to average interference power constraint, Q. The normalized ergodic capacity of the CR link is given as [3]

$$\frac{C_{\mathrm{erg}}}{B} = E_{h,g}\left[\log\left(1 + \frac{hP(h,g)}{N_o B}\right)\right], \tag{4}$$

where B [Hz] is the channel bandwidth, $P(h, g)$ represents the joint fading state (h, g), and N_o is the additive white Gaussian noise (AWGN) power spectral density at SU_{rx}. Since the maximum achievable ergodic capacity of the CR system is subject to average interference power constraint, the ergodic capacity can be formulated as an optimization problem as follows:

$$\underset{p(h,g)}{\text{maximize}} \quad E\left[\log\left(1 + \frac{hP(h,g)}{N_o B}\right)\right] \tag{5}$$

$$\text{subject to} \quad E_{h,g}\left[P(h,g)\,g\right] \le Q, \tag{6}$$

where Q is the average interference power constraint at the PU_{rx}. Then we can obtain the maximization solution for the above optimization problem using Lagrangian optimization technique. Hence, the solution for the optimization problem becomes

$$P(h,g)=\left(\frac{1}{\lambda g}-\frac{N_oB}{h}\right)^{+}, \quad (7)$$

where $(\cdot)^+$ depicts max $(\cdot,0)$ and the Lagrangian multiplier satisfying the condition $E_{h,g}[P(h,g)g]-Q=0$ is $\lambda\geq 0$. Therefore, putting the distributions of the channel gains in (2) and (3) into (5), with the interference power constraint, transmission experiencing Nakagami fading on both paths, and the SU_{rx} is implemented with MRC, the capacity per unit bandwidth (Hz) of the cognitive radio link can be expressed as

$$\begin{aligned}\frac{C_{\text{erg}}}{B}&=\frac{m_o^{m_o}m^{Km}(Km-1)!/\Gamma(m_o)\Gamma(Km)\gamma_o^{Km}}{(m/\gamma_o)^{Km}}\\&\quad\times\sum_{\alpha=0}^{Km-1}\frac{1}{\alpha!}\frac{(m/\gamma_o)^{\alpha}\Gamma(m_o+\alpha)}{m_o(m/\gamma_o+m_o)^{m_o+\alpha}}\\&\quad\times{}_2F_1\left(1,m_o+\alpha;m_o+1;\frac{m_o}{m/\gamma_o+m_o}\right),\end{aligned} \quad (8)$$

where $\gamma_o=1/\lambda$ for normalized case $(N_oB=1)$, $\Gamma(x)$ is the Gamma function, and ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function [5, 9]. The average interference power that maximizes the above capacity is obtained in (9) by integrating the power constraint expression in (6) over the distributions of the channel power gains in (2) and (3):

$$\begin{aligned}&\frac{Q}{N_oB}\\&=\frac{m_o^{m_o}m^{Km}}{\Gamma(m_o)\Gamma(Km)}\frac{\gamma_o^{m_o+1}\Gamma(Km+m_o)}{(m_o\gamma_o+m)^{Km+m_o}}\\&\quad\times\left[\frac{{}_2F_1(1,Km+m_o;m_o+1;m_o\gamma_o/(m_o\gamma_o+m))}{m_o{}^3}\right.\\&\quad-\frac{1}{(m_o+1)}\\&\quad\left.\times{}_2F_1\left(1,Km+m_o;m_o+2;\frac{m_o\gamma_o}{m_o\gamma_o+m}\right)\right].\end{aligned} \quad (9)$$

Again, where $\gamma_o=1/\lambda N_oB)$, $\Gamma(x)$ is the Gamma function, and ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function [5, 9]. In the next section, in order to complete our comparative analysis, we derive the mathematical expressions for analyzing the effective capacity of the CR system illustrated in Figure 1.

4. Effective Capacity of Spectrum Sharing Cognitive Radio with MRC over Nakagami-*m* Fading

The idea of effective capacity (EC) for time varying fading channels was first proposed in [12] as a wireless link model that characterizes radio channel in terms of quality of service (QoS) metrics (such as delay, data rate, and delay-violation probability) as opposed to link characterization based on fluctuations in the amplitude of radio signals. In other words, ergodic capacity measures achievable information rate at the physical layer while the effective capacity is a link-layer capacity model [12], which provides support for QoS requirements for analyzing the effect of queuing on channel capacity.

