Research Article

Green's Function for Dirac Particle in a Non-Abelian $SU(2)$ Field: A Path Integral Approach

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The Green function for a Dirac particle moving in a non-Abelian $SU(2)$ field and having a particular form is exactly determined by the path integral approach. The wave functions were deduced from the residues of Green's function. It is shown that the classical paths contributed mainly to the determination of the Green function.

1. Introduction

In the present paper, we suggest to determine the Green function for Dirac particle of mass M in a $SU(2)$ non-Abelian field by the path integral approach. The $SU(2)$ non-Abelian field in question has the form [1, 2]

$$A_\mu^a = [(k_1 x) b m^a + (k_2 x) c m^a + h^a] k_\mu, \quad (1)$$

where

- (i) k_1 , k_2 , and k are, respectively, the polarizations of color wave and the isotropic wave such as

$$k k_1 = k k_2 = k_1 k_2 = 0, \quad (2)$$

- (ii) $b = b(kx)$, $c = c(kx)$ are functions which depend only of $\varphi = kx$,

- (iii) m^a are generators of $SU(2)$ group which satisfy the condition

$$m^a(\varphi) \cdot m^a(\varphi) = 1, \quad (3)$$

- (iv) h^a is a gauge function [3] such as

$$h^a = \frac{1}{g} \varepsilon^{abc} \frac{dm^b(\varphi)}{d\varphi} m^c(\varphi) = \frac{1}{g} \varepsilon^{abc} m'^b(\varphi) m^c(\varphi), \quad (4)$$

- (v) and $a = 1, 2, 3$ is a color index.

With this form of field, the Lorentz condition is satisfied:

$$\partial^\mu A_\mu^a = 0. \quad (5)$$

From the point of view of the literature, we can notice that there exists only one limiting number of cases of interactions where the equation of Dirac has analytical and exact solution. We can cite, for example, the simple cases such as the constant field [4], the plane wave [5, 6], the crossed electric and magnetic fields [7, 8], and the combination of constant homogeneous electromagnetic field with a plane wave field [9, 10]. With the path integral formalism, we can cite [11–14], where the interactions have classical form.

For the non-Abelian case, at our knowledge, there exist no explicit calculations of propagators by the path integral formalism, except the formalism given by Fradkin et al. [15] and which we use in the present paper.

Note that for this problem treated in [1] the wave functions for a particle of spin 1/2 are determined by solving the Yang-Mills equation.

Also, in this paper we propose to consider the non-Abelian field described previously in order to determine analytically Green's function by using the approach of path integral, to extract then the wave functions.

2. Green's Function for Dirac Particle: Local Approach

First, let us proceed as indicated in the book [15] by replacing the generators of $SU(2)$ by a bilinear expression

$$m_a = \frac{1}{4} \Gamma_\alpha t_a^\alpha \Gamma_\beta, \quad (6)$$

where $t_{a\beta}^\alpha$ are traceless Hermitian matrices and Γ_α are the operators which satisfy the following anticommutation relations:

$$\{\Gamma_\alpha, \Gamma_\beta\} = 2\delta_{\alpha\beta}. \quad (7)$$

First, let us write the equation satisfied by Green's function S related to the Dirac particle submitted to the action of non-Abelian field [1]

$$\left[-(\gamma\hat{p}) + \frac{g}{8} (\gamma k) (b(k_1 x) + c(k_2 x)) (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) + \frac{g}{32g} (\gamma k) \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) - M \right] \times S(x_b, x_a) = \delta^4(x_b - x_a), \quad (8)$$

where γ^μ are usual Dirac matrices obeying to anticommutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad \mu, \nu = 0, 1, 2, 3 \quad (9)$$

with $g^{\mu\nu}$ is the metric tensor of Minkowski space (diag $g_{\mu\nu} = (1, -1, -1, -1)$).

In order to determine the Green function, we follow the usual construction method [11] of path integral form.

First, we know that the kernel

$$S(x_b, x_a) = \langle x_b | \hat{S} | x_a \rangle \quad (10)$$

is a matrix element of operator $\hat{S}$, the solution of the following symbolic equation:

$$\left(-(\gamma\hat{p}) + \frac{g}{8} (\gamma k) (b(k_1 x) + c(k_2 x)) \times (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) + \frac{1}{32g} (\gamma k) \times \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) - M \right) \hat{S} = I, \quad (11)$$

or

$$\hat{S} = I \times \left(\left(-(\tilde{\gamma} \cdot \hat{p}) + \frac{g}{8} (\tilde{\gamma} \cdot k) \times (b(k_1 \cdot x) + c(k_2 \cdot x)) (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) + \frac{(\tilde{\gamma} \cdot k)}{32g} \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) - M\gamma^5 \right) \right)^{-1} \gamma^5 = \tilde{S}\gamma^5, \quad (12)$$

where γ^5 is introduced in order to homogenize the variables

$$\gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3, \quad (\gamma^5)^2 = -1. \quad (13)$$

So, $\tilde{S}$ can be written in the exponential form

$$\tilde{S} = \int_0^\infty d\lambda \int d\chi \exp(-i\lambda \hat{H}_l(\chi, \hat{p}, \hat{x})), \quad (14)$$

where λ and χ are, respectively, bosonic and fermionic variables [16, 17]. The Grassmann variables have the following properties:

$$\chi^2 = 0, \quad \int d\chi = 0, \quad \int \chi d\chi = 1, \quad (15)$$

and the Hamiltonian system which governs the movement of the Dirac particle in the non-Abelian field is given by

$$H_l = p^2 - M^2 - \frac{1}{16} \left((\hat{p} \cdot \hat{k}) \times \left(\sigma \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) + (\hat{p} \cdot \hat{k}) (b(k_1 x) + c(k_2 x)) \right) \right)$$

$$\begin{aligned}
& \times \left(\sigma \left(\frac{1}{4} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right) - ig (\hat{k} \sigma \hat{k}) \\
& \times \left(\left(\frac{1}{32g} \frac{d\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}}{d\varphi} (\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}) \right) \right. \\
& \quad \left. + \left\{ (b(k_1 x) + c(k_2 x)) \left(\frac{1}{8} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right\}' \right) \\
& + \chi \left(-\hat{p} + \frac{g}{2} \hat{k} (b(k_1 x) + c(k_2 x)) \right. \\
& \quad \times \left(\sigma \left(\frac{1}{4} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right) + \frac{\hat{k}}{2} \\
& \quad \left. \times \left(\sigma \left(\frac{1}{16g} \frac{d\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}}{d\varphi} (\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}) \right) \right) - M\gamma^5 \right). \tag{16}
\end{aligned}$$

In the configuration space, the Green function is [4]

$$\tilde{S}(x_b, x_a) = \int_0^{\infty} d\lambda \int d\chi \int \langle x_b | \exp(-i\lambda \hat{H}_l(\chi, \hat{p}, \hat{x})) | x_a \rangle, \tag{17}$$

and the path integral formulation consists of

(i) subdividing the interval $[x_a, x_b]$ into N equals intervals of length $\Delta\tau = \lambda/N$,

(ii) inserting the projectors

$$\begin{aligned}
& \int d^4 x |x\rangle \langle x| = 1, \quad d^4 p |p\rangle \langle p| = 1, \\
& \int \langle x | p \rangle = \frac{1}{(2\pi)^4} \exp(ipx), \tag{18}
\end{aligned}$$

by using the actions on kets

$$\hat{x}^{\mu} |x\rangle = x^{\mu} |x\rangle, \quad \hat{p}^{\mu} |p\rangle = p^{\mu} |p\rangle, \tag{19}$$

in order to eliminate the operators $\hat{x}$, $\hat{p}$.

