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Research Article

Traveling Wave Solutions of Some Coupled Nonlinear Evolution Equations

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The modified simple equation (MSE) method is executed to find the traveling wave solutions for the coupled Konno-Oono equations and the variant Boussinesq equations. The efficiency of this method for finding exact solutions and traveling wave solutions has been demonstrated. It has been shown that the proposed method is direct, effective, and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics. Moreover, this procedure reduces the large volume of calculations.

1. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs has a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity, and so forth. Therefore, investigation, traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the modified simple equation method [1–4], the extended tanh method [5, 6], the Exp-function method [7–11], the Adomian decomposition method [12], the F-expansion method [13], the auxiliary equation method [14], the Jacobi elliptic function method [15], modified Exp-function method [16], the (G'/G)-expansion method [17–26], Weierstrass elliptic function method [27], the homotopy perturbation method [28–30], the homogeneous balance method [31, 32], the Hirota's bilinear transformation method [33, 34], the tanh-function method [35, 36] and so on.

The objective of this paper is to apply the MSE method to construct the exact and traveling wave solutions for nonlinear evolution equations in mathematical physics via coupled Konno-Oono equations and variant Boussinesq equations.

The paper is prepared as follows. In Section 2, the MSE method is discussed. In Section 3, we apply this method to the nonlinear evolution equations pointed out above, in Section 4, physical explanations, and in Section 5 conclusions are given.

2. The MSE Method

In this section, we describe the MSE method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x and t, is given by

$$\mathscr{R}\left(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \ldots\right) = 0, \tag{1}$$

where $u(\xi) = u(x,t)$ is an unknown function, $\mathscr{R}$ is a polynomial of $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [1–4].

Step 1. Combining the independent variables x and t into one variable $\xi = x \pm \omega t$, we suppose that

$$u(\xi) = u(x,t), \quad \xi = x \pm \omega t. \tag{2}$$

The traveling wave transformation equation (2) permits us to reduce (1) to the following ODE:

$$\mathscr{R}\left(u, u', u'', \ldots\right) = 0, \tag{3}$$

where $\mathscr{R}$ is a polynomial in $u(\xi)$ and its derivatives, while $u'(\xi) = du/d\xi$, $u''(\xi) = d^2u/d\xi^2$, and so on.

Step 2. We suppose that (3) has the formal solution

$$u(\xi) = C_0 + \sum_{k=1}^{n} C_k \left(\frac{\phi'(\xi)}{\phi(\xi)}\right)^k, \tag{4}$$

where C_k are constants to be determined, such that $C_n \neq 0$, and $\phi(\xi)$ is an unknown function to be determined later.

Step 3. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in (1) or (3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = n$ which gives rise to the degree of other expression as follows:

$$\begin{gathered} D\left(\frac{d^q u}{d\xi^q}\right) = n + q, \\ D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n+q). \end{gathered} \tag{5}$$

Therefore, we can find the value of n in (4), using (5).

Step 4. We substitute (4) into (3), and then we account the function $\phi(\xi)$. As a result of this substitution, we get a polynomial of $(\phi'(\xi)/\phi(\xi))$ and its derivatives. In this polynomial, we equate the coefficients of same power of $\phi^{-i}(\xi)$ to zero, where $i \geq 0$. This procedure yields a system of equations which can be solved to find α_k, $\phi(\xi)$ and $\phi'(\xi)$. Then the substitution of the values of α_k, $\phi(\xi)$ and $\phi'(\xi)$ into (4) completes the determination of exact solutions of (1).

3. Applications

3.1. The New Coupled Konno-Oono Equations. Now we will bring to bear the MSE method to find exact solutions, and then the solitary wave solutions of coupled Konno-Oono equations in the form [37],

$$u_{xt} - 2uv = 0, \qquad v_t + 2uu_x = 0. \tag{6}$$

Now let us suppose that the traveling wave transformation equation be

$$u(\xi) = u(x,t), \qquad v(\xi) = v(x,t), \qquad \xi = x - \omega t. \tag{7}$$

Equation (7) reduces (6) into the following ODEs:

$$-\omega u'' - 2uv = 0, \tag{8}$$

$$-\omega v' + 2uu' = 0. \tag{9}$$

By integrating (9) with respect to ξ, we obtain

$$v = \frac{1}{\omega}\left(u^2 + d\right), \tag{10}$$

where d is a constant of integration.

