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Research Article

A Model Matching STR Controller for High Performance Aircraft

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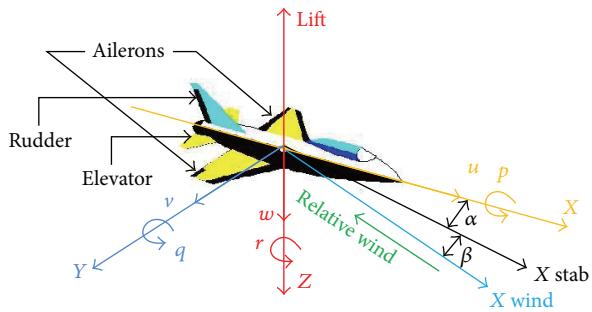
This paper presents a development, as well as an investigation of a Model Matching Controller (MMC) design based on the Self-Tuning Regulator (STR) framework for high performance aircraft with direct application to an F-16 aircraft flight control system. In combination with the Recursive Least Squares (RLS) identification, the MMC is developed and investigated for effectiveness on a detailed model of the aircraft. The popular robust Quantitative Feedback Theory (QFT) controller is also outlined and used to represent a baseline controller, for performance comparison during four simulated test flight maneuvers. In each of the four maneuvers, the proposed MMC provided consistently stable and satisfactory performance, including the challenging pull-up and pushover maneuvers. The baseline stationary controller has been found to become unstable in two of the four maneuvers tested. It also performs satisfactorily-to-arguably poorly in the remaining two as compared to the MMC. Simulation results presented in this investigation support a clear argument that the proposed MMC provides superior performance in the realm of automatic flight control.

1. Introduction

Challenges in automatic flight control are predominant over those in many systems due to the uncertainties that are involved in the aircraft itself, as well as its surroundings [1–5]. Nonlinearities are found in the dynamics of the plane and the actuators that control it. In addition, atmospheric conditions can always be given credit to the uncertainties in flight control. An aircraft's velocity, altitude, and orientation are all factors that decide how the plane will perform. Differences in these factors along with varying atmospheric conditions throughout the flight envelope can result in a less than optimum, or even unstable system. For the purpose of stability and control, the ability to cope with these different conditions cannot be compromised.

Current methods of flight control include dynamic inversion, gain scheduling, and QFT, among others [6–12]. These are stationary controllers, in which they incorporate a design that does not adapt to the many changes that an aircraft can encounter. Beyond the design phase, their behavior is fixed. For these reasons, the focus of most designs is robustness. This can prove successful, but maneuverability of the aircraft is usually sacrificed to some extent. The flight envelope may even be bounded by the restrictions of the controller itself. Additionally, the aircraft and its surroundings are modeled only in the design phase. The drawback to a design whose primary focus is robustness is that a more desirable response possibly could have been achieved for many situations. The benefit is a wider range of conditions by which the system can be controlled successfully. However, in the case of a high performance aircraft, both optimum maneuverability and stability are critical in most situations. Many studies have made an attempt to implement a linear controller with robustness that is sufficient enough to stabilize a flight control system [7, 8].

Nonlinear dynamic inversion is a commonly used method of controller design in the application of flight control and is quite different from QFT [8]. The basis of nonlinear dynamic inversion is to use feedback to linearize the system being controlled and to provide a desired response.



u, v, and w: velocity components along X, Y, and Z bodies
P, q , and r: roll, pitch, and rate along X-, Y-, Z-axes
α: angle of attack
β: sideslip angle

Figure 1: Aircraft three-dimensional representation with axes of control.

The method made several advancements, but its nature of linearizing the process meant that it was faced with the challenge of model uncertainties, which can prevent exact linearization. For this reason, dynamic inversion appears in many hybrid controllers that utilize some other types of control. These generally consist of Quantitative Feedback Theory, gain scheduling, and μ-synthesis.