In [5, 13], the effective capacity of delay-constrained CR relay and nonrelay systems has been studied, respectively. The case of MRC at the secondary receiver is not considered by the authors in [13]. Therefore, in this section we study the effective capacity of CR under Nakagami-m when MRC is implemented at the SU_{rx}, and assuming that the transmission mode of the SU_{tx} satisfies a statistical delay QoS and the average interference power constraints. The delay QoS exponent has been defined as [12]

$$\theta=-\lim_{\varphi\to\infty}\frac{\ln(\Pr\{q(\infty)>\varphi\})}{\varphi}, \quad (10)$$

where $q(x)$ is the probability that the queue length of the transmit buffer exceeds a certain threshold, φ, and decays exponentially as a function of x. Given this QoS constraint, the maximum arrival rate of the SU can be obtained. In the assumed CR system, there is no delay constraint when $\theta\to 0$ but a strict delay constraint exists when $\theta\to\infty$ [5, 13].

Considering the probability of the service queue length defined in the delay QoS constraint in (10), the EC can be obtained by assuming that the maximum achievable service rate $R[i]$ is a discrete time service rate, which is stationary and ergodic. Assuming the channel in **Figure 1** experiences block fading, which is also ergodic and stationary, the service rate of block i can be expressed as [5, 13]

$$\{R[i],\ i=1,2,3,\dots\}. \quad (11)$$

Therefore, without block fading the effective capacity subject to delay QoS exponent is

$$E_c(\theta)=-\lim_{t\to\infty}\frac{1}{\theta t}\ln\left(\varepsilon\left\{e^{-\theta\sum_{i=1}^{n}R[i]}\right\}\right). \quad (12)$$

However, for an uncorrelated sequence $i=1,2,\dots,j$, the effective capacity, E_c, for a block fading channel is given as

$$E_c(\theta)=-\frac{1}{\theta}\ln\left(\varepsilon\left\{e^{-\theta R[i]}\right\}\right). \quad (13)$$

Since transmit power constraint with respect to optimum power allocation is very vital in CR systems, the maximum achievable effective capacity is subject to the interference power constraint and the delay QoS exponent θ. Therefore, the maximum achievable instantaneous service rate $R[i]$ of block i becomes [13]

$$R[i]=TB\ln\left(1+\frac{p(\theta,h,g)g}{N_oB}\right), \quad (14)$$

where g denotes the channel power gain from the SU_{tx} to the PU_{rx}, h is the channel power gain from the SU_{tx} to the

$\mathrm{SU_{rx}}$, T is the time duration of a block, and B represents the channel bandwidth. Following this expression that the maximum effective capacity is subject to power and QoS constraints, the maximum effective capacity can be expressed as an optimization problem:

$$E_C = \text{maximize} \quad \left\{-\frac{1}{\theta}\ln\left(\varepsilon_{h,g}\left\{e^{-TB\ln(1+P(\theta,h,g)g/N_oB)}\right\}\right)\right\}$$
$$\text{s.t} \quad \varepsilon_{h,g}\left\{P(\theta,h,g)\,g\right\} \le Q, \quad P(\theta,h,g)\,g \ge 0. \tag{15}$$

The solution to the above optimization problem is the same as the solution for its minimization problem:

$$\text{minimize} \quad \left\{\varepsilon_{h,g}\left\{\left(1+\frac{P(\theta,h,g)\,g}{N_oB}\right)^{-\theta TB}\right\}\right\} \tag{16}$$

$$\text{s.t} \quad \varepsilon_{h,g}\left\{P(\theta,h,g)\,g\right\} \le Q, \quad P(\theta,h,g)\,g \ge 0. \tag{17}$$

The solution to the optimization problem of (16) can be obtained using the Lagrangian optimization technique as follows:

$$\begin{aligned}\mathcal{L}\left(P(\theta,h,g),\lambda_0\right) &= \lambda_0\left(\varepsilon_{h,g}\left\{P(\theta,h,g)\,g\right\}-Q\right)\\ &\quad+\varepsilon_{h,g}\left\{\left(1+\frac{P(\theta,h,g)\,g}{N_oB}\right)^{-\theta TB}\right\}.\end{aligned} \tag{18}$$