So the Green function can be written (with the midpoint prescription) as following

$$\begin{aligned}
& G^c(x_b, x_a) \\
& = iT_1 T_2 \int_0^{\infty} d\lambda \int Dx \int Dp \times \exp i \\
& \quad \times \int_0^{\lambda} d\tau \left[p\dot{x} + (p^2 - M^2) - \frac{1}{16} \right. \\
& \quad \times \left[(\hat{p}\hat{k}) \left(\sigma \left(\frac{d\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}}{d\varphi} (\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}) \right) \right) \right. \\
& \quad \left. + (\hat{p}\hat{k}) (b(k_1 x) + c(k_2 x)) \right. \\
& \quad \left. \times \left(\sigma \left(\frac{1}{4} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right) \right] \\
& \quad - ig (\hat{k} \sigma \hat{k}) \left(\left(\frac{1}{32g} \frac{d\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}}{d\varphi} (\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}) \right) \right. \\
& \quad \left. + \left\{ (b(k_1 x) + c(k_2 x)) \right. \right. \\
& \quad \left. \left. \times \left(\frac{1}{8} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right\}' \right) \\
& \quad \left. + \chi \left(-\hat{p} + \frac{g}{2} \hat{k} (b(k_1 \cdot x) + c(k_2 \cdot x)) \right. \right. \\
& \quad \times \left(\sigma \cdot \left(\frac{1}{4} \Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta} \right) \right) + \frac{\hat{k}}{2} \\
& \quad \times \left(\sigma \cdot \left(\frac{1}{16g} \frac{d\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}}{d\varphi} (\Gamma_{\alpha} t_{\beta}^{\alpha} \Gamma_{\beta}) \right) \right) \\
& \quad \left. \left. - M\gamma^5 \right) \right], \tag{20}
\end{aligned}$$

where the $T_1 T_2$ -products are introduced because of the presence of matrices γ^{μ} and operators Γ which do not commute. To eliminate the $T_1 T_2$ -products, we used the following form path integral:

$$\begin{aligned}
& T_1 \exp \left\{ \int_0^{\lambda} (\rho_1 \cdot \gamma) d\tau \right\} \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right)
\end{aligned}$$

$$\begin{aligned} & \times \int_{E_1} D\Psi \exp \left\{ \int_0^\lambda \left((\Psi \cdot \dot{\Psi}) - 2i(\rho_1 \cdot \Psi) \right) d\tau \right. \\ & \left. + (\Psi(\lambda) \cdot \Psi(0)) \right\} \Big|_{\Psi_0 + \Psi_1 = \theta}, \end{aligned} \quad (21)$$

$$\begin{aligned} & T_2 \exp \left\{ \int_0^\lambda (\rho_2 \cdot \Gamma) d\tau \right\} \\ & = \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\ & \times \int_{E_2} DY \exp \left\{ \int_0^\lambda \left((Y \cdot \dot{Y}) - 2i(\rho_2 \cdot Y) \right) d\tau \right. \\ & \left. + (Y(\lambda) \cdot Y(0)) \right\} \Big|_{Y_0 + Y_1 = \vartheta}, \end{aligned} \quad (22)$$

$E_1 = \{\Psi(\tau) \text{ where } \Psi(0) + \Psi(\lambda) = \theta\}$, $E_2 = \{Y(\tau) \text{ where } Y(0) + Y(\lambda) = \vartheta\}$ which are the domains of integration. ρ_1, ρ_2 are coupled currents to matrices γ and operators Γ , and θ, ϑ are Grassmann variables.

So, we obtain

$$\begin{aligned} & \tilde{S}(x_b, x_a) \\ & = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\ & \times \int_0^\infty d\lambda \int d\chi \int Dx \int Dp \int_{E_1} D\Psi \int_{E_2} DY \\ & \times \exp \left[i \int_0^\lambda d\tau \right. \\ & \times \left[p\dot{x} + \left(p^2 - M^2 - \frac{i}{16} (p\Psi)(k\Psi) \right. \right. \\ & \times (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) - i\frac{g}{4} (p\Psi)(k\Psi) \\ & \times (b(k_1x) + c(k_2x)) (t_\beta^\alpha \gamma_\alpha \gamma_\beta) \Big) \\ & \left. + \chi \left(- (p\Psi) + \frac{i}{32} (k\Psi)(\Psi(t_\beta^\alpha \gamma'_\alpha \gamma_\beta)) \right. \right. \\ & \left. \left. + i\frac{g}{8} (k\Psi)(b(k_1x) + c(k_2x)) \right) \right] \end{aligned}$$

$$\begin{aligned} & \times (t_\beta^\alpha \gamma_\alpha \gamma_\beta) - M\Psi^5 \Big) \\ & - i\Psi\dot{\Psi} - iY\dot{Y} \Big] + \Psi(\lambda) \Psi(0) \\ & + Y(\lambda) Y(0) \Big] \Big|_{\theta = \vartheta = 0}, \end{aligned} \quad (23)$$

which is the Green function to calculate in the local approach of path integral.

Having obtained the path integral form for the Green function $\tilde{S}$, let us perform the integrations.

We notice that the field depends on products kx and $k_i x$, with $i = 1, 2$. First, we put $\varphi = kx$ and $\varphi_i = k_i x$, and then the variables φ and φ_i are rendered, respectively, independent of kx and of $k_i x$ by introducing the following three identities [11, 12]:

$$\begin{aligned} & \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \\ & \times \int Dp_\varphi \exp \left\{ i \int_0^\lambda p_\varphi (\dot{\varphi} - k\dot{x}) d\tau \right\} = 1, \\ & \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \\ & \times \int Dp_{\varphi_i} \exp \left\{ i \int_0^\lambda p_{\varphi_i} (\dot{\varphi}_i - k_i \dot{x}) d\tau \right\} = 1. \end{aligned} \quad (24)$$

Then, $\tilde{S}$ is rewritten as

$$\begin{aligned} & \tilde{S}(x_b, x_a) \\ & = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\ & \times \int_0^\infty d\lambda \int d\chi \int Dx \int Dp \\ & \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\ & \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \\ & \times \int Dp_{\varphi_i} \int_{E_1} D\Psi \int_{E_2} DY \\ & \times \exp \left[i \int_0^\lambda d\tau \left[\left(p^2 - M^2 - i\frac{g}{32} (p\Psi)(k \cdot \Psi) (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) \right. \right. \right. \\ & \left. \left. + (p - kp_\varphi - k_1 p_{\varphi_1} - k_2 p_{\varphi_2}) \dot{x} \right. \right. \\ & \left. \left. + p_\varphi \dot{\varphi} + p_{\varphi_1} \dot{\varphi}_1 + p_{\varphi_2} \dot{\varphi}_2 \right) \right] \end{aligned}$$

$$\begin{aligned}
& -ig(p\Psi)(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \\
& -\chi \left[(p\Psi) - \frac{ig}{2}(k\Psi)(\Psi(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta)) \right. \\
& \quad \left. - i\frac{g}{2}(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right. \\
& \quad \left. + M\Psi^5 \right] - i\Psi\dot{\Psi} - i\Upsilon\dot{\Upsilon} \\
& \quad \left. + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \right] \Bigg|_{\theta=\vartheta=0}, \\
& + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \Bigg|_{\theta=\vartheta=0}.
\end{aligned} \tag{25}$$