Adaptive control methods, including Gain Scheduling, Self Tuning Regulators (STR), Model Reference Adaptive Controllers (MRAC) and Intelligent Adaptive Controllers have been recently proposed [1–3]. The complexity of the resulting algorithms and hence their control computational requirements have presented a limiting factor for their implementation. Because of the low computational requirements found in stationary designs, an increase in robustness has been their primary focus. Gain scheduling has also been implemented in hybrid designs [7]. This form of control has been a popular method and can be found in many finished designs in flight control today. While mildly related to adaptive methods, gain scheduling is another method that utilizes linear models in the design phase and is in many ways fixed. It does not directly face the nonlinearities that an aircraft has in its dynamics and encounters in its environment. Robustness is sometimes obtained by including high loop gains, which can become a problem in the presence of unmodeled dynamics, sensor noise, and unpredictable changes to the system or its environment. The use of an adaptive controller can be a significant improvement to automatic flight control. While preserving the robustness of other control methods, stability of the aircraft can be maintained while giving a faster and more accurate response by using a control scheme that features adaptability. Quantitative Feedback Theory has been proposed as a means of robust controller design. In addition to this, hybrid controllers have been discussed that include dynamic inversion, a nonlinear control method [6, 8]. QFT involves the selection of a feedback compensator and prefilter to ensure a response within acceptable limits. This is a stationary design whose structure inhibits the control from any adaptability once the design phase is complete. When applied to a nonlinear system, numerous models are formed to aid in the design process. These models are linear and time invariant in nature and do not fully represent the plant nonlinearities. For this reason, a response can only be predicted to the extent by which the designer tests the control and by the number of linear models that are used. QFT was originally devised for applications involving linear, time-invariant systems. However, efforts have been made to demonstrate the usefulness of this method of control for other systems that are nonlinear and time varying [6]. The objective of the controller design is to select two transfer functions, one to process the command and another to perform the closed loop performance requirements. The goal of both transfer functions is to change the elevator deflection angle such that the process output will track the desired angle of attack command. During the design, the plant is represented by a series of linear models. The linear models are used to determine the parameters for the two transfer functions. This is where a disadvantage of QFT control can be found for a flight control application. Just as in many other control methods, linearized models are used in the design phase, not in real time when the aircraft is in motion. In regards to an aircraft flight control system, the nonlinear parameters will vary throughout the flight envelope. It is conceivable that this system could be regarded as having an infinite number of parameter variations, thus creating many questions about the design of a controller of this type.

A real-time Model Matching Control (MMC) algorithm is proposed in this paper for implementation within the Self-Tuning Regulator (STR) structure of adaptive control framework. The proposed controller will be investigated on an F-16 aircraft simulated by a detailed dynamic model that has been developed and will be outlined in the paper. The popular QFT controller will be used as a baseline controller for the purpose of comparison and assessment of effectiveness of the proposed MMC.

2. Development of the Aircraft Model

A detailed model of the F-16 aircraft is given in the appendix. The aircraft model is based on the aircraft representation in Figure 1 and consists of a number of subsystems, each of which represent a significant part of the overall plant and its environment [1, 2]. First, the vehicle or aircraft itself has a manner in which it moves throughout the medium irrespective of any influential external action. These behaviors are defined by the flight dynamics. The flight dynamics have two components. They are governed by Newton's law, which identifies the translational dynamics experienced by the aircraft, and Euler's law, which relates the attitude dynamics or orientation of the aircraft.

There also exists a summation of forces that join with the equations of motion to deliver the resultant set of parameters that are meaningful to an analysis, such as speed, altitude, position, and orientation. All objects inherent to the model system other than the equations of motion can be linked to and have some relation to the forces. The primary components of these are propulsion, gravitation and aerodynamics.

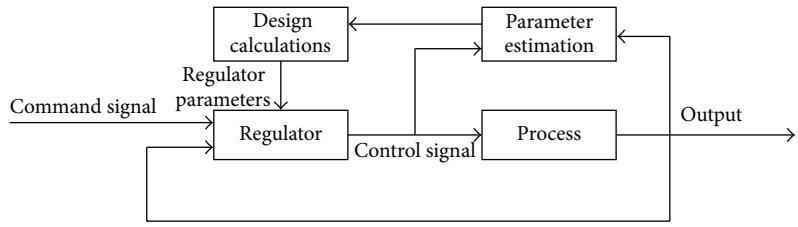


Figure 2: Self-Tuning Regulator framework.

The following nomenclature applies to the aircraft model for the states, variables, and parameters:

a_n = normal acceleration, gravity units
$\bar{c}$ = wing mean aerodynamic chord, m
b = wing span, m
C_{xt} = total aerodynamic x-axis force coefficient
ρ = air density, kg/m^3
C_{zt} = total aerodynamic z-axis force coefficient
C_{mt} = total aerodynamic pitching moment coefficient
F_X = aerodynamic force in x-direction, N
F_Z = aerodynamic force in z-direction, N
g = acceleration due to gravity, m/s^2
h = altitude, m
I_Y = moment of inertia about y-axis, kg·m^2
m = mass, kg
M = mach number
M_Y = aerodynamic pitching moment
q = pitch rate, rad/s
$\bar{q}$ = dynamic pressure, N/m^2
r = reference command, rad
S = wing area, m^2
T = thrust, N
u = velocity x-component, m/s
V = velocity magnitude, m/s
w = velocity z-component, m/s
x_{cg} = center of gravity location, fraction of $\bar{c}$
α = angle of attack, rad/s
δ_h = elevator deflection angle, rad
δ_{lef} = leading edge flap deflection angle, rad
η = elevator effectiveness factor
θ = pitch att angle, rad.

In this study, the angle of attack (AOA) of the aircraft will be controlled by manipulating the elevator deflection angle in the investigated flight maneuvers.

