Research Article

Improved Bounds on $m_r(2, q)$ $q = 19, 25, 27$

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An (n, r) -arc is a set of n points of a projective plane such that some r , but no $r + 1$ of them, are collinear. The maximum size of an (n, r) -arc in $\text{PG}(2, q)$ is denoted by $m_r(2, q)$. In this paper, a new $(286, 16)$ -arc in $\text{PG}(2, 19)$, a new $(341, 15)$ -arc, and a $(388, 17)$ -arc in $\text{PG}(2, 25)$ are constructed, as well as a $(394, 16)$ -arc, a $(501, 20)$ -arc, and a $(532, 21)$ -arc in $\text{PG}(2, 27)$. Tables with lower and upper bounds on $m_r(2, 25)$ and $m_r(2, 27)$ are presented as well. The results are obtained by nonexhaustive local computer search.

1. Introduction

Let $\text{GF}(q)$ denote the Galois field of q elements, and let $V(3, q)$ be the vector space of row vectors of length three with entries in $\text{GF}(q)$. Let $\text{PG}(2, q)$ be the corresponding projective plane. The points (x_1, x_2, x_3) of $\text{PG}(2, q)$ are the 1-dimensional subspaces of $V(3, q)$. Subspaces of dimension two are called *lines*. The number of points in $\text{PG}(2, q)$ is $q^2 + q + 1$ and so is the number of lines. There are $q + 1$ points on every line and $q + 1$ lines through every point.

Definition 1. An (n, r) -arc is a set of n points of a projective plane such that some r , but no $r + 1$ of them, are collinear.

The maximum size of a (k, r) -arc in $\text{PG}(2, q)$ is denoted by $m_r(2, q)$.

Definition 2. An (l, t) -blocking set S in $\text{PG}(2, q)$ is a set of l points such that every line of $\text{PG}(2, q)$ intersects S in at least t points, and there is a line intersecting S in exactly t points.

Note that an (n, r) -arc is the complement of a $(q^2 + q + 1 - n, q + 1 - r)$ -blocking set in a projective plane and conversely.

Definition 3. Let M be a set of points in any plane. An i -secant is a line meeting M in exactly i points. A 0-secant is also called skew line. Define τ_i as the number of i -secants to a set M .

In terms of τ_i , the definitions of an (n, r) -arc and an (l, t) -blocking set become the following:

an (n, r) -arc is a set of n points of a projective plane for which $\tau_i \geq 0$ for $i < r$, $\tau_r > 0$ and $\tau_i = 0$ when $i > r$;

an (l, t) -blocking set is a set of l points of a projective plane for which $\tau_i = 0$ for $i < t$, $\tau_t > 0$ and $\tau_i \geq 0$ when $i > t$.

In 1947, Bose [1] proved that

$$m_2(2, q) = \begin{cases} q + 1 & \text{for } q \text{ odd,} \\ q + 2 & \text{or } q \text{ even.} \end{cases} \quad (1)$$

From the result of Barlotti [2], it follows that for q odd and

$$r = \frac{(q + 1)}{2}, \quad r = \frac{(q + 3)}{2}, \quad (2)$$

there exists an

$$((r - 1)q + 1, r)\text{-arc.} \quad (3)$$

For background on $\text{PG}(2, q)$, see Hirschfeld [3].

A survey of (n, r) -arcs with the best known results was presented in [4]. After this publication, many improvements

TABLE 1: Values of $m_r(2, q)$.

r	q					
	3	4	5	7	8	9
2	4	6	6	8	10	10
3		9	11	15	15	17
4			16	22	28	28
5				29	33	37
6				36	42	48
7					49	55
8						65

were obtained in [5–7]. Summarizing these improvements, Ball and Hirschfeld [8] presented a new table with bounds on $m_r(2, q)$ for $q \leq 19$. As we can see from these tables, the exact values of $m_r(2, q)$ are known only for $q \leq 9$ (see Table 1).