Using a shift on p defined by

$$p - kp_\varphi - k_1 p_{\varphi_1} - k_2 p_{\varphi_2} \longrightarrow p, \tag{26}$$

we obtain

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp\left(i\gamma \frac{\delta_L}{\delta\theta}\right) \exp\left(i\Gamma \frac{\delta_L}{\delta\vartheta}\right) \\
& \quad \times \int_0^\infty d\lambda \int d\chi \int Dx \int Dp \\
& \quad \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \quad \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \\
& \quad \times \int D\varphi_i \int Dp_{\varphi_i} \int_{E_1} D\Psi \int_{E_2} D\Upsilon \\
& \quad \times \exp\left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left[\left(p\dot{x} + p^2 - M^2 \right. \right. \\
& \quad \left. \left. - i\frac{g}{32}(p\Psi)(k\Psi)(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \right. \right. \\
& \quad \left. \left. - ig(p\Psi)(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right) \right. \\
& \quad \left. + p_\varphi(\dot{\varphi} + 2kp) + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) \right. \\
& \quad \left. + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) \right. \\
& \quad \left. - \chi \left[(p\Psi) - \frac{ig}{2}(k\Psi)(\Psi(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta)) \right. \right. \\
& \quad \left. \left. - i\frac{g}{2}(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right. \right. \\
& \quad \left. \left. + M\Psi^5 \right] - i\Psi\dot{\Psi} - i\Upsilon\dot{\Upsilon} \right. \\
& \quad \left. + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \right] \Bigg|_{\theta=\vartheta=0},
\end{aligned}$$

where $b = b(\varphi)$, $c = c(\varphi)$.

If we integrate by part the first term in the action and then integrate the variables $x(\tau)$, a Dirac function $\delta(\dot{p})$ appeared, and the integrations on all the p_n reduce $\int Dp$ to a simple integral $\int d^4 p / (2\pi)^4$; that is, the momentum of the particle is conserved (constant) during the movement.

Thus, we have

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp\left(i\gamma \frac{\delta_L}{\delta\theta}\right) \exp\left(i\Gamma \frac{\delta_L}{\delta\vartheta}\right) \\
& \quad \times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4} \\
& \quad \times \exp\{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
& \quad \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \quad \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \\
& \quad \times \int D\varphi_i \int Dp_{\varphi_i} \int_{E_1} D\Psi \int_{E_2} D\Upsilon \\
& \quad \times \exp\left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left[\left(-i\frac{g}{32}(p\Psi)(k\Psi)(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \right. \right. \\
& \quad \left. \left. - ig(p\Psi)(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right) \right. \\
& \quad \left. + p_\varphi(\dot{\varphi} + 2kp) + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) \right. \\
& \quad \left. + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) \right. \\
& \quad \left. - \chi \left[(p\Psi) - \frac{ig}{2}(k\Psi)(\Psi(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta)) \right. \right. \\
& \quad \left. \left. - i\frac{g}{2}(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right. \right. \\
& \quad \left. \left. + M\Psi^5 \right] - i\Psi\dot{\Psi} - i\Upsilon\dot{\Upsilon} \right. \\
& \quad \left. + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \right] \Bigg|_{\theta=\vartheta=0},
\end{aligned}$$

$$\begin{aligned}
& -i\frac{g}{2}(k\Psi)(b\varphi_1 + c\varphi_2) \\
& \times \left(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta \right) + M\Psi^5 \Big] - i\Psi\dot{\Psi} - i\Upsilon\dot{\Upsilon} \Big] \\
& + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \Big] \Big|_{\theta=\vartheta=0}.
\end{aligned} \tag{28}$$

We notice also, that there is dependence of product of $k\Psi_a$ in the action. So if we put $\xi_a = k\Psi_a$, these new variables are rendered independent [11, 12] by the introduction of the following relation:

$$\begin{aligned}
& \int d\xi_a d\xi_b \delta(\xi_a - (k \cdot \Psi_a)) \\
& \times \int Dp_\xi \int D\xi \exp \left\{ i \int_0^\lambda p_\xi (\dot{\xi} - (k \cdot \dot{\Psi})) d\tau \right\} = 1,
\end{aligned} \tag{29}$$

with ξ and p_ξ being Grassmann variables.

Then $\tilde{S}$, with these new variables, is rewritten as

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\
& \times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4} \\
& \times \exp \{ ip(x_b - x_a) + i\lambda(p^2 - M^2) \} \\
& \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \\
& \times \int Dp_{\varphi_i} \int d\xi_a d\xi_b \delta(\xi_a - (k \cdot \Psi_a)) \\
& \times \int D\xi \int Dp_\xi \int_{E_1} D\Psi \int_{E_2} D\Upsilon \\
& \times \exp \left[i \int_0^\lambda d\tau \right. \\
& \times \left[\left(-i\frac{g}{32}(p\Psi)(k\Psi)(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \right. \right. \\
& \quad \left. \left. - ig(p\Psi)(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right) \right. \\
& \quad \left. + p_\varphi(\dot{\varphi} + 2k_1 p) + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) \right.
\end{aligned}$$

$$\begin{aligned}
& + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) + p_\xi(\dot{\xi} - (k\dot{\Psi})) \\
& - \chi \left[(p\Psi) - \frac{ig}{2}(k\Psi)(\Psi(t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta)) \right. \\
& \quad \left. - i\frac{g}{2}(k\Psi)(b\varphi_1 + c\varphi_2)(t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right. \\
& \quad \left. + M\Psi^5 \right] - i\Psi\dot{\Psi} - i\Upsilon\dot{\Upsilon} \Big] \\
& + \Psi(\lambda)\Psi(0) + \Upsilon(\lambda)\Upsilon(0) \Big] \Big|_{\theta=\vartheta=0}.
\end{aligned} \tag{30}$$

Let us remark that the integrations over the Grassmann variables Ψ are subjected to the boundary conditions. We can render free these integrations, if we use new variables ω (velocities) defined by [15, 18]

$$\omega(\tau) = \frac{d\Psi(\tau)}{d\tau}, \tag{31}$$

or by integration we have the relation

$$\Psi(\tau) = \frac{1}{2} \int_0^\lambda \varepsilon(\tau - \tau') \omega(\tau') d\tau' + \frac{\theta}{2} \tag{32}$$

between Ψ and ω .