Substituting (10) into (8), we get

$$\omega^2 u'' + 2ud + 2u^3 = 0. \tag{11}$$

Balancing the highest order derivative u'' and nonlinear term u^3 from (11), we obtain $3n = n + 2$, which gives $n = 1$.

Now for $n = 1$, using (4) we can write

$$u(\xi) = C_0 + C_1\left(\frac{\phi'(\xi)}{\phi(\xi)}\right), \tag{12}$$

where C_0 and C_1 are constants to be determined such that $C_1 \neq 0$, while $\phi(\xi)$ is an unknown function to be determined. It is trouble free to find that

$$\begin{gathered} u' = C_1\left(\frac{\phi''}{\phi} - \left(\frac{\phi'}{\phi}\right)^2\right), \\ u'' = C_1\left(\frac{\phi'''}{\phi}\right) - 3C_1\left(\frac{\phi''\phi'}{\phi^2}\right) + 2C_1\left(\frac{\phi'}{\phi}\right)^3, \\ u^3 = C_1^3\left(\frac{\phi'}{\phi}\right)^3 + 3C_1^2C_0\left(\frac{\phi'}{\phi}\right)^2 + 3C_1C_0^2\left(\frac{\phi'}{\phi}\right) + C_0^3. \end{gathered} \tag{13}$$

Now substituting the values of u, u^3, u'' into (11) and then equating the coefficients of ϕ^0, ϕ^{-1}, ϕ^{-2}, ϕ^{-3} to zero, we, respectively, obtain

$$2C_0^3 + 2AC_0 = 0, \tag{14}$$

$$\omega^2 C_1\phi''' + 6C_0^2C_1\phi' + 2AC_1\phi' = 0, \tag{15}$$

$$-3\omega^2 C_1\phi''\phi' + 6C_0C_1^2\left(\phi'\right)^2 = 0, \tag{16}$$

$$2\omega^2 C_1\left(\phi'\right)^3 + 2C_1^3\left(\phi'\right)^3 = 0. \tag{17}$$

Solving (14), we get

$$C_0 = 0, \pm\sqrt{-d}. \tag{18}$$

Solving (17), we get

$$C_1 = \pm I\omega, \quad C_1 \neq 0, \quad \text{where } I^2 = -1. \tag{19}$$

Solving (15) and (16) we get,

$$\phi'(\xi) = MA\exp(LM\xi). \tag{20}$$

Integrating (20) with respect to ξ, we obtain

$$\phi(\xi) = \frac{1}{L}\left(LB - A\exp(LM\xi)\right), \tag{21}$$

where $L = (6C_0^2 + 2d)/\omega^2$, $M = \omega^2/2C_1C_0$, and A, B are constants of integration.

Substituting the values of ϕ and ϕ' into (12), we obtain the following exact solution:

$$u(x,t) = C_0 + C_1 \frac{LMA \exp(-LM(x-\omega t))}{LB - A\exp(-LM(x-\omega t))}. \quad (22)$$

Case 1. When $C_0 = 0$, (22) yields trivial solution. So this case is discarded.

Case 2. When $C_0 = \pm\sqrt{-d}$ and $C_1 = \pm I\omega$, substituting the values of C_0, C_1, L, M into (22), we obtain

$$\begin{aligned} u(x,t) = {} & \pm I\sqrt{d} \\ & \times \left(1 + \left(2A\cosh\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right) \right.\right. \\ & \left. - \sinh\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right)\right) \\ & \times \left((LB-A)\cosh\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right)\right. \\ & \left.\left. + (LB+A)\sinh\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right)\right)^{-1}\right). \end{aligned} \quad (23)$$

We can freely choose the constants A and B. Therefore, setting $A = LB$, (23) reduces to

$$u_{1,2}(x,t) = \pm I\sqrt{d}\coth\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right), \quad \text{for } d > 0. \quad (24)$$

Again, if we set $A = -LB$, (23) reduces to

$$u_{3,4}(x,t) = \pm I\sqrt{d}\tanh\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right), \quad \text{for } d > 0. \quad (25)$$

Substituting (24) and (25) into (10), we get

$$v_1(x,t) = -\frac{d}{\omega}\operatorname{cosech}^2\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right), \quad \text{for } d > 0, \quad (26)$$

$$v_2(x,t) = \frac{d}{\omega}\operatorname{sech}^2\left(\frac{\sqrt{d}}{\omega}(x-\omega t)\right), \quad \text{for } d > 0, \quad (27)$$

respectively.