Some new improvements were made in recent years. A (79,6)-arc in PG(2,17) and a (126,8)-arc in PG(2,19) are given in [9]. A (95,7)-arc, and a (183,12)-arc, a (205,13)-arc in PG(2,17) and a (243,14)-arc and a (264,15)-arc in PG(2,19) have been presented in [10]. In 2010, Gulliver constructed an optimal (78,8)-arc in PG(2,11) (see [11]). A table for $m_r(2, q)$, $q \leq 19$, is maintained by Ball [11].

To obtain good (n, r) -arcs, we apply local search techniques. The neighborhood structure is a simple one. Given an arc, then its neighborhood consists of all arcs that can be obtained from the given arc by adding new points or deleting some points. The choice of a start solution is based on some heuristic observations. The cost function is chosen to favour as local optima arcs with a small number of r -secants. The computation times are in order of several minutes up to few hours on a PC. Similar techniques are employed for the construction of (l, t) -blocking sets.

Sum and product tables for the fields PG(2,25) and PG(2,27) are taken from [12]. In order to present the results in more concise form, the points in PG(2,25) and PG(2,27) are in lexicographic order, and each point is associated with its number. For example, some of the points in PG(2,25) and their numbers are given in Table 2.

In Table 3, some of the lines and their numbers are presented.

In [9], a (285,16)-arc in PG(2,19), a (339,15)-arc and a (387,17)-arc in PG(2,25), and a (393,16)-arc, a (499,20)-arc, and a (531,21)-arc in PG(2,27) are presented. In this paper, we improve these results by constructing six new large arcs.

2. A New Arc in PG(2, 19)

Theorem 4. *There exists a (286, 16)-arc in PG(2,19).*

Proof. The set of points having numbers

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 24 28 33
37 44 46 52 57 62 67 68 70 71 79 90 96 103 106 109 112
116 121 128 129 134 136 137 145 155 159 162 166 181 184
185 188 194 195 203 204 221 223 225 233 242 247 250
254 261 262 267 272 280 285 287 294 299 301 311 313

TABLE 2: Some points in PG(2, 25).

Number	Point
1	(0, 0, 1)
2	(0, 1, 0)
3	(0, 1, 1)
4	(0, 1, 2)
5	(0, 1, 3)
6	(0, 1, 4)
7	(0, 1, 5)
8	(0, 1, 6)
9	(0, 1, 7)
...	...
...	...
...	...
303	(1, 11, 1)
304	(1, 11, 2)
305	(1, 11, 3)
306	(1, 11, 4)
307	(1, 11, 5)
308	(1, 11, 6)
309	(1, 11, 7)
310	(1, 11, 8)
...	...
643	(1, 24, 16)
644	(1, 24, 17)
645	(1, 24, 18)
646	(1, 24, 19)
647	(1, 24, 20)
648	(1, 24, 21)
649	(1, 24, 22)
650	(1, 24, 23)
651	(1, 24, 24)

316 318 329 337 338 339 340 346 350 356 360 363 372
375 379

forms a (95, 4)-blocking set in PG(2, 19).

The complement of this blocking set is a (286, 16)-arc in PG(2, 19) with secant distribution

$$\begin{aligned} \tau_0 = 14, \quad \tau_1 = 1, \quad \tau_{13} = 10, \quad \tau_{14} = 60, \\ \tau_{15} = 147, \quad \tau_{16} = 159. \end{aligned} \quad (4)$$

In the next sections, we construct new large (n, r) -arcs in PG(2, 25) and PG(2, 27). These arcs were presented at ACCT 2012, Pomorie, Bulgaria [13]. $\square$

3. New Arcs in PG(2, 25)

Theorem 5. *There exist a (341, 15)-arc and a (388, 17)-arc in PG(2,25).*

Proof. (1) The set of points having numbers

6 9 10 15 16 17 20 22 23 25 26 29 30 31 32 33 34 35 36
37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 56

TABLE 3: Some lines in PG(2, 25).