For that purpose, the following reduced state space model that is relevant to those specific dynamics of the longitudinal aircraft model is developed from the overall detailed model of the appendix:

$$\begin{aligned}
\dot{u} &= -qw - g_c \sin(\theta) + \frac{\bar{q}\, SC_{xt} + T}{m}, \\
\dot{w} &= qu + g_c \cos(\theta) + \frac{\bar{q}\, SC_{zt}}{m}, \\
\dot{q} &= \frac{\bar{q}\, S\bar{c}C_{mt}}{I_Y}, \\
\dot{\theta} &= q, \\
\dot{h} &= u \sin(\theta) - w \cos(\theta), \\
\dot{\alpha} &= q + \frac{mg \cos(\theta) + \bar{q}\, SC_{zt}}{mu}.
\end{aligned} \tag{1}$$

3. Development of the STR MMC

A real-time Model Matching Control algorithm is proposed in this paper for implementation within the Self-Tuning Regulator (STR) structure of adaptive control framework. The control structure is shown in Figure 2, where an estimation of the plant parameters is used to update the controller parameters online.

The system has two loops, the first is a unity feedback loop, and the second loop includes a parameter estimation that is provided to the control law design algorithm. For system identification, the following nth order process is used:

$$\frac{y(k)}{u(k)} = \frac{b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n}}, \tag{2}$$

which is executed online with the difference equation:

$$\begin{aligned}
y(k) = {} & -a_1 y(k-1) - a_2 y(k-2) - \cdots - a_n y(k-n) \\
& + b_1 u(k-1) + b_2 u(k-2) + \cdots + b_n u(k-n).
\end{aligned} \tag{3}$$

The model parameters a's and b's are included in a parameter vector Θ of order $(2n * 1)$, and the following Recursive Least

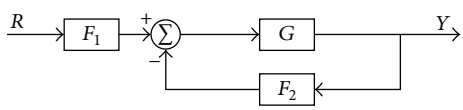


Figure 3: The Model Matching Controller structure.

Square (RLS) identification algorithm is used to identify the aircraft dynamics [13]:

$$\Theta(t) = \Theta(t-1) + K(t)\left[\phi(t) - x^t(t)\,\Theta(t-1)\right],$$
$$K(t) = \frac{P(t-1)\;x(t)}{\gamma + x^t(t)\;P(t-1)\;x(t)}, \tag{4}$$
$$P(t) = \frac{\left[I - K(t)\;x^t(t)\right]P(t-1)}{\gamma}.$$

A forgetting factor γ has been included in the algorithm for improved identification, which allows the estimation process to credit more recently obtained data.

3.1. The MMC Real-Time Algorithm. The proposed MMC structure is shown in Figure 3, where F_2 is the main feedback control transfer function and F_1 is updated online to satisfy the model matching condition. Consider G and F_2 as follows:

$$G(z) = \frac{B(z)}{A(z)},$$
$$F_2(z) = \frac{\beta(z)}{\alpha(z)}. \tag{5}$$

Plant G is of order n, and controller F_2 is of order $(n-1)$:

$$G(z) = \frac{B(z)}{A(z)} = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n},$$
$$F_2(z) = \frac{\beta(z)}{\alpha(z)} = \frac{\beta_0 z^{n-1} + \beta_1 z^{n-2} + \cdots + \beta_{n-2} z + \beta_{n-1}}{\alpha_0 z^{n-1} + \alpha_1 z^{n-2} + \cdots + \alpha_{n-2} z + \alpha_{n-1}}. \tag{6}$$

F_1 is chosen such that the overall response will match a specified design model:

$$\frac{Y}{R} = F_1(z)\cdot\frac{\alpha(z)\,B(z)}{\alpha(z)\,A(z) + \beta(z)\,B(z)} = \frac{B_m(z)}{A_m(z)}, \tag{7}$$

where the desirable model is given by

$$\frac{Y}{R} = \frac{B_m(z)}{A_m(z)}. \tag{8}$$

The closed form Diophantine equation solution (DES) [14] is then implemented online with a choice of the closed loop characteristic polynomial as

$$D(z) = \alpha(z)\,A(z) + \beta(z)\,B(z),$$
$$D(z) = H(z)\,F(z) = H_1(z)\,B(z)\,F(z), \tag{9}$$

and with the designer specifying H_1 and F, and B is the process nominator polynomial as determined by the identification algorithm. H is of order n, and F is of order $n-1$. If B is of order m, then the H_1 polynomial is of order $n-m$. If B is a stable polynomial, m can equal n and H_1 must be equal to 1, and $\alpha(z)$ and $\beta(z)$ are given by the DES as

$$M = E^{-1}D, \tag{10}$$

where M is the vector of the α and β polynomials. F_1 transfer is then given by

$$F_1(z) = \frac{H_1(z)\,F(z)}{\alpha(z)}\cdot\frac{B_m(z)}{A_m(z)}, \tag{11}$$

and the design requirement is satisfied as

$$\frac{Y}{R} = \frac{H_1(z)\,F(z)}{\alpha(z)}\cdot\frac{B_m(z)}{A_m(z)}\cdot\frac{\alpha(z)\,B(z)}{H_1(z)\,B(z)\,F(z)} = \frac{B_m(z)}{A_m(z)}. \tag{12}$$