Considering the fact that the optimum power allocation should satisfy $\partial\mathcal{L}(P(\theta,h,g),\lambda_o)/\partial P(\theta,h,g) = 0$ then

$$P(\theta,h,g) = N_oB\left[\frac{\beta^{1/(1+\alpha)}}{g^{1/(1+\alpha)}h^{\alpha/(1+\alpha)}}-\frac{1}{h}\right]^{+}, \tag{19}$$

where $\alpha = \theta TB$, $\beta = \gamma_o\alpha/N_oB$, $[x]^+$ denotes $\max\{0,x\}$; $\gamma_o = 1/\lambda_o$, $\lambda_o \ge 0$ is the Lagrange multiplier associated to the average interference power constraint; and γ_o must be determined in order to satisfy the interference power constraint with equality. Therefore, the optimum power allocation policy can be formulated as follows:

$$P(\theta,h,g) = \begin{cases} N_oB\left(\dfrac{\beta^{1/(1+\alpha)}}{g^{1/(1+\alpha)}h^{\alpha/(1+\alpha)}}-\dfrac{1}{h}\right) & \text{if } g \le \beta h,\\ 0 & \text{otherwise.}\end{cases} \tag{20}$$

The optimum cutoff ratio γ_o can be obtained by integrating the optimum power allocation function over the PDFs of the channel power gains, g and h, given in (2) and (3):

$$\begin{aligned}\frac{Q}{N_oB} &= \int_0^{\infty}\int_0^{\beta h}\left(\beta^{1/(1+\alpha)}\left(\frac{g}{h}\right)^{\alpha/(1+\alpha)}-\frac{g}{h}\right)\\ &\quad * P_g(g)\,P_{h\mathrm{MRC}}(h)\,dg\,dh.\end{aligned} \tag{21}$$

Let random variable v be defined as $v = g/h$ and using the fact that the distribution of the ratio between two Gamma distributed random variables with parameters Km and m_o is a beta prime distribution with parameters Km and m_o [14], the distribution of the random variable v becomes (22) where $z = Km/m_o$ and $B(\cdot,\cdot)$ is a beta function:

$$f_v(v) = \frac{Z^{Km}}{B(Km,m_o)}\frac{v^{m_o-1}}{(Z+v)^{m_o+Km}}. \tag{22}$$

The proof of this joint probability distribution in (22) was obtained using Jacobian method. Therefore, with this joint distribution defined above, the integrals in (21) above can be evaluated to determine the cutoff ratio γ_o as follows:

$$\begin{aligned}&\frac{Q}{N_oB}\\ &= \frac{z^{Km-1}}{B(Km,m_o)}\int_0^{\beta}\left(\beta^{1/(1+\alpha)}v^{\alpha/(1+\alpha)}-v\right)\frac{v^{m_o-1}}{(v+Z)^{Km+m_o}}dv.\end{aligned} \tag{23}$$

Then with the help of the Gauss hypergeometric function defined in [9, 13] to express the integrals in (21), the expression for the average interference power is obtained as

$$\begin{aligned}\frac{Q}{N_oB} &= \frac{\beta^{m_o+1}}{B(Km,m_o)\,Z^{m_o}\left(m_o+\alpha/(1+\alpha)\right)}\\ &\quad\times {}_2F_1\left(Km+m_o; m_o+\frac{\alpha}{1+\alpha}; m_o+\frac{\alpha}{1+\alpha}+1, -\frac{\beta}{Z}\right)\\ &\quad-\frac{\beta^{m_o+1}}{B(Km,m_o)(m_o+1)\,Z^{m_o}}\\ &\quad\times {}_2F_1\left(Km+m_o; m_o+1; m_o+2, -\frac{\beta}{Z}\right).\end{aligned} \tag{24}$$

In order to obtain the closed form expression, the Gauss Hypergeometric function, ${}_2F_1(\cdot)$, can be transformed using equation (9.131) in [15] given as follows:

$$F(\alpha,\beta;\gamma;z) = (1-z)^{-\alpha}F\left(\alpha,\gamma-\beta;\gamma;\frac{z}{z-1}\right). \tag{25}$$

