Letter to the Editor

Comment on “Adomian Decomposition Method for a Class of Nonlinear Problems”

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Sánchez Cano in his paper “Adomian Decomposition Method for a Class of Nonlinear Problems” in application part pages 8, 9, and 10 had made some mistakes in context; in this paper we correct them.

1. Introduction

Adomian [1, 2] proposed a powerful method for solving nonlinear functional equation. The technique uses a decomposition of the nonlinear operator as a series of functions; each term of this series is a generalized polynomial called Adomian polynomial.

We will see that using the Adomian decomposition method together with some properties of the nested integral [3, 4] the solution of nonlinear ordinary differential equations system is obtained.

2. Correct Equations

In page 8, he showed that the solution $x_n(t)$ is given by

$$\begin{aligned} x(t) &= x_0(t) + x_1(t) + x_2(t) + \cdots + x_n(t) + \cdots \\ &= \alpha + \int_0^t f(u) du + a\beta t + \left[-a^2 \propto \frac{t^2}{2!} - a^2 \frac{1}{\Gamma(3)} \right. \\ &\quad \left. \times \int_0^t (t-u)^2 f(u) du \right] \end{aligned}$$

$$\begin{aligned} -a^3 \beta \frac{t^3}{3!} + \left[a^4 \propto \frac{t^4}{4!} + a^4 \frac{1}{\Gamma(5)} \right. \\ \left. \times \int_0^t (t-u)^4 f(u) du \right] + \cdots \end{aligned} \quad (1)$$

By rearranging he obtained

$$\begin{aligned} x(t) &= \alpha \left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \cdots (-1)^n \frac{(at)^{2n}}{2n!} + \cdots \right] \\ &+ \beta \left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} \right. \\ &\quad \left. + \cdots (-1)^n \frac{(at)^{2n+1}}{(2n+1)!} + \cdots \right] \\ &+ \int_0^t \left[\frac{(a(t-u))^0}{\Gamma(1)} - \frac{(a(t-u))^2}{\Gamma(3)} \right. \\ &\quad \left. + \cdots (-1)^n \frac{(a(t-u))^{2n}}{\Gamma(2n+1)} + \cdots \right] f(u) du. \end{aligned} \quad (2)$$

And similarly for $y_n(t)$ we will have

$$\begin{aligned} y(t) &= y_0(t) + y_1(t) + y_2(t) + \cdots \\ &+ y_n(t) + \cdots = \beta - \alpha at - a \frac{1}{\Gamma(2)} \\ &\times \int_0^t (t-u) f(u) du \\ &- a^2 \beta \frac{t^2}{2!} + a^3 \alpha \frac{t^3}{3!} + a^3 \frac{1}{\Gamma(4)} \\ &\times \int_0^t (t-u)^3 f(u) du + a^4 \beta \frac{t^4}{4!} + \cdots \end{aligned} \quad (3)$$

By rearranging he obtained

$$\begin{aligned} y(t) &= \beta \left[1 - \frac{(at)^2}{2!} + \frac{(at)^4}{4!} + \cdots (-1)^n \frac{(at)^{2n}}{2n!} + \cdots \right] \\ &- \alpha \left[(at) - \frac{(at)^3}{3!} + \frac{(at)^5}{5!} + \cdots (-1)^n \frac{(at)^{2n+1}}{(2n+1)!} + \cdots \right] \\ &- \int_0^t \left[\frac{(a(t-u))^1}{\Gamma(2)} - \frac{(a(t-u))^3}{\Gamma(4)} \right. \\ &\quad \left. + \cdots (-1)^n \frac{(a(t-u))^{2n+1}}{\Gamma(2n)} + \cdots \right] f(u) du. \end{aligned} \quad (4)$$

Continuing in this fashion, he concluded the following formulas:

$$\begin{aligned} x_n(t) &= (-1)^n \left[a^n a \frac{t^n}{n!} + a^n \frac{1}{\Gamma(n+1)} \int_0^t (t-u)^n f(u) du \right], \\ y_n(t) &= (-1)^{n+1} \left[a^n a \frac{t^n}{n!} + a^n \frac{1}{\Gamma(n+1)} \int_0^t (t-u)^n f(u) du \right]. \end{aligned} \quad (5)$$

But the correct formulas are given by

$$\begin{aligned} x_{2n-1}(t) &= (-1)^{n+1} \left(\frac{a^{2n-1} \beta t^{2n-1}}{(2n-1)!} \right), \\ x_{2n}(t) &= (-1)^n \left[\frac{a^{2n} \alpha t^{2n}}{(2n)!} + a^n \frac{1}{\Gamma(2n+1)} \right. \\ &\quad \left. \times \int_0^t (t-u)^{2n} f(u) du \right], \\ y_{2n-1}(t) &= (-1)^n \left[\frac{a^{2n-1} \alpha t^{2n-1}}{(2n-1)!} + a^n \frac{1}{\Gamma(2n)} \right. \\ &\quad \left. \times \int_0^t (t-u)^{2n-1} f(u) du \right], \\ y_{2n}(t) &= (-1)^n \left(\frac{a^{2n} \beta t^{2n}}{(2n)!} \right). \end{aligned} \quad (6)$$