Number	a, b, c	Line— $ax + by + cz = 0$
1	0, 0, 1	626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651
110	1, 3, 8	31 66 69 84 87 167 176 187 211 260 265 299 303 322 325 394 411 432 467 488 503 518 580 587 606 628
310	1, 11, 8	5 38 66 89 112 140 153 181 204 227 255 293 321 344 367 395 408 436 459 482 510 548 576 599 622 650
444	1, 16, 17	8 30 93 112 121 144 160 175 196 201 216 253 301 339 357 408 423 448 480 532 537 539 578 594 606 648
651	1, 24, 24	59 61 63 65 67 122 124 126 128 130 132 142 193 195 197 199 201 203 265 267 269 271 273 275 291 651

TABLE 4: Upper bounds on $m_r(2, 25)$.

r	$(r - 1)q + r - 2$ (13)	$(r - 1)q + \lfloor r^2/q \rfloor$ (14) or (15)	$(r - 1)q + (r, q)$ (16)	Upper bounds
2	26	25	26	26
3	51	50	51	51
4	77	75	76	76
5	103	100	105	103
6	129	126	126	126
7	155	151	151	151
8	181	177	176	177
9	207	203	201	203
10	233	228	230	230
11	259	254	251	254
12	285	280	276	280
13	311	306	301	306
14	337	332	326	332
15	363	358	355	358
16	389	385	376	385
17	415	411	401	411
18	441	437	426	437
19	467	464	451	464
20	493	480	480	480
21	519	517	501	517
22	545	544	526	544
23	571	571	551	571
24	597	598	576	597

60 63 65 66 68 69 71 74 77 78 79 80 81 85 87 89 92 95
97 99 102 103 104 105 107 108 110 113 121 122 124 126 129
130 131 134 135 136 138 146 148 149 150 154 155 157 160
164 165 169 170 172 174 176 178 181 182 187 188 189 192
195 196 197 200 202 207 210 211 212 214 215 217 219 220
223 224 228 229 230 231 232 239 241 243 245 246 247
248 252 257 258 259 263 266 268 270 271 272 275 276
279 280 284 287 290 291 292 293 294 296 300 307 309

310 313 315 319 320 321 322 324 325 328 331 333 334 335
336 340 344 348 349 350 351 352 353 356 358 359 364
365 368 369 370 376 377 379 380 382 384 386 387 390
424 425 427 428 431 432 436 437 440 441 442 443 444
447 452 454 455 458 459 461 464 468 470 473 475 476
479 480 486 488 489 490 491 493 494 495 498 508 510
511 512 513 516 517 518 521 523 524 526 528 531 533 536
537 538 539 542 545 546 551 552 559 560 562 564 566

TABLE 5: Bounds on $m_r(2, 25)$.

r	
2	26
3	38–51
4	64–76
5	85–103
6	126
7	135–151
8	168–177
9	189–203
10	210–230
11	231–254
12	266–280
13	301–306
14	326–332
15	341–358
16	362–385
17	388–411
18	414–437
19	447–464
20	480
21	501–517
22	527–544
23	558–571
24	577–597

567 568 570 575 576 578 579 580 581 583 584 587 590
 592 594 600 601 602 607 608 611 613 615 619 621 622
 623 626 629 630 632 633 635 637 641 642 643 647 649
 651

forms a (310,11)-blocking set in PG(2,25) with secant distribution

$$\begin{aligned} \tau_{11} = 229, \quad \tau_{12} = 247, \quad \tau_{13} = 122, \quad \tau_{14} = 28, \\ \tau_{15} = 1, \quad \tau_{23} = 7, \quad \tau_{24} = 8, \quad \tau_{25} = 3, \quad \tau_{26} = 6. \end{aligned} \quad (5)$$

The complement of this blocking set is a (341,15)-arc in PG(2,25).