The controller transfer function and its execution difference equation are given by:

$$u = -\frac{\beta(z)}{\alpha(z)}y + R\cdot\frac{H_1(z)\,F(z)}{\alpha(z)}\cdot\frac{B_m(z)}{A_m(z)},$$
$$\alpha A_m u = H_1 F B_m R - \beta A_m Y. \tag{13}$$

If the identification algorithm happens to yield an unstable pole within the process polynomial B, a modified controller algorithm is used, starting with the expression for the Diophantine equation:

$$D(z) = \alpha(z)\,A(z) + \beta(z)\,B(z). \tag{14}$$

In the case of the unstable pole, we state that:

$$B(z) = B^-(z)\,B^+(z),$$
$$B_m(z) = B^-(z)\,B_1(z), \tag{15}$$

where B^+ is stable and B_1 is the only polynomial specified in the design stage

$$D(z) = H(z)\,F(z) = H_1(z)\,B^+(z)\,F(z). \tag{16}$$

The Diophantine equation is solved for $\alpha(z)$ and $\beta(z)$ with the new definition for $D(z)$, and the front-end transfer function is stated in (17) to satisfy the design requirement as in (18)

$$F_1(z) = \frac{H_1(z)\,F(z)}{\alpha(z)}\cdot\frac{B_1(z)}{A_m(z)}, \tag{17}$$

$$\frac{Y}{R} = \frac{H_1(z)\,F(z)}{\alpha(z)}\cdot\frac{B_1(z)}{A_m(z)}\cdot\frac{\alpha(z)\,B^-(z)\,B^+(z)}{H_1(z)\,B^+(z)\,F(z)}$$
$$= \frac{B_1(z)\,B^-(z)}{A_m(z)} = \frac{B_m(z)}{A_m(z)}. \tag{18}$$

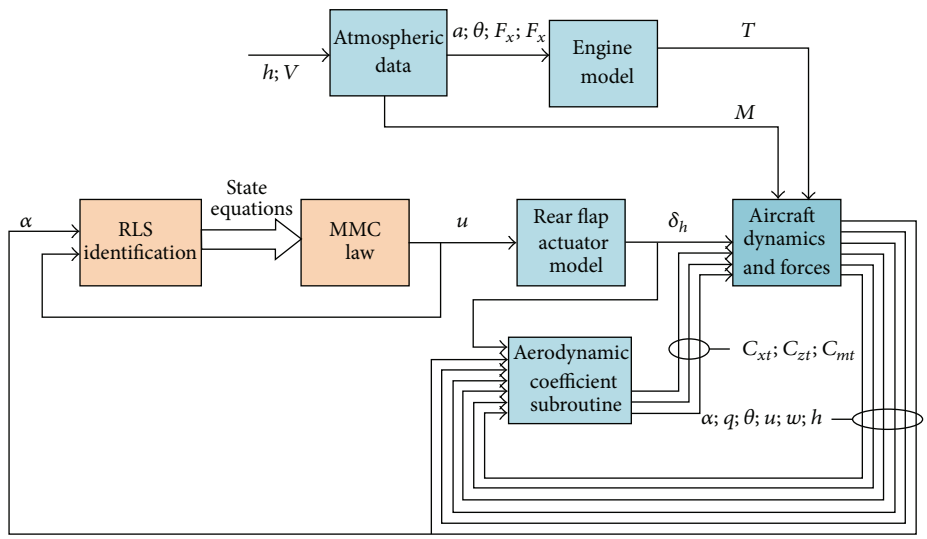


Figure 4: Complete aircraft system simulation layout.

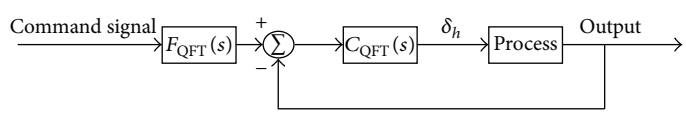


Figure 5: QFT controller configuration.

And the corresponding controller equations are then given by

$$u = -\frac{\beta(z)}{\alpha(z)} y + R \cdot \frac{H_1(z) F(z)}{\alpha(z)} \cdot \frac{B_1(z)}{A_m(z)},$$
$$\alpha A_m u = H_1 F B_1 R - \beta A_m Y. \tag{19}$$

The online model matching control algorithm incorporates both cases by checking the poles of $B(z)$ during each iteration and using the appropriate control method in each case.