Since

$$\begin{aligned}
& \int_0^\lambda d\tau \Psi \dot{\Psi} + \Psi(\lambda)\Psi(0) \\
& = \int_0^\lambda \int_0^{\lambda'} d\tau d\tau' \omega(\tau) \varepsilon(\tau - \tau') \omega(\tau') \\
& = -\frac{1}{2} \omega * \varepsilon * \omega,
\end{aligned} \tag{33}$$

$$\int_E D\Psi = \int_E D\omega,$$

where the following notation is used:

$$a * b * c = \int \int a(\tau) b(\tau, \tau') c(\tau') d\tau d\tau', \tag{34}$$

we obtain for $\tilde{S}$

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\
& \times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4}
\end{aligned}$$

$$\begin{aligned}
& \times \exp \{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
& \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \int Dp_{\varphi_i} \\
& \times \int d\xi_a \int d\xi_b \int Dp_\xi \int D\xi \\
& \times \int_{E_1} D\omega \delta\left(\xi_a + \frac{1}{2}(k(\omega - \theta))\right) \int_{E_2} DY \\
& \times \exp \left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left(-\frac{i}{2} g p(\varepsilon * \omega) \xi \right. \\
& \quad - \frac{i}{2} g p(\varepsilon * \omega) \xi (b\varphi_1 + c\varphi_2) \\
& \quad - \frac{i}{2} g(p\theta) (b\varphi_1 + c\varphi_2) \xi + p_\varphi(\dot{\varphi} + 2kp) \\
& \quad + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) \\
& \quad + p_\xi \dot{\xi} - iY\dot{Y} \\
& \quad \left. + \chi \left[-\frac{1}{2} p(\varepsilon * \omega) \right. \right. \\
& \quad - \frac{1}{2} (p\theta) + \frac{i}{4} g(\varepsilon * \omega) (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) \\
& \quad + \frac{i}{4} g(\varepsilon * \omega) \xi (b\varphi_1 + c\varphi_2) \\
& \quad + \frac{i}{4} g(\theta\xi) (b\varphi_1 + c\varphi_2) \\
& \quad \left. \left. \times (t_\beta^\alpha \gamma_\alpha \gamma_\beta) + M(\varepsilon * \omega^5 + \theta^5) \right] \right] \\
& \quad \left. - \frac{1}{2} (\omega * \varepsilon * \omega) + (Y(\lambda) \cdot Y(0)) \right] \Big|_{\theta=\theta=0}.
\end{aligned} \tag{35}$$

By using a shift for ω

$$\begin{aligned}
\bar{\omega}(\tau) & \longrightarrow \omega(\tau) + ik(\varepsilon^{-1} * p_\xi) \\
& + ik_1(\varepsilon^{-1} * p_{\xi_1}) + ik_2(\varepsilon^{-1} * p_{\xi_2}),
\end{aligned} \tag{36}$$

the Green function can be simplified and take the following form:

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\theta} \right)
\end{aligned}$$

$$\begin{aligned}
& \times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4} \\
& \times \exp \{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
& \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \\
& \times \int D\varphi_i \int Dp_{\varphi_i} \int d\xi_a \int d\xi_b \int Dp_\xi \int D\xi \\
& \times \int_{E_1} D\omega \delta\left(\xi_a + \frac{1}{2}(k \cdot (\omega - \theta))\right) \int_{E_2} DY \\
& \times \exp \left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left(-\frac{i}{2} g(p(\varepsilon * \omega - i(k \cdot p_\xi) - i(k_1 \cdot p_{\xi_1}) \right. \\
& \quad \left. \left. - i(k_2 \cdot p_{\xi_2})) \xi \right) \right. \\
& \quad - \frac{i}{2} g p(\varepsilon * \omega - i(k \cdot p_\xi) - i(k_1 \cdot p_{\xi_1}) \\
& \quad \left. \left. - i(k_2 \cdot p_{\xi_2})) \right) \right. \\
& \quad \times \xi(b \cdot \varphi_1 + c \cdot \varphi_2) - \frac{i}{2} g(p \cdot \theta) \\
& \quad \times (b\varphi_1 + c\varphi_2) \xi + p_\varphi(\dot{\varphi} + 2(k \cdot p)) \\
& \quad + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) \\
& \quad + p_\xi \dot{\xi} - iY\dot{Y} \\
& \quad \left. + \chi \left[-\frac{1}{2} p \right. \right. \\
& \quad \times (\varepsilon * \omega - i(k p_\xi) - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2})) \\
& \quad - \frac{1}{2} (p\theta) + \frac{i}{4} g \\
& \quad \times (\varepsilon * \omega - i(k \cdot p_\xi) \\
& \quad \left. \left. - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2})) \right) \right. \\
& \quad \times (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) + \frac{i}{4} g \\
& \quad \times (\varepsilon * \omega - i(k p_\xi) - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2})) \\
& \quad \times \xi(b\varphi_1 + c\varphi_2) \\
& \quad + \frac{i}{4} g(\theta\xi) (b\varphi_1 + c\varphi_2) (t_\beta^\alpha \gamma_\alpha \gamma_\beta) \\
& \quad \left. \left. + M(\varepsilon * \omega^5 + \theta^5) \right] \right]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \left(\omega * \varepsilon * \omega - 2i(k\omega) p_\xi \right. \\
& \quad \left. - 2i(k_1\omega) p_{\xi_1} - 2i(k_1\omega) p_{\xi_2} \right) \\
& \quad + \left. Y(\lambda) Y(0) \right] \Big|_{\theta=\vartheta=0}.
\end{aligned} \tag{37}$$

At this level, to integrate velocities ω , first, we insert the integral representation of the Dirac function

$$\begin{aligned}
& \delta \left(\xi_a + \frac{1}{2} (k(\omega - \theta)) \right) \\
& = \int dp_{\xi_a} \exp \left\{ ip_{\xi_a} \left(\xi_a + \frac{1}{2} (k(\omega - \theta)) \right) \right\},
\end{aligned} \tag{38}$$

with p_{ξ_a} being Grassmann variable; then we have

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \exp \left(i\Gamma \frac{\delta_L}{\delta\vartheta} \right) \\
& \quad \times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4} \\
& \quad \times \exp \left\{ ip(x_b - x_a) + i\lambda(p^2 - M^2) \right\} \\
& \quad \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \quad \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \\
& \quad \times \int Dp_{\varphi_i} \int d\xi_a \int dp_{\xi_a} \int d\xi_b \int Dp_\xi \int D\xi \\
& \quad \times \int_{E_1} D\omega \delta \left(\xi_a + \frac{1}{2} (k(\omega - \theta)) \right) \int_{E_2} DY \\
& \quad \times \exp \left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left(-\frac{i}{2} g \right. \\
& \quad \times \left(p \left(\varepsilon * \omega - i(kp_\xi) - i(k_1 p_{\xi_1}) \right. \right. \\
& \quad \quad \left. \left. - i(k_2 p_{\xi_2}) \right) \xi \right) \\
& \quad \left. - \frac{i}{2} gp \left(\varepsilon * \omega - i(kp_\xi) \right. \right. \\
& \quad \quad \left. \left. - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2}) \right) \right]
\end{aligned}$$