Applying this Gauss Hypergeometric *transformation* formula, the closed form expression for the power constraint in (24) was obtained as

$$\begin{aligned}\frac{Q}{N_oB} &= \frac{\beta^{m_o+1}Z^{Km}}{B(Km,m_o)\left(m_o+\alpha/(1+\alpha)\right)(z+\beta)^{Km+m_o}}\\ &\quad\times {}_2F_1\left(Km+m_o; 1; m_o+\frac{\alpha}{1+\alpha}+1, \frac{\beta}{\beta+Z}\right)\\ &\quad-\frac{\beta^{m_o+1}Z^{Km}}{B(Km,m_o)(m_o+1)(z+\beta)^{Km+m_o}}\\ &\quad\times {}_2F_1\left(Km+m_o; m_o+1; m_o+2, \frac{\beta}{\beta+Z}\right).\end{aligned} \tag{26}$$

The effective capacity of a spectrum sharing cognitive radio channel as a function of the delay exponent can be formulated in terms of power allocation constraint as (27), where $p := p(\theta, h, g)$:

$$E_c(\theta) = -\frac{1}{\theta}\ln\left(\varepsilon_v\left\{\left(1+\left[\beta^{1/(1+\alpha)}v^{-1/(1+\alpha)}-1\right]^+\right)^{-\alpha}\right\}\right)$$
$$= -\frac{1}{\theta}\ln\left(\frac{z^{Km-1}\beta^{-\alpha/(1+\alpha)}}{B(Km,m_o)}\int_0^\beta \frac{v^{m_o-1/(1+\alpha)}}{(v+Z)^{Km+m_o}}dv + \frac{z^{Km-1}}{B(Km,m_o)}\int_0^\beta \frac{v^{m_o-1}}{(v+Z)^{Km+m_o}}dv\right). \quad (27)$$

Using the fact that $\int_0^\infty f_v(v)dv = 1$, the closed form expression for the effective capacity of the secondary user operating in a spectrum sharing (*underlay*) cognitive radio channel subject to interference power constraint has been expressed as (28) below, where m_o is the fading parameter of the path between SU_{tx} and PU_{rx}, m is the fading parameter at each MRC K antenna, and ${}_2F_1(\cdot)$ is a Gaussian hypergeometric function whose general integral representation is given in equation (9.111) in [16]:

$$E_c(\theta) = -\frac{1}{\theta}\ln\left(\frac{\beta^{m_o}}{B(Km,m_o)Z^{m_o}(m_o+\alpha/(1+\alpha))} \times {}_2F_1\left(Km+m_o; m_o+\frac{\alpha}{1+\alpha}; m_o+\frac{\alpha}{1+\alpha}+1, -\frac{\beta}{Z}\right)+1 - \left[\frac{\beta^{m_o}}{B(Km,m_o)m_oZ^{m_o}} \times {}_2F_1\left(Km+m_o; m_o; m_o+1, -\frac{\beta}{Z}\right)\right]\right). \quad (28)$$

The closed form expression for the effective capacity in (28) can be obtained by using the same previous Gauss Hypergeometric *transformation* formula equation (9.131) in [15]. The resultant effective capacity after this transformation was obtained as given below:

$$E_c(\theta) = -\frac{1}{\theta}\ln\left(\frac{\beta^{m_o}Z^{Km}}{B(Km,m_o)(z+\beta)^{Km+m_o}(m_o+\alpha/(1+\alpha))} \times {}_2F_1\left(Km+m_o; 1; m_o+\frac{\alpha}{1+\alpha}+1, \frac{\beta}{Z+\beta}\right)+1 - \left[\frac{\beta^{m_o}Z^{Km}}{B(Km,m_o)m_o(z+\beta)^{Km+m_o}} \times {}_2F_1\left(Km+m_o; 1; m_o+1, \frac{\beta}{Z+\beta}\right)\right]\right). \quad (29)$$

