Research Article

Correlation between Shear Wave Velocity and Porosity in Porous Solids and Rocks

J. Kováčik¹ and Š. Emmer²

¹ Institute of Materials & Machine Mechanics, Slovak Academy of Sciences, Račianska 75, SK—831 02 Bratislava 3, Slovakia

² Slovak University of Technology, IVMA STU, Institute of Technologies & Materials STU, Pionierska 15, SK—831 02 Bratislava 3, Slovakia

Correspondence should be addressed to J. Kováčik, ummsjk@savba.sk

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The shear wave velocity dependence on porosity was modelled using percolation theory model for the shear modulus porosity dependence. The obtained model is not a power law dependence (no simple scaling with porosity), but a more complex equation. Control parameters of this equation are shear wave velocity of bulk solid, percolation threshold of the material and the characteristic power law exponent for shear modulus porosity dependence. This model is suitable for all porous materials, mortars and porous rocks filled with liquid or gas. In the case of pores filled with gas the model can be further simplified: The term for the ratio of the gas density to the density of solid material can be omitted in the denominator (the ratio is usually in the range of 10^{-4} , 10^{-3}) for all solids). This simplified equation was then tested on the experimental data set for porous ZnO filled with air. Due to lack of reasonable data the scientists are encouraged to test the validity of proposed model using their experimental data.

1. Introduction

The porous materials are usually prepared by various powder metallurgy methods from powders, which composition, particle size, and shape can vary significantly. During the powder consolidation different porosity can be achieved by varying of the technological parameters: such as temperature, external pressure, or time. Compacting starts from just touching powder particles and goes to the lower porosity by the creation and growth of the necks between particles. The subsequent closure of the pore channels leads to the elimination of the pores. Three various porosity ranges can be usually identified, for example, for sintered iron the following porosity ranges can be observed ([1] and references therein):

- (i) porosity <3%: fully isolated pores of nearly spherical or elliptical shape,
- (ii) porosity >20%: fully interconnected pores of complex shape, and

- (iii) porosity between 3% and 20%: both isolated and interconnected pores are present in various amounts.

This indicates that the powder consolidation is in general a connectivity problem, which is studied by the percolation theory [2]. According to the percolation theory a critical volume fraction exists, called a percolation threshold, at which a solid phase forms a continual network spanning across the whole system. Below this threshold it is just a pile of powders or mud of powders mixed with liquid. At and above the percolation threshold p_c the geometrical, physical, and mechanical properties of the system behave in the form of the power law dependence $\propto ((p_c - p)/p_c)^t$ for $p \leq p_c$, where p is the porosity within the solid material and t is the critical exponent for the investigated property.

The percolation theory expects that the values of the critical exponents are universal, that is, they do not depend on the structure and geometrical properties of the system, but only depends on the dimension of the problem [2]. On the other hand, the value of the percolation threshold depends significantly on the structure.

FIGURE 1: Shear wave velocity dependence on porosity for porous ZnO [3].

TABLE 1: Fitting results for shear wave velocity dependence on porosity for porous ZnO [3]. There χ^2 is a characteristic value of the nonlinear fitting.

Porosity range [—]	c_{s0} [ms ⁻¹]	p_c [—]	f_G [—]	χ^2 [—]
0–0.33	2844 ± 16	0.52 ± 0.04	1.22 ± 0.14	611

Today nondestructive method of the determination of mechanical properties of porous solids is usually required instead of destructive tensile test and/or bending test. Acoustic or ultrasonic measurements are often used in this case [4, 5]. Measurements are usually conducted at sonic (20–50 kHz) and ultrasonic (300–700 kHz) frequencies using pulse transmission technique. The longitudinal and shear wave velocities are then determined and used to calculate the mechanical properties of the porous materials.

The aim of this paper is to model the dependence of the shear wave velocity on the porosity for porous solids and rocks.

2. Theoretical

The shear wave velocity, c_s , depends on the following material properties [2]:

$$c_s = \sqrt{\frac{G}{\rho}}, \quad (1)$$

where G is the shear modulus and ρ is the density of the material.

Recently [1, 6–8], in the case of homogeneous isotropic porous materials, modulus of elasticity E , shear modulus G , and Poisson's ratio ν were modelled using power law dependence on porosity on the basis of the percolation theory using the idea of percolation threshold [2, 9, 10]. As was mentioned above, the percolation threshold is a porosity at which the mechanical properties of porous material become zero (the porous material does not exist anymore as entity). It

was further showed [1, 6] how the powder size, preparation method (various amounts of sintering additives), the powder shape, and the investigated porosity range influence the values of characteristic exponent and percolation threshold for elastic modulus and shear modulus of porous materials prepared by powder metallurgy.