$$\begin{aligned}
& \times \xi(b\varphi_1 + c\varphi_2) - \frac{i}{2} g(p\theta)(b\varphi_1 + c\varphi_2)\xi \\
& + p_\varphi(\dot{\varphi} + 2kp) + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) \\
& + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) + p_\xi \dot{\xi} + p_{\xi_a} \\
& \times \left(\xi_a + \frac{1}{2} (k(\omega - \theta)) \right) - iY\dot{Y} \\
& + \chi \left[-\frac{1}{2} p \left(\varepsilon * \omega - i(kp_\xi) \right. \right. \\
& \quad \left. \left. - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2}) \right) \right. \\
& \quad \left. - \frac{1}{2} (p\theta) + \frac{i}{4} g \right. \\
& \quad \times \left(\varepsilon * \omega - i(kp_\xi) \right. \\
& \quad \left. - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2}) \right) \\
& \quad \times (t_\beta^\alpha Y'_\alpha Y_\beta) + \frac{i}{4} g \\
& \quad \times \left(\varepsilon * \omega - i(kp_\xi) \right. \\
& \quad \left. - i(k_1 p_{\xi_1}) - i(k_2 p_{\xi_2}) \right) \\
& \quad \times \xi(b\varphi_1 + c\varphi_2) + \frac{i}{4} g(\theta\xi)(b\varphi_1 + c\varphi_2) \\
& \quad \times (t_\beta^\alpha Y_\alpha Y_\beta) + M(\varepsilon * \omega^5 + \theta^5) \Big] \\
& - \frac{1}{2} \left(\omega * \varepsilon * \omega - 2i(k\omega) p_\xi \right. \\
& \quad \left. - 2i(k_1\omega) p_{\xi_1} - 2i(k_1\omega) p_{\xi_2} \right) \\
& \quad + \left. Y(\lambda) Y(0) \right] \Big|_{\theta=\vartheta=0}.
\end{aligned} \tag{39}$$

We notice that the integration over ω^5 can be eliminated thanks to the following result:

$$\int D\omega^5 \exp \left\{ -\frac{1}{2} \omega_5 * \varepsilon * \omega^5 - \frac{iM}{2} \varepsilon * \omega^5 \chi \right\} = 1, \tag{40}$$

and for other variables $\omega^\mu (\mu = 0, 1, 2, 3)$, the integrations have the following forms:

$$\int D\omega \exp \left\{ \int_0^\lambda \left[-\frac{1}{2} \omega_\mu \varepsilon \omega^\mu + L_\mu \omega^\mu \right] d\tau \right\}, \tag{41}$$

where

$$\begin{aligned}
 L_\mu(\tau) = & -\frac{ig}{2} \int_0^\lambda p_\mu \xi(s) (1 + (b\varphi_1 + c\varphi_2)) \varepsilon(\tau - s) ds \\
 & + \chi \int_0^\lambda \left(-\frac{1}{2} p_\mu + \frac{i}{4} g k_\mu \right. \\
 & \quad \times \left((t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) + \xi(s) (b\varphi_1 + c\varphi_2) \right) \\
 & \quad \times \varepsilon(\tau - s) ds + \frac{1}{2} k_\mu p_{\xi_a}.
 \end{aligned} \tag{42}$$

After we have integrated over the ω , $\bar{S}$ is reduced to

$$\begin{aligned}
 \bar{S}(x_b, x_a) &= \exp\left(i\gamma \frac{\delta_L}{\delta\theta}\right) \exp\left(i\Gamma \frac{\delta_L}{\delta\vartheta}\right) \\
 &\times \int_0^\infty d\lambda \int d\chi \\
 &\times \int \frac{d^4 p}{(2\pi)^4} \exp\{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
 &\times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \\
 &\times \int D\varphi \int DP_\varphi \prod_{i=1}^2 \int d\varphi_{ib} \\
 &\times \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \int DP_{\varphi_i} \\
 &\times \int d\xi_a \int dp_{\xi_a} \int d\xi_b \int dp_\xi \int D\xi \int_{E_2} DY \\
 &\times \exp\left[i \int_0^\lambda d\tau \right. \\
 &\quad \times \left(p_\varphi(\dot{\varphi} + 2kp) + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) \right. \\
 &\quad \left. + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) + p_\xi \dot{\xi} + p_{\xi_a} \right. \\
 &\quad \times \left(\xi_a - \frac{1}{2} k\theta \right) - iY\dot{Y} \\
 &\quad \left. + \chi \left[-\frac{1}{2} (p\theta) + \frac{i}{4} g (t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) \right. \right. \\
 &\quad \left. \left. + \frac{i}{4} g \xi (b\varphi_1 + c\varphi_2) + \frac{i}{4} g (\theta\xi) (b\varphi_1 + c\varphi_2) \right. \right. \\
 &\quad \left. \left. \times (t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) + M\theta^5 \right] \right) \\
 &\quad \left. + Y(\lambda) Y(0) \right] \Bigg|_{\theta=\vartheta=0},
 \end{aligned} \tag{43}$$

and the integrations on variables p_ξ give

$$\dot{\xi} = 0, \quad \text{that is, } \xi_a = \xi_b = c^{st}. \tag{44}$$

Thus, $\bar{S}$ becomes

$$\begin{aligned}
 \bar{S}(x_b, x_a) &= \exp\left(i\gamma \frac{\delta_L}{\delta\theta}\right) \exp\left(i\Gamma \frac{\delta_L}{\delta\vartheta}\right) \\
 &\times \int_0^\infty d\lambda \int d\chi \\
 &\times \int \frac{d^4 p}{(2\pi)^4} \exp\{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
 &\times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int DP_\varphi \\
 &\times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \int D\varphi_i \\
 &\times \int DP_{\varphi_i} \int d\xi_a \int dp_{\xi_a} \int D\xi \int_{E_2} DY \\
 &\times \exp\left[i \int_0^\lambda d\tau \right. \\
 &\quad \times \left(p_\varphi(\dot{\varphi} + 2(kp)) \right. \\
 &\quad \left. + p_{\varphi_1}(\dot{\varphi}_1 + 2k_1 p) + p_{\varphi_2}(\dot{\varphi}_2 + 2k_2 p) \right. \\
 &\quad \left. + p_{\xi_a} \left(\xi_a - \frac{1}{2} k\theta \right) - iY\dot{Y} \right. \\
 &\quad \left. + \chi \left[-\frac{1}{2} (p\theta) + \frac{i}{4} g (t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \right. \right. \\
 &\quad \left. \left. + \frac{i}{4} g \xi (b\varphi_1 + c\varphi_2) + \frac{i}{4} g (\theta\xi) \right. \right. \\
 &\quad \left. \left. \times (b\varphi_1 + c\varphi_2) (t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) + M\theta^5 \right] \right) \\
 &\quad \left. + Y(\lambda) Y(0) \right] \Bigg|_{\theta=\vartheta=0}.
 \end{aligned} \tag{45}$$