If $d < 0$, using hyperbolic function identities, from (24)–(27), we get the following periodic travelling wave solutions:

$$u_{5,6}(x,t) = \pm\sqrt{d}\cot\left(\frac{\sqrt{-d}}{\omega}(x-\omega t)\right), \quad (28)$$

$$u_{7,8}(x,t) = \pm\sqrt{d}\tan\left(\frac{\sqrt{-d}}{\omega}(x-\omega t)\right), \quad (29)$$

$$v_3(x,t) = \frac{d}{\omega}\operatorname{cosec}^2\left(\frac{\sqrt{-d}}{\omega}(x-\omega t)\right), \quad (30)$$

$$v_4(x,t) = \frac{d}{\omega}\sec^2\left(\frac{\sqrt{-d}}{\omega}(x-\omega t)\right). \quad (31)$$

3.2. The Variant Boussinesq Equations. In this section, we will apply the modified simple equation method to find the exact solutions and then the solitary wave solutions of the variant Boussinesq equation [24] in the form

$$u_t + H_x + uu_x = 0, \qquad H_t + (uH)_x + u_{xxx} = 0, \quad (32)$$

The traveling wave transformation is

$$u(\xi) = u(x,t), \qquad H(\xi) = H(x,t), \quad \xi = x - \omega t. \quad (33)$$

Using traveling wave equation (33), (32) reduces into the following ODEs:

$$\begin{aligned} -\omega u' + H' + uu' &= 0. \\ -\omega H' + (uH)' + u''' &= 0. \end{aligned} \quad (34)$$

Integrating (34) with respect to ξ, choosing constant of integration as zero, we obtain the following ODEs:

$$-\omega u + H + \frac{1}{2}u^2 = 0, \quad (35)$$

$$-\omega H + uH + \frac{1}{3}u'' = 0. \quad (36)$$

From (35), we get

$$H = \omega u - \frac{1}{2}u^2. \quad (37)$$

Substituting (37) into (36) yields

$$u'' - \omega^2 u + \frac{3}{2}\omega u^2 - \frac{1}{2}u^3 = 0. \quad (38)$$

Now balancing the highest order derivative u'' and nonlinear term u^3, we get $n = 1$.

Now for $n = 1$, $u(\xi) = C_0 + \sum_{k=1}^{n} C_k(\phi'(\xi)/\phi(\xi))^k$ becomes

$$u(\xi) = C_0 + C_1\left(\frac{\phi'(\xi)}{\phi(\xi)}\right), \quad (39)$$

where C_0 and C_1 are constants to be determined such that $C_1 \neq 0$, while $\phi(\xi)$ is an unknown function to be determined. It is easy to see that

$$\begin{aligned} u' &= C_1\left(\frac{\phi''}{\phi} - \left(\frac{\phi'}{\phi}\right)^2\right), \\ u'' &= C_1\left(\frac{\phi'''}{\phi}\right) - 3C_1\left(\frac{\phi''\phi'}{\phi^2}\right) + 2C_1\left(\frac{\phi'}{\phi}\right)^3, \\ u^2 &= C_0^2 + 2C_0C_1\left(\frac{\phi'}{\phi}\right) + C_1^2\left(\frac{\phi'}{\phi}\right)^2, \\ u^3 &= C_1^3\left(\frac{\phi'}{\phi}\right)^3 + 3C_1^2C_0\left(\frac{\phi'}{\phi}\right)^2 + 3C_1C_0^2\left(\frac{\phi'}{\phi}\right) + C_0^3. \end{aligned} \tag{40}$$