According to percolation theory the shear modulus porosity dependence is expressed as follows [6]:

$$G = G_0 \left(\frac{p_c - p}{p_c} \right)^{f_G} \quad \text{for } p \leq p_c, \quad (2)$$

where G is the effective shear modulus of porous material with porosity p , G_0 is shear modulus of solid material, p_c is a percolation threshold, and f_G is the characteristic exponent for the shear modulus of porous material.

The effective density of porous solid is also dependent on porosity. This dependence can be simply expressed by the rule of mixture as follows:

$$\rho = \rho_0 \cdot (1 - p) + \rho_f \cdot p, \quad (3)$$

where ρ_0 is the density of the solid material and ρ_f is the density of pore filling material (gas, liquid) at given temperature and pressure.

Now, (2) and (3) can be put into (1) thus obtaining

$$c_s = c_{s0} \sqrt{\frac{((p_c - p)/p_c)^{f_G}}{1 - p + (\rho_f/\rho_0) \cdot p}} \quad \text{for } p \leq p_c, \quad (4)$$

where c_{s0} is the shear wave velocity of bulk material.

In the case when the porous solid is filled with gas or air the density of air/gas is always significantly smaller (e.g., at 20°C and 101.325 kPa, dry air has a density of 1.2041 kg·m⁻³ [11]) in comparison to the density of usual solid materials. Therefore, the term containing the density of air/gas in (4) can be neglected. Thus, the proposed equation can be simplified in the following way:

$$c_s = c_{s0} \sqrt{\frac{((p_c - p)/p_c)^{f_G}}{1 - p}} \quad \text{for } p \leq p_c. \quad (5)$$

3. Experimental Results and Discussion

To prove the hypothesis it is necessary to do experiments or to use already published data. Unfortunately the “proper” experimental data (almost the same technology of the preparation, as wide as possible investigated porosity range, data also in the vicinity of the percolation threshold) is very hard to find over the scientific journals for the shear wave velocity of porous materials filled with air/gas. To authors' knowledge, all these requirements were fulfilled only for the experimental data for porous ZnO [3]. Therefore, the proposed equation was tested only using the experimental data of porous ZnO [3], what is also the main lack of this article. As Table 1 and Figure 1 indicate (5) describes the investigated experimental data relatively well. Besides determination of the value of

percolation threshold for porous ZnO to 0.52 ± 0.04 it also estimates the velocity of shear waves in solid ZnO to $2844 \pm 16 \text{ ms}^{-1}$ and the characteristic exponent f_G 1.22 ± 0.14 . The percolation threshold value coincides with the percolation threshold value obtained for modulus of elasticity and shear modulus of ZnO [1, 6]: 0.51 ± 0.06 and 0.51 ± 0.04 , respectively. The shear wave velocity of solid ZnO is comparable to the value of 2810 ms^{-1} for solid material in the scientific literature [3]. The value of the characteristic exponent f_G is higher than the corresponding value of 1.18 ± 0.15 determined from the percolation model for shear modulus [6].

4. Conclusions

Summarising, the model proposed on the basis of the percolation theory enables to describe and explain the complex shear wave velocity dependence on porosity of porous ZnO using only 3-model parameters: shear wave velocity of the solid material, percolation threshold, and power law characteristic exponent for shear modulus. However, further testing of the model is required to prove its validity for other porous ceramics and solids. It is expected that all model parameters will vary due to the preparation method, the powder size, powder shape, and the investigated porosity range influence.

It is evident that the shear wave velocity dependence is more complex function of porosity in comparison with simple power law scaling of Young's and shear modulus with porosity. Moreover it indicates that not all properties of the porous material ought to fulfil the simple scaling power law dependence of the investigated mechanical property on porosity.

It must be further noted, that one can start with the shear wave velocity model in the scaling form identical with (2), but after putting this formula into the (1) and accepting (3) the shear modulus dependence on porosity ought to become more complex. However, this is in contradiction with the generally accepted theoretical results and also with the observed experimental results.

Therefore, the scientists are encouraged to test the validity of proposed model using their experimental data.

Appendix

There is a lot of data across literature about mechanical properties of porous solids, but only small part of them deal with shear wave velocities and most of them did not fulfil the basic requirements for reasonable fitting to any model, such as: Almost the same technology of the preparation, as wide as possible investigated porosity range, and data also in the vicinity of the percolation threshold. To be honest most of the data in the literature are from narrow porosity range where simple linear regression is more precise as any model.

For this reason we have used only the data for porous ZnO as they were the only one we were able to find in the literature and which approximately fulfilled requirements for reasonable fitting. Therefore we write this paper, to encourage

other scientists to create proper data set and to test the validity of the proposed model.

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