The integrations over p_{ξ_a} give the following result:

$$\xi_a - \frac{1}{2} (k\theta) = 0, \quad \text{that is, } \xi_a = \xi_b = \frac{1}{2} (k\theta), \tag{46}$$

and the Green function is reduced to

$$\begin{aligned}
 \bar{S}(x_b, x_a) &= \exp\left(i\gamma \frac{\delta_L}{\delta\theta}\right) \exp\left(i\Gamma \frac{\delta_L}{\delta\vartheta}\right) \\
 &\times \int_0^\infty d\lambda \int d\chi \int \frac{d^4 p}{(2\pi)^4}
 \end{aligned}$$

$$\begin{aligned}
& \times \exp \{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
& \times \int d\varphi_b \int d\varphi_a \delta(\varphi_a - kx_a) \int D\varphi \int Dp_\varphi \\
& \times \prod_{i=1}^2 \int d\varphi_{ib} \int d\varphi_{ia} \delta(\varphi_{ia} - k_i x_a) \\
& \times \int D\varphi_i \int Dp_{\varphi_i} \int d\xi_a \int D\xi \int_{E_2} DY \\
& \times \exp \left[i \int_0^\lambda d\tau \right. \\
& \quad \times \left(p_\varphi (\dot{\varphi} + 2kp) \right. \\
& \quad + p_{\varphi_1} (\dot{\varphi}_1 + 2k_1 p) \\
& \quad + p_{\varphi_2} (\dot{\varphi}_2 + 2k_2 p) - i\Upsilon \dot{\Upsilon} \\
& \quad + \chi \left[-\frac{1}{2} (p\theta) + \frac{i}{4} g (t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \right. \\
& \quad \left. + \frac{i}{4} g \xi (b\varphi_1 + c\varphi_2) + \frac{i}{4} g (\theta\xi) \right. \\
& \quad \left. \left. \times (b\varphi_1 + c\varphi_2) (t_\beta^\alpha \Upsilon_\alpha \Upsilon_\beta) + M\theta^5 \right] \right] \\
& \quad \left. + \Upsilon(\lambda) \Upsilon(0) \right] \Big|_{\theta=\vartheta=0}.
\end{aligned} \tag{47}$$

By integrations on p_φ , p_{φ_1} , and p_{φ_2} again, three Dirac functions appear also

$$\delta(\dot{\varphi} + 2(kp)), \quad \delta(\dot{\varphi}_i + 2(k_i p)), \quad (i = 1, 2); \tag{48}$$

that is, the equations

$$\frac{d\varphi}{d\tau} = -2(kp), \quad \frac{d\varphi_i}{d\tau} = -2(k_i p), \tag{49}$$

for solutions

$$\begin{aligned}
\varphi(\tau) &= -2(kp)\tau + c^{te}, \\
\varphi_i(\tau) &= -2(k_i p)\tau + c^{te}
\end{aligned} \tag{50}$$

contribute to the determination of the Green function.

Now, we introduce the integral representations of three δ :

$$\begin{aligned}
& \delta(\varphi_a - kx_a - (kp)\lambda) \\
&= \frac{1}{2\pi} \int dp_\varphi \\
& \quad \times \exp \{ip_\varphi (\varphi_a - kx_a - (kp)\lambda)\}, \\
& \delta(\varphi_{ia} - k_i x_a - (k_i p)\lambda) \\
&= \frac{1}{2\pi} \int dp_{\varphi_i} \\
& \quad \times \exp \{ip_{\varphi_i} (\varphi_{ia} - k_i x_a - (k_i p)\lambda)\}.
\end{aligned} \tag{51}$$

After having changed p into $p - kp_{\varphi_b} - k_1 p_{\varphi_{1b}} - k_2 p_{\varphi_{2b}}$, the relations (22) are reused to reintroduce the generators Γ_α of $SU(2)$ group since there is no order problem; the T_2 -product is omitted, then the Green function $\tilde{S}$ symmetrized with respect to positions x_a and x_b takes the following form:

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
&= \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \int_0^\infty d\lambda \int d\chi \\
& \quad \times \int \frac{d^4 p}{(2\pi)^4} \exp \{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
& \quad \times \exp \left[i \int_{\varphi_a}^{\varphi_b} d\varphi(k\theta) \right. \\
& \quad \times \left(-\frac{ig}{8(kp)} (kp) \right. \\
& \quad \times \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) - \frac{1}{4(kp)^2} \\
& \quad \times \frac{p^2}{(1 - (1/2)\chi)} \chi \\
& \quad \left. - \frac{ig}{16} \left(\frac{d\Gamma_\alpha t_\beta^\alpha \Gamma_\beta}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \chi \right) \\
& \quad \left. - \frac{i}{2} ((p\theta) + M\theta^5) \chi \right. \\
& \quad \left. + \frac{(k\theta)}{(kp)} \left(-\frac{1}{32} \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \right. \right. \\
& \quad \times (k\theta) \chi + \frac{k^2}{128} \int_{\varphi_a}^{\varphi_b} d\varphi \frac{p^2}{(kp)} \\
& \quad \times \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right)^2 + \frac{ig}{16} \\
& \quad \times \int_{\varphi_a}^{\varphi_b} d\varphi \frac{p^2}{(kp)} \\
& \quad \times \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \chi \\
& \quad \left. + \frac{g}{64} \times \int_{\varphi_a}^{\varphi_b} d\varphi \frac{1}{(kp)} \right]
\end{aligned}$$

$$\begin{aligned}
& \times \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right)^2 \chi \Bigg) \\
& + \frac{g^2}{64} \int_{\varphi_{1a}}^{\varphi_{1b}} d\varphi_1 \\
& \times \int_{\varphi_{2a}}^{\varphi_{2b}} d\varphi_2 \frac{1}{(kp)} (b\varphi_1 + c\varphi_2) (k\theta) (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \Bigg] \Bigg|_{\theta=\vartheta=0}. \quad (52)
\end{aligned}$$

The variable χ [16, 19] can be eliminated by integration, and we obtain

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
& = \exp \left(i\gamma \frac{\delta_L}{\delta\theta} \right) \int_0^\infty d\lambda \\
& \times \int \frac{d^4 p}{(2\pi)^4} \exp \{ ip(x_b - x_a) + i\lambda(p^2 - M^2) \} \\
& \times \left(\frac{1}{2} ((p\theta) + M\theta^5) \right) \\
& \times \exp \left[-\frac{g}{16} \left((k\theta) + i \frac{(k\theta)}{(kp)^2} p^2 + \frac{g}{8} (k\theta) \right) \right. \\
& \times \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \\
& + \frac{1}{32} \frac{(k\theta)}{(kp)} \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \\
& \times (k \cdot \vartheta) + \frac{(k\theta)}{(kp)} \left(-\frac{g}{64(kp)} + \frac{(kp)}{128} \right) \\
& \times \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right)^2 + \frac{g^2}{64} \frac{(k\theta)}{(kp)} \\
& \times \int_{\varphi_{1a}}^{\varphi_{1b}} d\varphi_1 \\
& \times \int_{\varphi_{2a}}^{\varphi_{2b}} d\varphi_2 (b\varphi_1 + c\varphi_2) (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \Bigg] \Bigg|_{\theta=\vartheta=0}, \quad (53)
\end{aligned}$$

where the calculations have been simplified thanks to proprieties of field.