Now substituting the values of u, u^2, u^3, u'' into (38) and then equating the coefficients of ϕ^0, ϕ^{-1}, ϕ^{-2}, ϕ^{-3} to zero, we, respectively, obtain

$$-\frac{1}{2}C_0^3 + \frac{3}{2}\omega C_0^2 - \omega^2 C_0 = 0, \tag{41}$$

$$C_1\phi''' - \omega^2 C_1\phi' - \frac{3}{2}C_0^2C_1\phi' + 3\omega C_0C_1\phi' = 0, \tag{42}$$

$$-3C_1\phi''\phi' - \frac{3}{2}C_0C_1^2(\phi')^2 + \frac{3}{2}\omega C_1^2(\phi')^2 = 0, \tag{43}$$

$$2C_1(\phi')^3 - \frac{1}{2}C_1^3(\phi')^3 = 0. \tag{44}$$

Solving (41), we get

$$C_0 = 0, \omega, 2\omega. \tag{45}$$

Solving (44), we get

$$C_1 = \pm 2, \qquad C_1 \neq 0. \tag{46}$$

From (42) and (43), we get

$$\phi'(\xi) = -ME\exp(-LM\xi). \tag{47}$$

Integrating (47), we obtain

$$\phi(\xi) = \frac{E\exp(LM\xi) + LF}{L}, \tag{48}$$

where $L = (\omega^2 + (3/2)C_0^2 - 3\omega C_0)$, $M = 2/(\omega - C_0)C_1$, and E, F are constants of integration.

Substituting ξ, $\phi(\xi)$ and $\phi'(\xi)$ from (47) and (48) into (39), we obtain

$$u(x,t) = C_0 - C_1\left(\frac{LME\exp(-LM(x-\omega t))}{E\exp(-LM(x-\omega t)) + LF}\right). \tag{49}$$

Case 1. When $C_0 = \omega$, (49) yields trivial solution. So, this case is rejected.

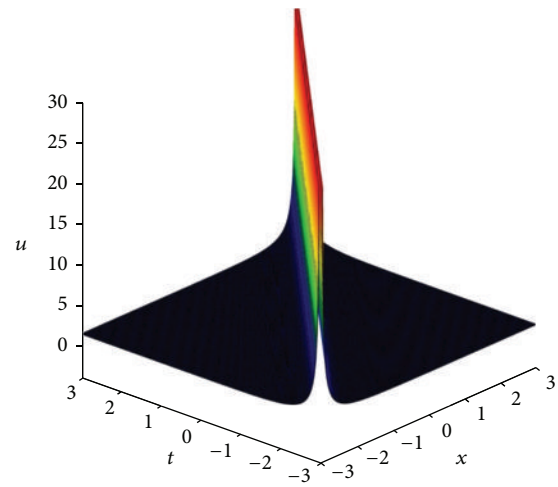

Figure 1: Singular soliton profile of (24) with wave speed $\omega = 1$, $d = 1$, and $-3 \leq x, t \leq 3$.

Case 2. When $C_0 = 0$ and $C_1 = \pm 2$, executing the parallel course of action described in Section 3.1 (Case 2), putting the values of L and M (49) yields,

$$u_{1,2}(x,t) = \omega\left(1 \pm \tanh\left(\frac{\omega}{2}(x - \omega t)\right)\right), \tag{50}$$

$$u_{3,4}(x,t) = \omega\left(1 \pm \coth\left(\frac{\omega}{2}(x - \omega t)\right)\right). \tag{51}$$

Substituting (49) and (50) into (36), we obtain

$$H_1(x,t) = \frac{\omega^2}{2}\operatorname{sech}^2\left(\frac{\omega}{2}(x - \omega t)\right), \tag{52}$$

$$H_2(x,t) = -\frac{\omega^2}{2}\operatorname{cosech}^2\left(\frac{\omega}{2}(x - \omega t)\right). \tag{53}$$

Case 3. When $C_0 = 2\omega$ and $C_1 = \pm 2$, we get the same results like (50)–(53).

4. Physical Explanation

In this section, we will put forth the physical explanation and the graphical representation of determined traveling wave solutions of nonlinear evolution equations through coupled Konno-Oono equations and the variant Boussinesq equations.