4. Controller Assessment and Simulation Investigation Results

The investigation simulation system layout, including detailed representation of the aircraft dynamics is shown in Figure 4. The aircraft, earth atmosphere, and RLS data are as follows:

$$m = 9295, \quad I_Y = 75674, \quad S = 27.87,$$
$$\bar{c} = 3.45, \quad x_{cg} = 0.35, \quad g_c = 9.81, \quad R = 287$$
$$n = 2, \quad \gamma = 0.95, \quad f = 10000, \quad t_f = 0.02. \tag{20}$$

The aircraft is considered in a steady flight path prior to a command step in angle of attack. Initial conditions for each simulation are

$$u = 163, \quad w = 6.0, \quad q = 0.17,$$
$$\theta = 2.02, \quad \alpha = 4.0, \quad h = 9144. \tag{21}$$

The altitude and velocities were chosen based on other studies [13]. The orientation values were found by trial and error to provide a short initialization period for simulation purposes. These values exist prior to a 2-second initialization that stabilizes the aircraft to a +5 degree angle of attack from initial conditions. During the 1st second, the model for the elevator actuator is bypassed to allow the plane to move unrestricted by physical limitations. This is merely for the purpose of establishing a true set of states for the system and is necessary due to the simulation setting. In the next second, the real dynamics of the actuator are included, and the aircraft is steadied. Following this is time zero, or the beginning of the relevant simulation.

For comparison, a QFT controller was designed on the basis of a given F-16 aircraft model [6, 8]. The objective of the controller design is to select the two transfer functions in the control configuration shown in Figure 5. The first of these, $F_{QFT}(s)$, is an input transducer. The second, $C_{QFT}(s)$, is a front-end transfer function to the process whose output is the elevator deflection angle, δ_h, in the case of longitudinal flight control. The goal of both transfer functions is to change

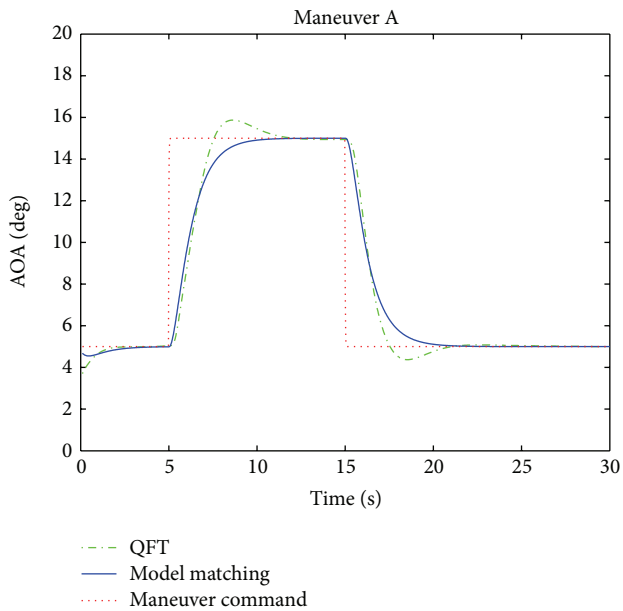


Figure 6: AOA response: maneuver A.

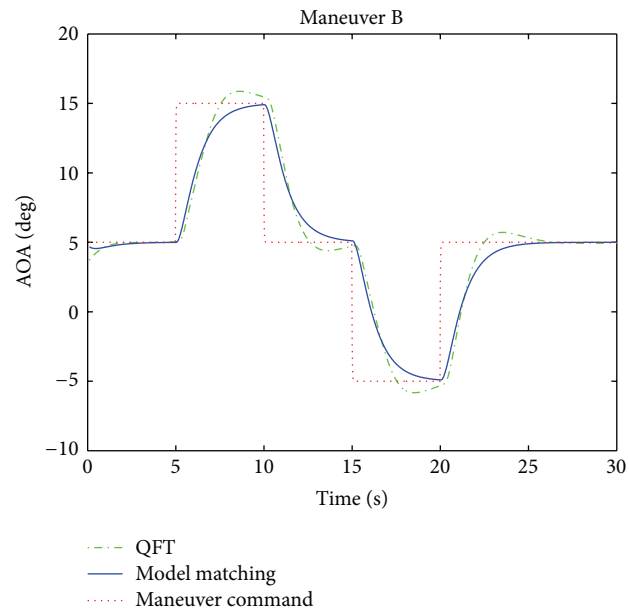


Figure 7: AOA Response: Maneuver B.

the elevator deflection angle such that the process output will track the command signal, the desired angle of attack. As shown in the figure, unity feedback is employed.