The matrices γ^μ are reintroduced by using the following relations:

$$\begin{aligned}
& \exp \left(i \left(\gamma \frac{\delta_L}{\delta\theta} \right) \right) f(\theta) \Big|_{\theta=0} \\
& = f \left(\frac{\partial_L}{\partial\theta} \right) \exp(i(\gamma\theta)) \Big|_{\theta=0}, \\
& \frac{\partial_L^2}{\partial\theta^\alpha \partial\theta^\beta} \theta_\mu \theta_\nu \gamma^\mu \gamma^\nu = -2i\sigma_{\alpha\beta}, \\
& \exp(i(\gamma\theta)) = 1 + i(\gamma\theta) \\
& - \frac{1}{2} \theta_\mu \theta_\nu \gamma^\mu \gamma^\nu \\
& + \frac{i}{6} \theta_\mu \theta_\nu \theta_\sigma \gamma^\mu \gamma^\nu \gamma^\sigma \\
& + \theta_0 \theta_1 \theta_2 \theta_3 \gamma^5, \quad (54)
\end{aligned}$$

where

$$\begin{aligned}
& f(\theta) \\
& = \frac{1}{2} ((p\theta) + M\theta^5) \\
& \times \exp \left\{ -\frac{g}{16} \left((k\theta) + i \frac{(k\theta)}{(kp)^2} p^2 + \frac{g}{8} (k\theta) \right) \right. \\
& \times \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \\
& + \frac{1}{32} \frac{(k\theta)}{(kp)} \\
& \times \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right) \\
& \times (k \cdot \vartheta) + \frac{(k\theta)}{(kp)} \left(-\frac{g}{64(kp)} + \frac{(kp)}{128} \right) \\
& \times \int_{\varphi_a}^{\varphi_b} d\varphi \left(\frac{d(\Gamma_\alpha t_\beta^\alpha \Gamma_\beta)}{d\varphi} (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \right)^2 \\
& + \frac{g^2}{64} \frac{(k\theta)}{(kp)} \int_{\varphi_{1a}}^{\varphi_{1b}} d\varphi_1 \\
& \times \int_{\varphi_{2a}}^{\varphi_{2b}} d\varphi_2 (b\varphi_1 + c\varphi_2) (\Gamma_\alpha t_\beta^\alpha \Gamma_\beta) \Bigg\}. \quad (55)
\end{aligned}$$

Then

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
&= \int_0^\infty d\lambda \\
&\times \int \frac{d^4 p}{(2\pi)^4} \exp \{ip(x_b - x_a) + i\lambda(p^2 - M^2)\} \\
&\times \exp \left\{ \frac{i}{(kp)} \right. \\
&\quad \times \left(\frac{g(p\sigma k)}{64(kp)} \right. \\
&\quad \times \left. \left. \int_{kx_a}^{kx_b} d\varphi \left(b \frac{k_1}{k} + c \frac{k_2}{k} \right) \varphi \frac{dm(\varphi)}{d\varphi} \right) \right\}, \tag{56}
\end{aligned}$$

and after having eliminated λ by integration, we have

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
&= \frac{1}{(2\pi)^4} \\
&\times \int \frac{d^4 p (-\gamma p + M)}{p^2 - M^2 + i0} \exp \{ip(x_b - x_a)\} \\
&\times \exp \left\{ \frac{i}{(kp)} \right. \\
&\quad \times \left(\frac{g(p\sigma k)}{64(kp)} \right. \\
&\quad \times \left. \left. \int_{kx_a}^{kx_b} d\varphi \left(b \frac{k_1}{k} + c \frac{k_2}{k} \right) \varphi \frac{dm(\varphi)}{d\varphi} \right) \right\}, \tag{57}
\end{aligned}$$

and an integration by part of the exponent of exponential $\tilde{S}(x_b, x_a)$ has the following form:

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
&= \frac{1}{(2\pi)^4} \int \frac{d^4 p (-\gamma p + M)}{p^2 - M^2 + i0^+} \exp \{ip(x_b - x_a)\} \\
&\times \exp \left\{ \frac{ig}{64(kp)^2} (p\sigma k) \right.
\end{aligned}$$

$$\begin{aligned}
& \times \left[\left(\left(\frac{k_1}{k} b(\varphi_b) + \frac{k_2}{k} c(\varphi_b) \right) \varphi_b m(\varphi_b) \right. \right. \\
&\quad \left. \left. - \left(\frac{k_1}{k} b(\varphi_a) + \frac{k_2}{k} c(\varphi_a) \right) \varphi_a m(\varphi_a) \right) \right. \\
&\quad \left. - \int_{kx_a}^{kx_b} d\varphi \right. \\
&\quad \times \left(\frac{k_1}{k} (b(\varphi) + \varphi b'(\varphi)) \right. \\
&\quad \left. \left. + \frac{k_2}{k} (c(\varphi) + \varphi c'(\varphi)) \right) m(\varphi) \right] \Bigg\}, \tag{58}
\end{aligned}$$

and after some manipulations, the Green function for the Dirac particles moving in the non-Abelian field in the local approach is finally

$$\begin{aligned}
& \tilde{S}(x_b, x_a) \\
&= \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 - M^2 + i0^+} \exp \{ip(x_b - x_a)\} \\
&\times \frac{g}{64(kp)^2} \\
&\times \left[\left(1 + (p\sigma k) \left(\frac{k_1}{k} b(\varphi_b) + \frac{k_2}{k} c(\varphi_b) \right) \varphi_b m(\varphi_b) \right) \right. \\
&\quad \times (-\gamma p + M) \\
&\quad \times \left(1 - (p\sigma k) \left(\frac{k_1}{k} b(\varphi_a) + \frac{k_2}{k} c(\varphi_a) \right) \varphi_a m(\varphi_a) \right) \Bigg] \\
&\times \exp \left\{ i \left[p(x_b - x_a) - \frac{g(p\sigma k)}{64(kp)^2} \right. \right. \\
&\quad \times \left(\int_{kx_a}^{kx_b} d\varphi \right. \\
&\quad \times \left[\frac{k_1}{k} (b(\varphi) + \varphi b'(\varphi)) + \frac{k_2}{k} \right. \\
&\quad \times \left. \left. \left. (c(\varphi) + \varphi c'(\varphi)) \right] m(\varphi) \right) \right] \Bigg\}, \tag{59}
\end{aligned}$$

which is obviously symmetrical with respect to x_a and x_b .
From the residues related to two poles

$$p_\pm^0 = \pm\omega - i0^+ = \pm(\mathbf{p}^2 + M^2)^{-1/2} - i0^+, \tag{60}$$

and after having introduced the projectors on states of positive and negative energy [20],

$$\begin{aligned}\Lambda_+ &= \sum_{\pm s} u(p, s) \bar{u}(p, s) = \frac{\gamma P + M}{2M}, \\ \Lambda_- &= \sum_{\pm s} v(p, s) \bar{v}(p, s) = \frac{-\gamma P + M}{2M},\end{aligned}\quad (61)$$

we obtain the following spectral decomposition:

$$\begin{aligned}S(x_b, x_a) &= -i\theta(t_b - t_a) \\ &\times \int d^3p \sum_{\pm s} \Psi_{s,p}^+(x_b) \bar{\Psi}_{s,p}^+(x_a) \\ &+ i\theta(t_b - t_a) \int d^3p \\ &\times \sum_{\pm s} \Psi_{s,p}^-(x_b) \bar{\Psi}_{s,p}^-(x_a),\end{aligned}\quad (62)$$