4.1. Explanations

(i) The equations (24) and (25) are complex soliton solutions. The shape of (24) is known as singular soliton, and the shape of (25) is known as kink soliton. Figures 1 and 2 represent the modulus shape of (24) and (25) with wave speed $\omega = 1$, $d = 1$ and wave speed $\omega = 1$, $d = 2$, respectively, within the interval $-3 \leq x, t \leq 3$. The disturbance of (24) and (25) is in the positive x-direction for positive values of wave speed ω. If we take negative values of wave speed ω,

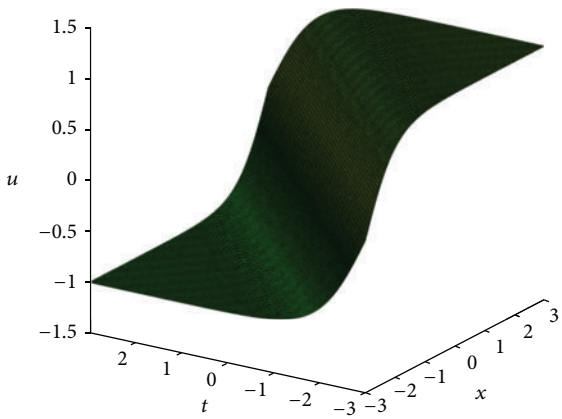

Figure 2: Kink wave profile of (25) with wave speed $\omega = 1$, $d = 2$, and $-3 \le x, t \le 3$.

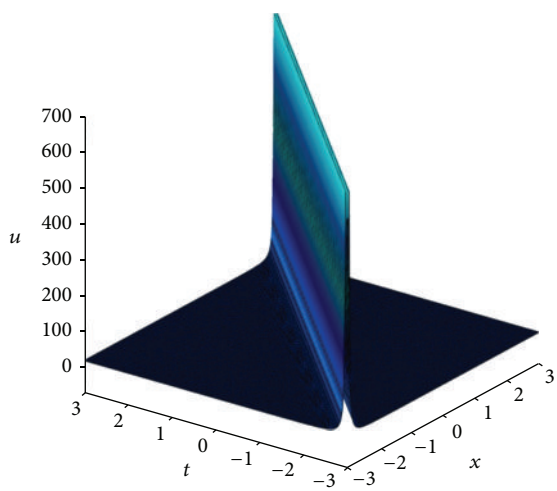

Figure 3: Soliton wave of (26) with wave speed $\omega = -2$, $d = 4$, and $-3 \le x, t \le 3$.

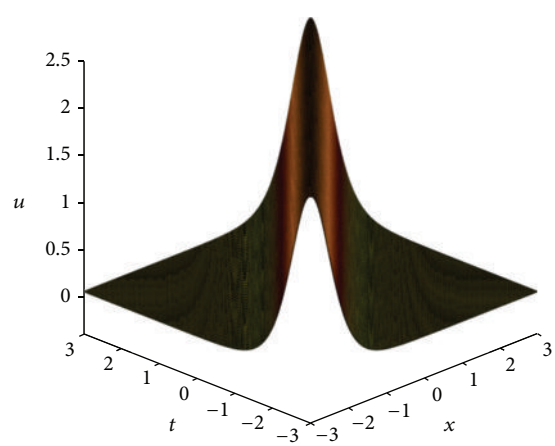

Figure 4: Bell-shaped wave profile of (27) with wave speed $\omega = 1$, $d = 2$, and $-3 \le x, t \le 3$.

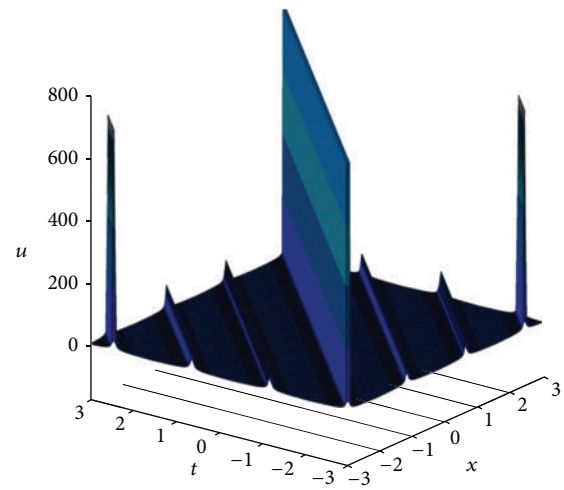

Figure 5: Modulus plot of periodic wave, shape of (28) with wave speed $\omega = 1$, $d = -3$, and $-3 \le x, t \le 3$.