During the design, the plant is represented by a series of linear models. The linear models are used to determine the parameters for $F_{\mathrm{QFT}}(s)$ and $C_{\mathrm{QFT}}(s)$, and the following controller transfer functions have been obtained:

$$C_{\mathrm{QFT}}(s) = \frac{-48,800\left(s^2 + 1.2(0.38)s + 0.38^2\right)(s+2.1)}{s(s+0.05)\left(s^2 + 1.4(20)s + 20^2\right)},$$

$$F_{\mathrm{QFT}}(s) = \frac{(1.15)^2}{s^2 + 1.4(1.15)s + 1.15^2}. \tag{22}$$

The model matching control design was based on the following reference model and polynomials:

$$\frac{B_m(z)}{A_m(z)} = \frac{0.051z - 0.05}{z^2 - 1.93z + 0.931},$$
$$H_1(z) = z + 0.05, \tag{23}$$
$$F(z) = z.$$

The performance of the proposed MMC in comparison with the robust QFT design has been investigated under *four aircraft maneuvers* that have been synthesized to demonstrate the relative effectiveness of both techniques.

In the first *maneuver A*, the command reference is a +10 degree angle of attack step increase followed by a return to steady flight. This movement is consistent with a positive pitch angle and increase in altitude. And in *maneuver B*, the command reference involves a number of changes. First, a +10 degree step up in angle of attack is set, followed by a −10 degree return to level flight, then another −10 degree reduction to pitch the plane downward and followed by +10 degrees to level off. The *MMC* shows an outstanding response in maneuver A as shown in Figure 6. The response is just as fast as the baseline controller yet has no overshoot and promptly converges with the step command. Similar activity is found in the return from the step. Simulation of maneuver B shows the same features as shown in Figure 7, although there are periods where a more abrupt response by the baseline controller is noted. These are only found in a negative AOA command and are considered minor differences.

The third *maneuver C* investigated is the opposite of A. It is a −10 degree pushover followed by a command to level off and come out of a dive. And the fourth *maneuver D* is similar to C in its intent. It differs in that the initial pushover is 15 degrees, which is more aggressive, but held 5 seconds rather than 10. In these two maneuvers C and D, which denote instability using the QFT controller, are shown in Figures 8 and 9, and in both cases, the MMC was shown to recognize a difference between the response of the system at extreme pitch angles and that of the same system at level flight. The model matching adaptive controller performed well in every simulation and features a more consistent result in both a pull-up and pushover situations. In all simulations, it is found that the baseline controller overshoots the set point most of the time. This is not always a negative trait, but maneuvers C and D clearly demonstrate the manner in which this controller behaves differently when the aircraft is in a unique situation or orientation. The overshoot develops into instability in these cases. In contrast to the MMC response in every situation, the overshoot is unnecessary as the model determined online by the identification algorithm manipulates the control variable in a way that provides for a response that is both fast and dampened enough to eliminate overshoot.

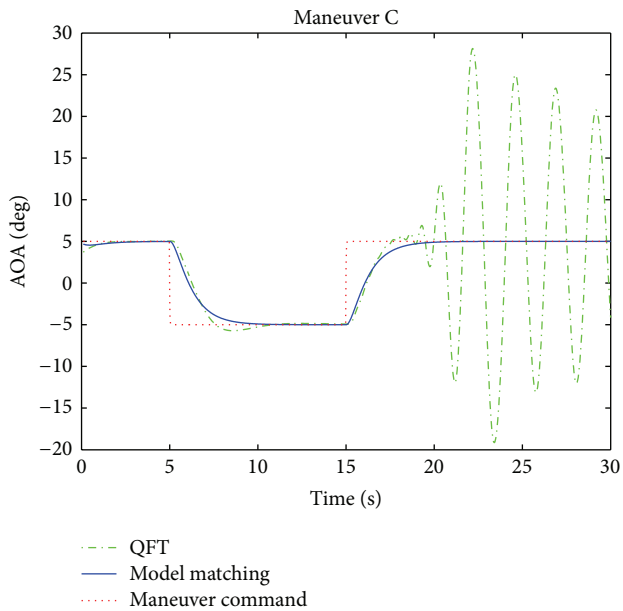


Figure 8: AOA response: maneuver C.

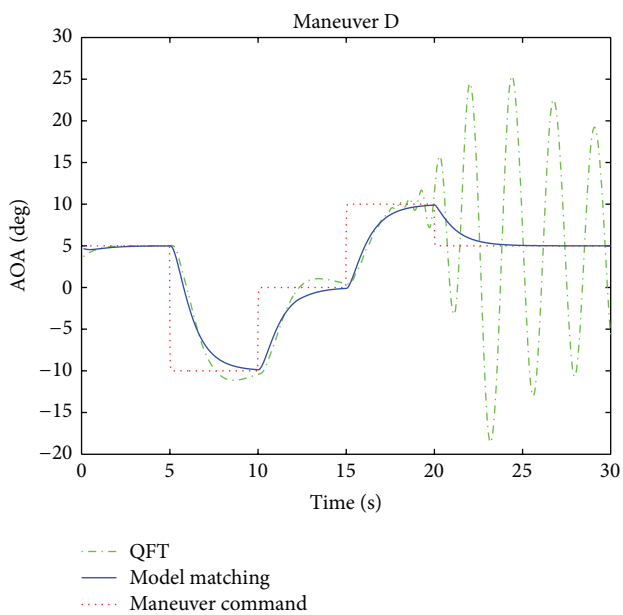


Figure 9: AOA response: maneuver D.