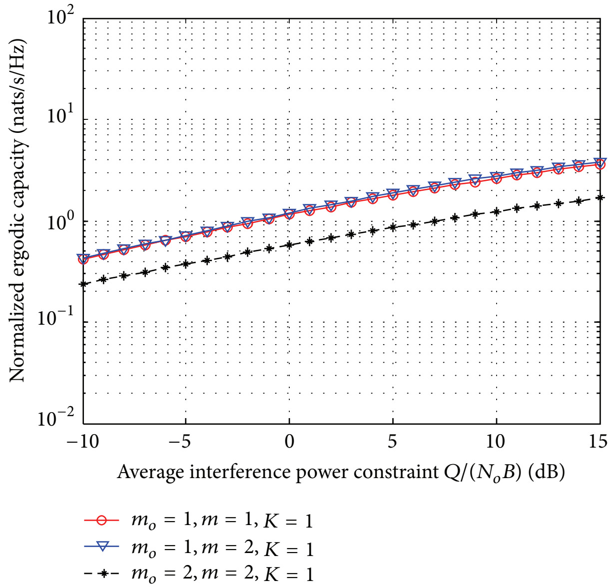

FIGURE 2: Normalized ergodic capacity per unit bandwidth for the same degree of fading on both $SU_{tx} \rightarrow SU_{rx}$ and $SU_{tx} \rightarrow PU_{rx}$ paths, $m_o = m = 1$ and $m_o = m = 2$, subject to transmit power constraint, Q, without MRC ($K = 1$) at the SU_{rx}.

5. Numerical Results and Analysis

In the preceding section, the ergodic capacity and the effective capacity of Nakagami-m CR fading channels subject to average interference power constraint have been mathematically illustrated, and in addition, the effective capacity is subject to delay QoS constraint and assumed Rayleigh block fading channel. In this section, we present the numerical results. It is assumed that the *underlay* spectrum sharing approach is being used, where the secondary users share the spectrum with the primary user such that the transmission of the SU does not interfere with that of the PU. The results for the ergodic capacity and effective capacity analyses are normalized by assuming that $N_oB = 1$ with various values of average interference power, $Q/(N_oB)$ (dB), and different values of delay QoS constraints in the case of effective capacity.

The ergodic capacity of the SU_{rx} with K branch increases when the degree of Nakagami-m fading parameter m increases. The numerical result reveals that the interference to the primary system increases when the Nakagami fading parameter m_o along $SU_{tx} \rightarrow PU_{rx}$ path is less severe (m_o increases). From Figure 2, it is apparent that when the interference channel ($SU_{tx} \rightarrow PU_{rx}$) gets better, $m_o = 2$, the ergodic capacity of the secondary channel decreases, even more severely without MRC irrespective of the fact the $SU_{tx} \rightarrow SU_{rx}$ channel is equally getting better, $m = 2$.

The benefit of MRC in CR system design can be inferred from comparing Figures 2 and 3. Without MRC ($K = 1$) and the average interference power $Q = -5$ dB in Figure 2, the maximum achievable capacity is 0.4 nats/s/Hz while with MRC ($K = 2$) in Figure 3, the capacity under −5 dB power constraint gives a capacity of 1 nat/s/Hz. Also, in Figure 3, using 0 dB power constraint as reference point, it is obvious

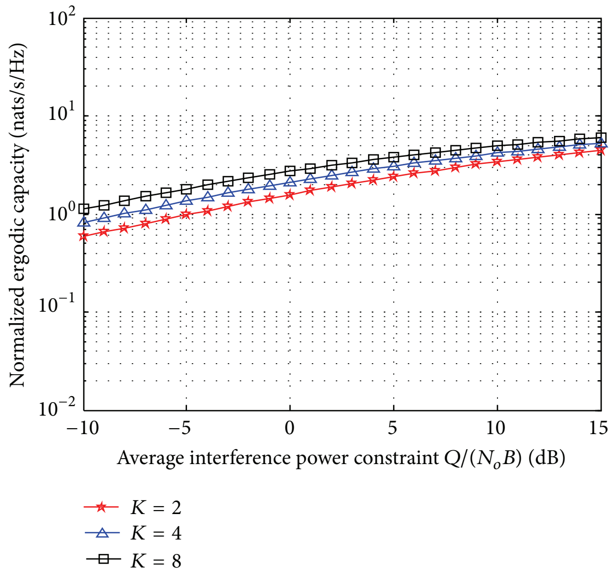

FIGURE 3: Normalized ergodic capacity per unit bandwidth for same degree of fading on both $SU_{tx} \rightarrow SU_{rx}$ paths $m_o = m = 1$ and $SU_{tx} \rightarrow PU_{rx}$ subject to transmit power constraint, Q (dB), and with MRC ($K = 2$, 4, and 8) at the SU_{rx}.