(2) The set of points having numbers

1 3 6 12 13 14 17 18 20 21 24 27 28 31 35 38 40 46 49 52
 60 62 64 67 70 71 76 77 78 79 80 81 82 83 84 85 86 87
 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104
 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119
 120 121 122 123 124 125 126 127 133 137 138 139 142 145
 149 157 158 160 164 166 168 172 174 176 182 183 185 190
 194 197 199 201 205 206 211 213 214 215 216 219 220 221
 223 225 227 229 230 231 233 238 246 247 255 256 259
 262 263 266 267 268 269 271 275 278 281 286 289 291
 293 295 298 302 303 304 305 306 310 311 313 324 325
 328 331 334 337 340 342 344 350 354 355 358 361 364
 373 375 376 379 380 382 384 386 392 397 400 403 406
 411 414 416 418 420 423 429 430 432 434 437 447 448
 450 454 455 458 459 461 470 473 476 478 481 484 487
 490 492 494 500 502 503 504 505 506 509 510 523 524

532 533 535 541 543 547 549 551 553 554 559 562 563
 565 566 567 568 571 575 577 578 579 580 582 588 596
 601 603 604 609 611 613 614 615 618 619 620 621 623
 632 633 635 640 644 647 649 651

forms a (263,9)-blocking set in PG(2,25) with secant distribution

$$\begin{aligned} \tau_9 = 220, \quad \tau_{10} = 226, \quad \tau_{11} = 96, \quad \tau_{12} = 58, \quad \tau_{13} = 16, \\ \tau_{14} = 17, \quad \tau_{15} = 4, \quad \tau_{16} = 2, \quad \tau_{25} = 4, \quad \tau_{26} = 8. \end{aligned} \quad (6)$$

The complement of this blocking set is a (388,17)-arc in PG(2,25). $\square$

4. New Arcs in PG(2, 27)

Theorem 6. *There exist a (394, 16)-arc, a (501, 20)-arc, and a (532, 21)-arc in PG(2,27).*

Proof. (1) The set of points having numbers

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 29 57 58 59 62 67 69
 71 72 73 75 77 78 79 82 83 84 85 86 88 89 90 93 94 96
 97 102 103 108 109 110 113 114 116 118 119 120 121 124 126
 128 129 130 133 135 138 139 141 145 149 150 151 152 153
 158 160 161 162 163 164 165 166 173 174 176 178 179 180
 182 184 185 187 188 189 196 198 201 202 203 205 207 208
 210 211 213 214 215 216 219 220 223 225 227 229 231 232
 233 236 237 239 243 244 245 249 253 255 256 258 260
 261 262 264 265 266 267 268 271 273 274 275 276 277
 278 279 280 281 288 289 290 294 295 299 300 301 303
 304 306 307 311 313 315 316 321 322 323 324 326 331 333
 335 337 338 339 340 343 344 345 348 350 351 352 353
 356 358 359 360 364 365 368 369 370 372 374 375 376
 377 383 385 386 387 389 390 394 395 396 398 400 401
 403 405 408 409 410 411 413 415 417 418 421 424 426
 427 429 432 438 442 443 444 445 446 447 450 452 453
 454 457 458 460 464 465 467 469 470 473 474 475 478
 479 483 485 486 487 491 492 493 494 495 496 497 499
 500 503 506 507 509 512 515 518 521 524 528 529 530
 531 532 533 536 537 538 540 541 546 547 549 550 552
 555 556 557 559 562 563 564 567 568 570 571 572 574
 575 576 579 581 582 585 589 592 593 595 596 597 598
 599 602 605 606 608 609 613 614 616 618 620 622 623
 624 625 627 628 630 631 632 634 636 639 641 642 646
 649 650 651 652 653 654 656 658 661 662 664 665 669
 671 674 675 678 679 686 687 688 689 692 694 695 696
 697 698 699 701 704 705 706 708 709 711 712 713 714
 719 721 722 724 725 726 731 734 735 736 737 738 739
 741 743 744 746 751 752 755 757

forms a (394,16)-arc in PG(2,27) with secant distribution

$$\tau_1 = 12, \quad \tau_2 = 15, \quad \tau_{14} = 156, \quad \tau_{15} = 378, \quad \tau_{16} = 196. \quad (7)$$

(2) The set of points having numbers

1 2 13 17 19 21 23 24 40 42 46 48 51 54 55 59 61 62 63 67
 70 75 81 86 88 89 90 94 97 102 108 110 122 124 125 127

TABLE 6: Upper bounds on $m_r(2, 27)$.