5. Conclusions

A development as well as investigation of a Model Matching Controller for a high performance (F-16) aircraft based on the STR framework has been presented. The proposed controller includes Recursive Least Squares identification algorithm and an MMC technique, both to be implemented in real time to produce adaptive systems that will track a reference signal optimally as the model undergoes various thirty second flight simulations. The MMC has been investigated and compared to a widely accepted and popular robust controller design, the QFT controller. This design was referred to as the baseline controller to assess the performance of the proposed controller. Simulations were performed for both controllers during four separate flight maneuvers. The baseline controller was found to be unstable in two of those maneuver demonstrations. Investigation results have shown that the proposed MMC can be a significant improvement to automatic flight control. While preserving the robustness of other control methods, stability of the aircraft can be maintained, giving a faster and more accurate response by using a control scheme that features adaptability. The ability to track a reference command well and remain stable throughout the flight envelope is necessary in design of a highly maneuverable aircraft. Challenges in automatic flight control are predominant over those in many systems due to the uncertainties that are involved in the aircraft itself, as well as its surroundings. Nonlinearities are found in the dynamics of the plane and the behavior of the actuators that control it. In addition, atmospheric conditions can always be given credit to the uncertainties in flight control. An aircraft's velocity, altitude, and orientation are all factors that decide how the plane will perform. Differences in these factors along with varying atmospheric conditions throughout the flight envelope can result in a less than optimum, or even unstable system. For the purpose of stability and control, the ability to cope with these different conditions cannot be compromised, and the proposed MMC has been shown to accomplish that.

Appendix

A. Six Degree of Freedom (DOF) F-16 Fighter Aircraft Dynamic Model

In developing a rigid body six DOF model of the F-16 aircraft, the aircraft flight mechanics can be aggregated from the object body (fuselage, wing, and stabilizers), engine, aerodynamics, gravity, atmosphere, and winds [1, 2, 13]. The dynamics of the aircraft are due to the force in the translational direction, which in effect creates the velocity. The velocity vector can be divided into three components based on three axes. Each has the effect of the moments on the other two axes, a gravitational effect developed based on the kinematic components and the force in each direction. Thus, the velocity vector components will constitute the three states of the dynamics. The kinematical equation shows the Euler angles of the motion. They are the effect of the moments and the attitude of the aircraft. In the x-axis, the angle is related to the moment "P," and the other two angles are due to the moments "Q" and "R." The z-axis component of the angle will have the effect of "Q" and "R" along with the angles in the x and y directions. The principles of flight dynamics involve the study of vehicle motions through air or space, where the aircraft experiences six degrees of freedom. These include three translational degrees which describe the movement of the center of mass, or the trajectory, and three attitude degrees that depict the orientation of the vehicle [2, 13].

A.1. Translational Dynamics.

$$\begin{aligned}\dot{u} &= rv - qw - g_c \sin(\theta) + \frac{\bar{q}\, SC_{xt}}{m} + \frac{T}{m},\\ \dot{v} &= pw - ru + g_c \cos(\theta)\sin(\varphi) + \frac{\bar{q}SC_{yt}}{m},\\ \dot{w} &= qu - pv + g_c \cos(\theta)\cos(\varphi) + \frac{\bar{q}SC_{zt}}{m}.\end{aligned} \tag{A.1}$$

A.2. Attitude Dynamics.

$$\begin{aligned}\dot{p} &= \frac{(I_Y - I_Z)\, qr}{I_X} + \frac{I_{XZ}(\dot{r} + pq)}{I_X} + \frac{\bar{q}\, SbC_{lt}}{I_X},\\ \dot{q} &= \frac{(I_Z - I_X)\, pr}{I_Y} + \frac{I_{XZ}\left(r^2 - p^2\right)}{I_Y} + \frac{\bar{q}\, S\bar{c}C_{mt}}{I_Y} - \frac{l_E r}{I_Y},\\ \dot{r} &= \frac{(I_X - I_Y)\, pq}{I_Z} + \frac{I_{XZ}(\dot{p} - rq)}{I_Z} + \frac{\bar{q}\, SbC_{nt}}{I_Z} + \frac{l_E q}{I_Z}.\end{aligned} \tag{A.2}$$

A.3. Kinematic Equations.