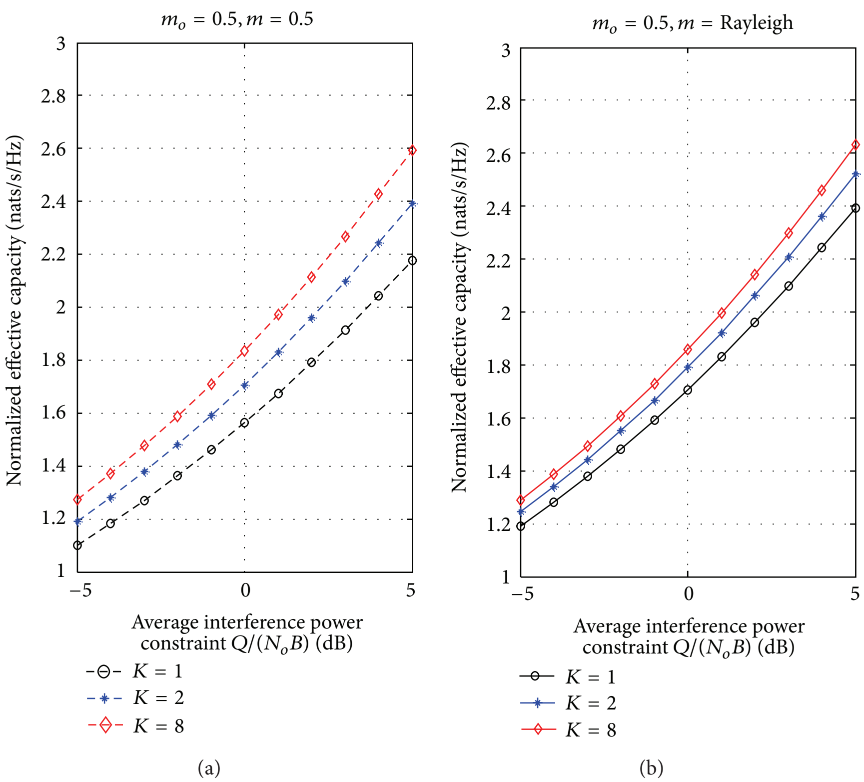

FIGURE 4: (a) Normalized effective capacity per unit bandwidth subject to transmit power constraint, Q (dB), with MRC ($K = 1$, 2, and 8) at the SU_{rx} and delay QoS exponent $\theta = 0.01$ when $SU_{tx} \rightarrow SU_{rx}$ Nakagami path $m = 0.5$ while $SU_{tx} \rightarrow PU_{rx}$ Nakagami path $m_o = 0.5$. (b) Normalized effective capacity per unit bandwidth subject to transmit power constraint, Q (dB), with MRC ($K = 1$, 2, and 8) at the SU_{rx} and delay QoS exponent $\theta = 0.01$ when $SU_{tx} \rightarrow SU_{rx}$ Nakagami path $m =$ *Rayleigh* while $SU_{tx} \rightarrow PU_{rx}$ Nakagami path $m_o = 0.5$.

that increasing the number of MRC antennas ($K = 2 \rightarrow 4 \rightarrow 8$) at the SU_{rx}, yields an increase in ergodic capacity; although, not so significant or having much increase, there is a gain in capacity when the number of antennas is increased.