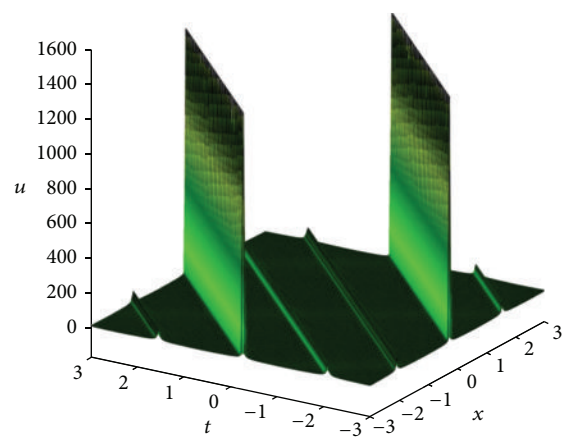

Figure 6: Modulus plot of periodic wave, profile of (29) with wave speed $\omega = 1$, $d = -3$, and $-3 \le x, t \le 3$.

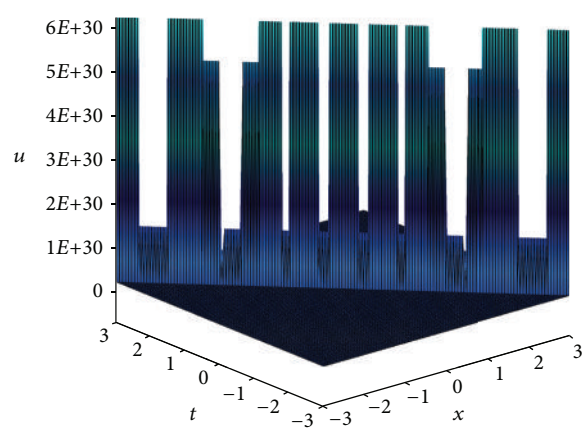

Figure 7: 3d plot of periodic wave solution, shape of (30) with wave speed $\omega = -1$, $d = -4$, and $-3 \le x, t \le 3$.

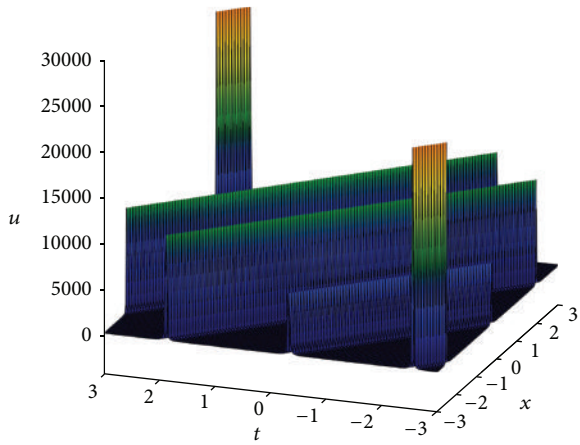

Figure 8: 3d plot of periodic wave solution, profile of (31) with wave speed $\omega = -1$, $d = -2$, and $-3 \le x, t \le 3$.

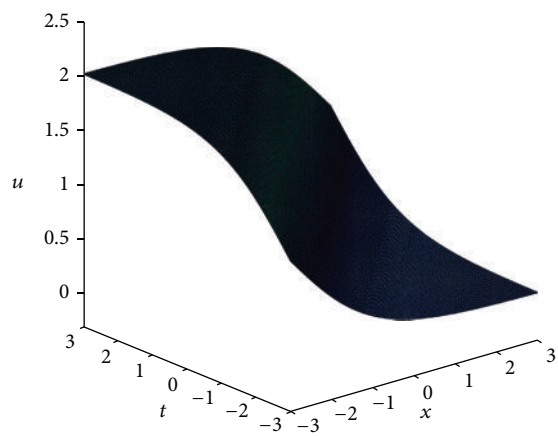

Figure 9: Kink wave, profile of (50) with wave speed $\omega = 1$ and $-3 \le x, t \le 3$.

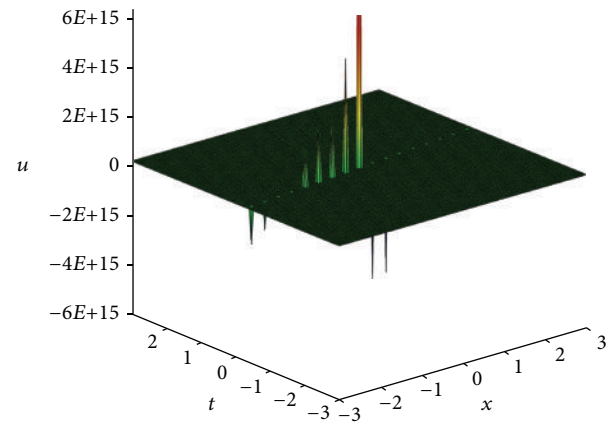

Figure 10: Singular kink soliton, shape of (51) with wave speed $\omega = 9$ and $-3 \le x, t \le 3$.