where finally the wave functions related to particle and to antiparticle are, respectively,

$$\begin{aligned}\Psi_{s,p}^+(x) &= \frac{1}{(2\pi)^{3/2}} \left(\frac{M}{p^0} \right)^{1/2} \\ &\times \left(1 + (p\sigma k) \right. \\ &\times \left(\frac{k_1}{k} b(kx) + \frac{k_2}{k} c(kx) \right) \varphi_b m(kx) \Big) u(p, s) \\ &\times \exp \left\{ i \left[px - \frac{g(p\sigma k)}{64(kp)^2} \right. \right. \\ &\times \left[\int^{kx} d\varphi \right. \\ &\times \left(\frac{k_1}{k} (b(\varphi) + \varphi b'(\varphi)) \right. \\ &\quad \left. \left. + \frac{k_2}{k} (c(\varphi) + \varphi c'(\varphi)) \right) \right. \\ &\quad \left. \left. \times m(\varphi) \right] \right\},\end{aligned}$$

$$\begin{aligned}\Psi_{s,p}^-(x) &= \frac{1}{(2\pi)^{3/2}} \left(\frac{M}{p^0} \right)^{1/2} \\ &\times \left(1 - (p\sigma k) \right. \\ &\times \left(\frac{k_1}{k} b(kx) + \frac{k_2}{k} c(kx) \right) \\ &\times \varphi_a m(kx) \Big) v(p, s) \\ &\times \exp \left\{ -i \left[px + \frac{g(p\sigma k)}{64(k \cdot p)^2} \right. \right. \\ &\times \left[\int^{kx} d\varphi \right. \\ &\times \left(\frac{k_1}{k} (b(\varphi) + \varphi b'(\varphi)) \right. \\ &\quad \left. \left. + \frac{k_2}{k} (c(\varphi) + \varphi c'(\varphi)) \right) m(\varphi) \right] \right\}.\end{aligned}\quad (63)$$

$u(p, s)$ and $v(p, s)$ are the spinors solutions of the free Dirac equation and are

$$\begin{aligned}\bar{u}(p, s) u(p, s) &= 1, \\ \bar{v}(p, s) v(p, s) &= -1.\end{aligned}\quad (64)$$

3. Conclusion

In the present paper, the Green function related to Dirac particle moving in a non-abelian field has been determined in the local approach of path integral. The obtained wave functions from the spectral decomposition are in accordance with those obtained by direct resolution of Dirac equation [1].

Using only shifts related to properties of field, the integrations have been easily performed, and it is remarkable to notice that the selected equations by arguments of δ functions which are appeared during calculation can be also obtained from the classical mechanics (cf. Appendix).

Appendix

In local approach, the action is

$$\begin{aligned}A &= \int_0^\lambda d\tau \\ &\times \left[p\dot{x} + p^2 - M^2 - \frac{i}{16} \right. \\ &\times (p\Psi)(k\Psi) (t_\beta^\alpha \Upsilon'_\alpha \Upsilon_\beta) \\ &\quad \left. - i \frac{g}{4} (p\Psi)(k\Psi) \right]\end{aligned}$$

$$\begin{aligned}
& \times (b(k_1 x) + c(k_2 x)) (t_\beta^\alpha \gamma_\alpha \gamma_\beta) \\
& + \chi \left(-(p\Psi) + \frac{i}{32} (k\Psi) (\Psi (t_\beta^\alpha \gamma'_\alpha \gamma_\beta)) \right. \\
& \quad \left. + i \frac{g}{8} (k\Psi) (b(k_1 x) + c(k_2 x)) \right. \\
& \quad \left. \times (t_\beta^\alpha \gamma_\alpha \gamma_\beta) - M\Psi^5 \right) - i\Psi\dot{\Psi} - i\dot{\Psi}\Psi \Big], \\
\end{aligned} \tag{A.1}$$

and the equations of movement are, respectively,

$$\begin{aligned}
\frac{\delta A}{\delta x^\mu(s)} = 0 : \dot{p} &= -(k\Psi) \\
& \times \left\{ -i \frac{g}{4} (k \cdot \Psi) \right. \\
& \quad \times \left[(k_1 b'(k_1 x) + k_2 c'(k_2 x)) \right. \\
& \quad \times (t_\beta^\alpha \gamma_\alpha \gamma_\beta) \\
& \quad + (b(k_1 x) + c(k_2 x)) \\
& \quad \times (t_\beta^\alpha k \gamma'_\alpha \gamma_\beta + t_\beta^\alpha \gamma_\alpha k \gamma'_\beta) \Big] \\
& + \chi \left(\frac{i}{32} ((k\Psi) (t_\beta^\alpha k \gamma''_\alpha \gamma_\beta + t_\beta^\alpha \gamma'_\alpha k \gamma'_\beta)) + i \frac{g}{8} k \right. \\
& \quad \times \left[(k_1 b'(k_1 x) + k_2 c'(k_2 x)) \right. \\
& \quad \times (t_\beta^\alpha \gamma_\alpha \gamma_\beta) \\
& \quad + (k_1 b(k_1 x) + k_2 c(k_2 x)) \\
& \quad \times (t_\beta^\alpha k \gamma'_\alpha \gamma_\beta + t_\beta^\alpha \gamma_\alpha k \gamma'_\beta) \Big] \Big\}, \\
\frac{\delta A}{\delta p^\mu(s)} = 0 : \dot{x} &= -2p \\
& + i \frac{g}{4} \Psi (k\Psi) \\
& \times (b(k_1 x) + c(k_2 x)) (t_\beta^\alpha \gamma'_\alpha \gamma_\beta). \\
\frac{\delta A}{\delta \Psi^\mu} = 0 : \dot{\Psi} &= \frac{1}{16} \Psi (k\Psi) \\
& \times (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) + \frac{g}{8} \Psi (k\Psi)
\end{aligned}$$

$$\times (b(k_1 x) + c(k_2 x)) (t_\beta^\alpha \gamma_\alpha \gamma_\beta),$$

$$\dot{\Psi}^5 = iM\chi : \Psi^5 = iM\tau\chi + \text{const.},$$

$$\frac{\delta A}{\delta Y^\mu} = 0 : \dot{Y} = \frac{1}{16} (p\Psi) (k\Psi)$$

$$\begin{aligned}
& \times (t_\beta^\alpha \gamma'_\alpha \gamma_\beta) + \frac{g}{4} (p\Psi) (k\Psi) \\
& \times (b(k_1 x) + c(k_2 x)) (t_\beta^\alpha \gamma_\alpha \gamma_\beta).
\end{aligned} \tag{A.2}$$

After multiplication by k_μ and k_i ($i = 1, 2$), we obtain the following relations:

$$\begin{aligned}
k\dot{p} &= 0, \quad kp = \text{const.} \\
\dot{\phi} &= k\dot{x} = -2kp, \\
\dot{\phi}_i &= k_i \dot{x} = -2k_i p, \\
k\dot{\Psi} &= 0.
\end{aligned} \tag{A.3}$$

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