In analyzing the CR effective capacity, it is further assumed that $N_oB = 1$ and $TB = 1$, thereby normalizing the effective capacity results. Figures 4(a) and 4(b) illustrate the normalized achievable effective capacity of CR channel under

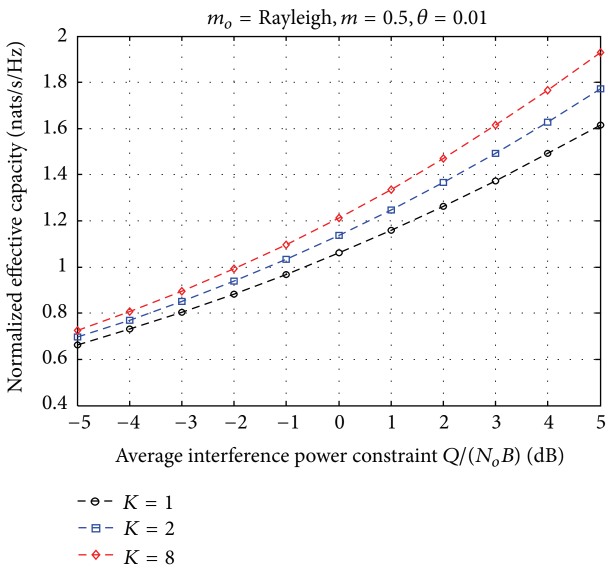

FIGURE 5: Normalized effective capacity per unit bandwidth subject to transmit power constraint, Q (dB), with MRC (K = 1, 2, and 8) at the SU_{rx} and delay QoS exponent $\theta = 0.01$ when $SU_{tx} \rightarrow PU_{rx}$ Nakagami path has fading parameter m_o being Rayleigh while $SU_{tx} \rightarrow SU_{rx}$ has Nakagami path fading parameter $m = 0.5$.

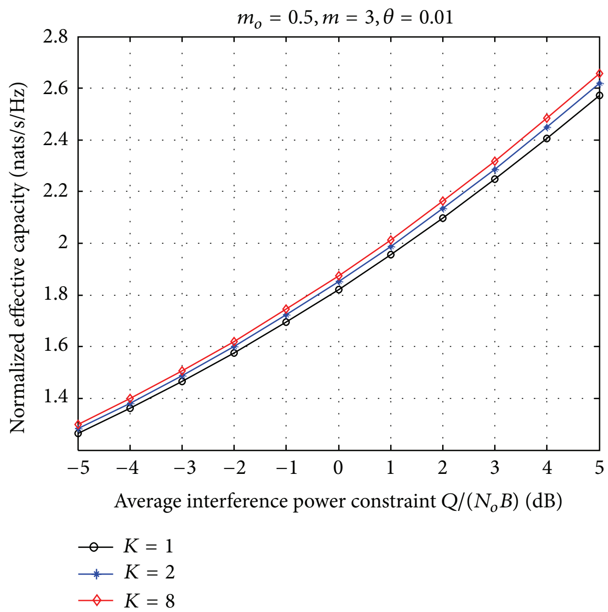

FIGURE 6: Normalized effective capacity per unit bandwidth subject to transmit power constraint, Q (dB), with MRC (K = 1, 2, 8) at the SU_{rx} and delay QoS exponent $\theta = 0.01$ when $SU_{tx} \rightarrow SU_{rx}$ Nakagami path parameter $m = 3$ while $SU_{tx} \rightarrow PU_{rx}$ Nakagami path parameter increases as $m_o = 0.5$.

different average interference power, Q, with assumed delay QoS exponent $\theta = 0.01$ (1/nat). From these Figures, it is obvious that the effective capacity increases with increase in number of MRC antennas K at the SU_{rx}. In Figure 4(a), the increase in capacity is significant when the Nakagami-m parameter of fading $m = 0.5$ along the $SU_{tx} \rightarrow SU_{rx}$ path.

On the contrary, in Figure 4(b), with better secondary channel, m is Rayleigh and increases in MRC antennas K at the secondary receiver; the increase in effective capacity is less significant but more than that when $m = 0.5$. Taking 0 dB interference power as reference point in Figure 4(a), when $m_o = m = 0.5$ and $K = 2$, the achievable transmission rate under 0.01 delay QoS exponent is 1.7 nats/s/Hz, and at the same interference power reference in Figure 4(b) where $K = 2$ and m = Rayleigh, the achievable capacity is 1.8 nats/s/Hz due to better channel.