r	$(r - 1)q + r - 2$ (13)	$(r - 1)q + \lfloor r^2/q \rfloor$ (14) or (15)	$(r - 1)q + (r, q)$ (16)	Upper bounds
2	28	28	28	28
3	55	54	57	55
4	83	81	82	82
5	111	108	109	109
6	139	136	138	138
7	167	163	163	163
8	195	191	190	191
9	223	219	225	223
10	251	246	244	246
11	279	274	271	274
12	307	302	300	302
13	335	330	325	330
14	363	358	352	358
15	391	386	381	386
16	419	414	406	414
17	447	442	433	442
18	475	471	468	471
19	503	499	487	499
20	531	527	514	527
21	559	556	543	556
22	587	584	568	584
23	615	613	595	613
24	643	642	624	642
25	671	671	649	671
26	699	700	676	699

131 132 134 135 140 141 142 143 144 145 146 148 155 156
 167 170 174 175 178 183 184 189 191 203 205 206 207 208
 212 213 214 215 216 219 220 221 224 228 229 232 237 238
 243 248 249 251 253 254 256 260 263 264 266 281 282
 285 288 290 292 295 298 300 301 310 313 314 318 320
 324 327 328 330 331 333 334 336 343 346 348 353 357
 361 362 365 370 371 375 377 381 382 390 392 396 400
 403 404 410 412 413 414 417 419 421 422 423 427 428
 430 431 432 438 442 445 447 449 453 455 460 461 462
 463 474 475 476 477 482 484 486 487 491 493 494 495
 500 503 509 512 519 523 524 525 528 531 533 535 538
 541 542 546 547 549 550 551 552 559 560 562 564 570
 571 582 583 584 590 594 595 596 600 604 607 609 611
 615 617 622 623 628 630 635 638 640 644 645 647 650
 663 665 666 667 671 672 673 676 681 685 686 687 694
 695 697 699 707 710 714 716 717 720 724 727 728 730
 732 733 740 743 747 749 754 755 706

forms a (256,8)-blocking set in PG(2,27) with secant distribution

$$\begin{aligned} \tau_8 &= 252, & \tau_9 &= 236, & \tau_{10} &= 152, & \tau_{11} &= 73, \\ \tau_{12} &= 21, & \tau_{13} &= 10, & \tau_{14} &= 2, & \tau_{15} &= 1, & \tau_{28} &= 10. \end{aligned} \quad (8)$$

The complement of this blocking set is a (501,20)-arc in PG(2,27).

(3) The set of points having numbers

1 2 13 16 17 21 23 27 29 30 31 39 41 42 46 51 55 59 61
 62 63 67 70 75 81 86 88 89 90 94 97 102 108 110 111 112
 114 122 124 127 132 134 135 137 139 140 141 143 145 146
 148 155 156 165 166 167 170 174 175 178 183 184 189 191
 192 193 195 203 205 208 213 215 216 219 220 221 224 228
 229 232 237 238 243 245 247 248 249 251 253 254 256
 263 264 281 282 285 288 290 292 295 298 300 301 313
 314 320 324 330 331 333 334 336 343 346 348 357 361
 362 370 371 375 381 382 392 394 396 403 404 408 410
 413 417 419 423 427 430 431 438 442 445 447 449 453
 455 460 461 462 474 476 477 482 484 487 493 495 500
 503 509 512 524 525 528 531 533 535 538 541 546 547
 549 550 552 559 562 564 570 571 583 584 590 594 600
 604 607 609 611 615 617 622 628 630 635 638 644 647
 650 651 663 665 666 671 673 676 681 685 686 694 695
 699 705 707 710 714 716 720 724 727 728 732 733 743
 745 747 754 755

forms a (225,7)-blocking set in PG(2,27) with secant distribution

TABLE 7: Bounds on $m_r(2, 27)$.

r	
2	28
3	42–55
4	66–82
5	88–109
6	116–138
7	142–163
8	169–191
9	198–223
10	223–246
11	253–274
12	281–302
13	315–330
14	352–358
15	379–386
16	394–414
17	419–442
18	468–471
19	483–499
20	501–527
21	532–556
22	558–584
23	586–613
24	624–642
25	652–671

$$\begin{aligned} \tau_7 &= 308, & \tau_8 &= 208, & \tau_9 &= 130, & \tau_{10} &= 53, \\ \tau_{11} &= 46, & \tau_{12} &= 3, & \tau_{14} &= 1, & \tau_{28} &= 8. \end{aligned} \quad (9)$$