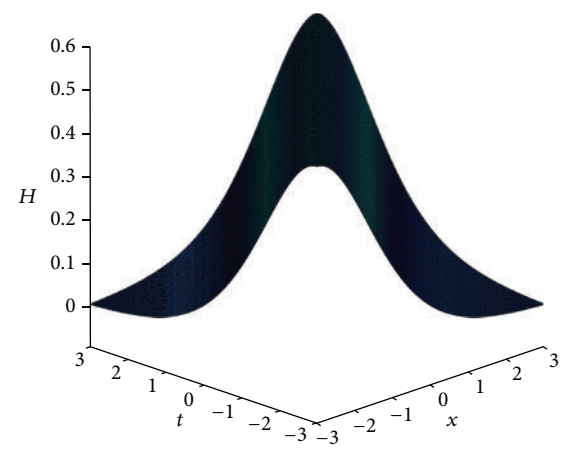

Figure 11: Bell-shaped soliton, profile of (52) with wave speed $\omega = 1$ and $-3 \le x, t \le 3$.

then the disturbance of (24) and (25) will be in the negative x-direction.

(ii) Equations (26) and (27) are soliton solutions. Figure 3 shows the shape of singular soliton of (26) with wave speed $\omega = -2$, $d = 4$, and $-3 \le x, t \le 3$, and Figure 4 shows bell-shaped soliton of (27) with wave speed $\omega = 1$, $d = 2$, and $-3 \le x, t \le 3$. The propagation or disturbance of (26), represented in Figure 3, is in the negative x-direction. And the propagation or disturbance of (27), represented in Figure 4, is in the positive x-direction.

(iii) Figures 5, 6, 7, and 8 corresponding to the shape of (28)–(31) are traveling wave solutions, which are periodic.

(iv) Figure 9 represents the profile of (50) that is kink solution with wave speed $\omega = 1$ and $-3 \le x, t \le 3$.

(v) Figure 10 represents the silhouette of (51) that is singular kink solution with wave speed $\omega = 9$ and $-3 \le x, t \le 3$.

(vi) Figure 11 represents the shadow of (52) that is bell-shaped solution with wave speed $\omega = 1$ and $-3 \le x, t \le 3$.

(vii) Figure 12 represents the profile of (53) that is soliton solution with wave speed $\omega = 1$ and $-3 \le x, t \le 3$.

The disturbances represented in Figures 9–12 are in the positive x-direction.

4.2. Graphical Representation. Some of our obtained traveling wave solutions are represented in the following figures with the aid of commercial software Maple.

5. Conclusions

In this paper, the MSE method has been employed for analytic treatment of two nonlinear coupled partial differential equations. The MSE method requires wave transformation formulae. Via MSE method traveling wave solutions, kink solutions, bell-shaped solutions of coupled Konno-Oono equations, and the variant Boussinesq equations were derived. The

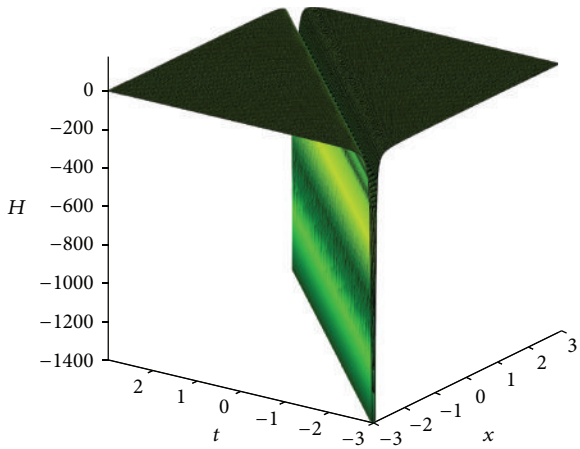

Figure 12: Singular soliton, shape of (53) with wave speed $\omega = 1$ and $-3 \leq x, t \leq 3$.

procedure is simple, direct, and constructive. Without the help of a computer algebra system all examples in this paper show the efficiency of MSE method.

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