Figure 5 illustrates the normalized effective capacity of CR channel when the Nakagami fading parameter m_o ($SU_{tx} \rightarrow PU_{rx}$) is Rayleigh and that of the secondary channel $m = 0.5$. Therefore, when the primary or interference channel gets better, the capacity of CR decreases significantly compared to the achievable capacity in Figure 4(b) where the secondary channel is better. Although, when the interference channel is better than the CR channel, the capacity can still be increased by increasing the MRC antennas but not as significant as the achievable capacity when the secondary channel is better than the primary channel. Figures 4(b) and 6 show improved CR capacity when CR channel improves, m is Rayleigh, and $m = 3$, respectively. But, it is worth to point out that in Figure 6, even though the CR channel gets better with $m = 3$, the increase in capacity is not very significant due to transmit power limitation that hinders the use of all the opportunities to transmit.

In Figure 7, the numerical result of $E_c(\theta)$ against various delay QoS exponents is illustrated at $Q = -5$ dB. It is obvious that the effective capacity of spectrum sharing CR decreases as the delay QoS exponent θ increases. Hence, increasing the delay QoS exponent reduces the maximum information transmission rate of CR systems. This result justifies the capability of CR systems in satisfying various QoS specifications for different CR users or different wireless applications such as real-time or delay-sensitive applications. In other words, different QoS requirements for different users or applications can be supported by cognitive radio under the same power constraint for all CR users.

Lastly, in comparison with the ergodic capacity at the same $Q = -5$ dB, it is apparent that as the delay QoS exponent decreases, the effective capacity approaches ergodic capacity. Although, delay QoS exponent is not defined for ergodic capacity, we can comparatively analyze ergodic and effective capacity on the same logarithm scale. With MRC, the increase in effective capacity is more significant than that in ergodic capacity, but when the delay QoS exponent decreases, for instance to $\theta = 0.01$ at $Q = -5$ dB and $K = 8$ in Figure 7, the effective capacity decreases and approaches the ergodic capacity. And when the QoS exponent increases towards infinity, the effective capacity approaches outage capacity.

6. Conclusion

The maximum achievable capacity of spectrum sharing CR with MRC under Nakagami fading has been studied in this

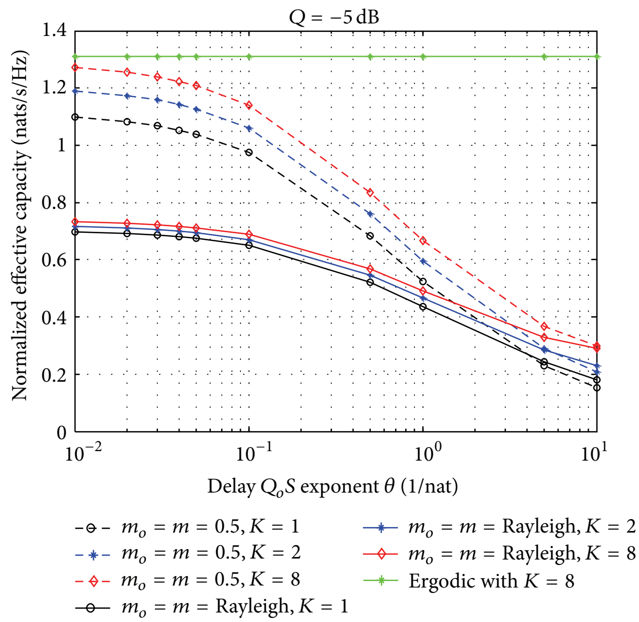

Figure 7: Normalized effective capacity versus ergodic per unit bandwidth subject to transmit power constraint, Q (dB), with MRC ($K = 1$, 2, and 8) at the SU_{rx} and various delay QoS exponent values at $Q = -5$ dB.

paper. We presented the mathematical analyses for both the ergodic capacity and the effective capacity. Our numerical results show that with MRC at the secondary receiver, there is a gain in capacity in both ergodic and effective capacity. This implies that MRC can be implemented to combat the effect of fading in CR systems. This is true for other types of fading since Rayleigh and AWGN can be derived as special cases in Nakagami ($m = 1$). Also, it has been numerically demonstrated that under the same average interference power constraint, different QoS requirements can be supported in cognitive radio. Finally, it is proven that the secondary user can share spectrum with the primary user and still achieve a significant capacity gains provided the average interference power constraint is specified.

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