The complement of this blocking set is a $(532, 21)$ -arc in $\text{PG}(2, 27)$. $\square$

There exists a projective $[n, 3, d]_q$ code if and only if there exists an $(n, n - d)$ -arc in $\text{PG}(2, q)$ (see [14]). So, there exist projective $[286, 3, 270]_{19}$, $[341, 3, 326]_{25}$ Griesmer's codes and projective codes, having parameters $[388, 3, 371]_{25}$, $[394, 3, 378]_{27}$, $[501, 3, 481]_{27}$, and $[532, 3, 511]_{27}$.

5. New Upper Bounds on $m_r(2, 25)$ and $m_r(2, 27)$

When q is a prime, there are good upper bounds on $m_r(2, q)$ obtained by Ball [15] (see also [4]) and Daskalov [16].

Theorem 7. *Let K be an (n, r) -arc in $\text{PG}(2, q)$, where q is prime.*

- (1) If $r \leq (q + 1)/2$, then $m_r(2, q) \leq (r - 1)q + 1$.
- (2) If $r \geq (q + 3)/2$, then $m_r(2, q) \leq (r - 1)q + r - (q + 1)/2$.

Theorem 8. *Let K be an (n, r) -arc in $\text{PG}(2, q)$ with $r > (q + 3)/2$ and prime $q \leq 29$. Then,*

$$m_r(2, q) \leq (r - 1)q + r - \frac{(q + 3)}{2}. \quad (10)$$

From Theorem 7 and Barlotti's constructions, it follows that, for prime q ,

$$\begin{aligned} m_r(2, q) &= (r - 1)q + 1, \text{ where} \\ r &= \frac{(q + 1)}{2}, \quad r = \frac{(q + 3)}{2}. \end{aligned} \quad (11)$$

But when q is an odd prime power, these bounds are not valid. For example, a $(48, 6)$ -arc in $\text{PG}(2, 9)$ was constructed by Mason [17]. So, when q is an odd prime power, in order to obtain good upper bounds on $m_r(2, q)$, we have to combine some bounds given in [8]—consider the following (12), (13), (14), (15) and (16).

Let $\mathcal{A}$ be an (n, r) -arc and P a point of $\mathcal{A}$. Each line incident with P contains at most $(r - 1)$ points of $\mathcal{A} \setminus P$, and the next trivial upper bound follows

$$n \leq (r - 1)(q + 1) + 1 = (r - 1)q + r. \quad (12)$$

If $(r, q) \neq (2^t, 2^h)$ and $2 < r < q$, then (Barlotti, [18])

$$n \leq (r - 1)q + r - 2. \quad (13)$$

Corollary 2.2 in [8]. An (n, r) -arc $\mathcal{A}$ in a projective plane of order q which has no skew line satisfies

$$n \leq (r - 1)q + \left\lfloor \frac{r^2}{q} \right\rfloor, \quad (14)$$

and if $\sqrt{q}$ divides r , then

$$n < (r - 1)q + \left\lfloor \frac{r^2}{q} \right\rfloor. \quad (15)$$

It follows from [19] that an (n, r) -arc $\mathcal{A}$ in $\text{PG}(2, q)$ which has a skew line satisfies

$$n \leq (r - 1)q + (r, q). \quad (16)$$

Corollary 2.2 in combination with (16) can always be used to provide an upper bound on $m_r(2, q)$.

In [9], upper bounds on $m_r(2, 25)$, based on the trivial bound (12), are given. Now, we will improve these bounds, using (13), (14), (15), and (16) (see Table 4).

Combining these upper bounds with the lower bounds from [9] and the new arcs, presented in Sections 3 and 4, we obtain the next bounds on $m_r(2, 25)$ (see Table 5).

In $\text{PG}(2, 27)$, we have Tables 6 and 7.

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