$$\begin{aligned}\dot{\varphi} &= p + q\sin(\varphi)\tan(\theta) + r\cos(\varphi)\tan(\theta),\\ \dot{\theta} &= q\cos(\varphi) - r\sin(\varphi),\\ \dot{\psi} &= q\frac{\sin(\varphi)}{\cos(\theta)} + r\frac{\cos(\varphi)}{\cos(\theta)}.\end{aligned} \tag{A.3}$$

A.4. Navigation Equations.

$$\begin{aligned}\dot{p}_n &= u\cos(\psi)\cos(\theta)\\ &\quad + v\left(\cos(\psi)\sin(\theta)\sin(\varphi) - \sin(\psi)\cos(\varphi)\right)\\ &\quad + w\left(\cos(\psi)\sin(\theta)\cos(\varphi) + \sin(\psi)\sin(\varphi)\right),\\ \dot{p}_e &= u\sin(\psi)\cos(\theta)\\ &\quad + v\left(\sin(\psi)\sin(\theta)\sin(\varphi) + \cos(\psi)\cos(\varphi)\right)\\ &\quad + w\left(\sin(\psi)\sin(\theta)\cos(\varphi) - \cos(\psi)\sin(\varphi)\right),\\ \dot{h} &= u\sin(\theta) - v\cos(\theta)\sin(\varphi) - w\cos(\theta)\sin(\varphi).\end{aligned} \tag{A.4}$$

The 6 degree of freedom aircraft dynamic model and navigation dynamics are given by the four sets of equations (A.1)–(A.4). In most cases, these 12 equations, which have very close relation to spacecraft and missile dynamics, joined by a robust depiction of geophysical and aerodynamic data, as well as thrust and control deflector subsystems, complete a thorough and accurate simulation in flight control.

References

[1] S. Kamalasadan and A. A. Ghandakly, "Multiple fuzzy reference model adaptive controller design for pitch-rate tracking," *IEEE Transactions on Instrumentation and Measurement*, vol. 56, no. 5, pp. 1797–1808, 2007.

[2] S. Kamalasadan and A. A. Ghandakly, "A neural network parallel adaptive controller for fighter aircraft pitch-rate tracking," *IEEE Transactions on Instrumentation and Measurement*, vol. 60, no. 1, pp. 258–267, 2011.

[3] S. E. Talole, A. Ghosh, and S. B. Phadke, "Proportional navigation guidance using predictive and time delay control," *Control Engineering Practice*, vol. 14, no. 12, pp. 1445–1453, 2006.

[4] M. Pachter, P. R. Chandler, and L. Smith, "Maneuvering flight control," *Journal of Guidance, Control, and Dynamics*, vol. 21, no. 3, pp. 368–374, 1998.

[5] R. Bhattacharya, G. J. Balas, M. A. Kaya, and A. Packard, "Nonlinear receding horizon control of an F-16 aircraft," *Journal of Guidance, Control, and Dynamics*, vol. 25, no. 5, pp. 924–931, 2002.

[6] T. Wagner and J. Valasek, "Digital autoland control laws using quantitative feedback theory and direct digital design," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 5, pp. 1399–1413, 2007.

[7] I. Fialho, G. J. Balas, A. K. Packard, J. Renfrow, and C. Mullaney, "Gain-scheduled lateral control of the F-14 aircraft during powered approach landing," *Journal of Guidance, Control, and Dynamics*, vol. 23, no. 3, pp. 450–458, 2000.

[8] S. A. Snell and P. W. Stout, "Robust longitudinal control design using dynamic inversion and quantitative feedback theory," *Journal of Guidance, Control, and Dynamics*, vol. 20, no. 5, pp. 933–940, 1997.

[9] C.-H. Lee, M.-G. Seo, M.-J. Tahk, J.-I. Lee, and B.-E. Jun, "Missile acceleration controller design using pi and time-delay adaptive feedback linearization methodology," *Proceedings of the Institution of Mechanical Engineers G*, vol. 226, no. 8, pp. 882–897, 2012.

[10] C. H. Lee, T. H. Kim, and M. J. Tahk, "Missile autopilot design for agile turn using time delay control with nonlinear observer," *International Journal of Aeronautical and Space Science and Technology*, vol. 12, no. 3, pp. 266–273, 2011.

[11] L. Bruyere, A. Tsourdos, and B. A. White, "Robust augmented lateral acceleration flight control design for a quasi-linear parameter-varying missile," *Proceedings of the Institution of Mechanical Engineers G*, vol. 219, no. 2, pp. 171–181, 2005.

[12] Q. Wang and R. F. Stengel, "Robust nonlinear control of a hypersonic aircraft," *Journal of Guidance, Control, and Dynamics*, vol. 23, no. 4, pp. 577–585, 2000.

[13] W. Gonsalves, *A multi-input multi-output self tuning regulator for nonlinear high performance aircraft control [M.S. thesis]*, ECE Department, California State University, Chico, Calif, USA, 2010.

[14] B. C. Kuo, *Automatic Control Systems*, Prentice Hall, 7th edition, 1995.