Writing (2) and (4) as a single integral, he had

$$\begin{aligned} x(t) &= \alpha \cos(at) + \beta \sin(at) + \int_0^t \cos(a(t-u)) f(u) du \\ &= \alpha \cos(at) + \beta \sin(at) + f(t) * \cos(at). \end{aligned} \quad (7)$$

Similarly,

$$\begin{aligned} y(t) &= \beta \cos(at) - \alpha \sin(at) - \int_0^t \sin(a(t-u)) f(u) du \\ &= \beta \cos(at) - \alpha \sin(at) + f(t) * \sin(at). \end{aligned} \quad (8)$$

In page 9, he uses

$$\begin{aligned} \frac{dx}{dt} - ay(t) &= f(t), & \frac{dy}{dt} + ax(t) &= 0 \\ (a \in \mathbb{R}, a \neq 0). \end{aligned} \quad (9)$$

With $x(0) = \alpha$ and $y(0) = \beta$, he obtained

$$\begin{aligned} \frac{d^2 y}{dt^2} + a^2 y(t) &= f(t), \\ y(0) &= \alpha, & \dot{y}(0) &= \beta. \end{aligned} \quad (10)$$

In fact, the correct solution is given by

$$\begin{aligned} \frac{dx}{dt} - ay(t) &= f(t) \longrightarrow \frac{dx}{dt} = ay(t) + f(t), \\ \frac{dy}{dt} + ax(t) &= 0 \longrightarrow \frac{d^2 y}{dt^2} + a \frac{dx}{dt} = 0, \\ &\Downarrow \\ \frac{d^2 y}{dt^2} + a^2 y(t) &= -af(t), \end{aligned} \quad (11)$$

$$y(0) = \beta, \quad \dot{y}(0) = \alpha.$$

In page 10, he showed two cases for the solutions $x(t)$ and $y(t)$ using (2), (4), and $f(t) = \cos(t)$.

Case 1 ($a \neq 1$). In this case, he obtained the solutions

$$\begin{aligned} x(t) &= \alpha \cos(at) + \beta \sin(at) + \frac{1}{1-a^2} \sin(t), \\ y(t) &= \beta \cos(at) - \alpha \sin(at) + \frac{a}{1+a^2} \cos(t). \end{aligned} \quad (12)$$

But the correct method is the following:

$$\begin{aligned} x(t) &= \alpha \cos(at) + \beta \sin(at) + \int_0^t \cos(a(t-u)) f(u) du \\ &\longrightarrow \int_0^t \cos(a(t-u)) f(u) du \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \left[\int_0^t \cos(at - au + u) \cos(at - au - u) du \right] \\
&= \left[\frac{1}{2(1-a)} \sin(at - au + u) \right. \\
&\quad \left. + \frac{1}{2(-1-a)} \sin(at - au - u) \right]_0^t \\
&= \frac{1}{1-a^2} \sin t - \frac{a}{1-a^2} \sin at \\
&\longrightarrow x(t) = \alpha \cos(at) + \beta \sin(at) + \frac{1}{1-a^2} \sin t \\
&\quad - \frac{a}{1-a^2} \sin at, \\
x(t) &= \alpha \cos at + \left(\beta - \frac{a}{1-a^2} \right) \sin at + \frac{1}{1-a^2} \sin t.
\end{aligned} \tag{13}$$

Similarly,

$$y(t) = \left(\beta - \frac{a}{1-a^2} \right) \cos at - \alpha \sin at + \frac{a}{1-a^2} \cos t. \tag{14}$$

Case 2 ($a = 1$). He arrived at the formulas $x(t)$ and $y(t)$ as follows:

$$\begin{aligned}
x(t) &= \alpha \cos(t) + \beta \sin(t) + \frac{1}{2}t \cos t, \\
y(t) &= \left(\alpha - \frac{1}{2} \right) \cos t + \beta \sin t - \frac{1}{2}t \sin t.
\end{aligned} \tag{15}$$

But the correct formulas are given by

$$\begin{aligned}
x(t) &= \alpha \cos(t) + \beta \sin(t) + \int_0^t \cos((t-u)) f(u) du \\
&\longrightarrow \int_0^t \cos((t-u)) \cos u du \\
&= \frac{1}{2} \left[\int_0^t \cos t + \cos(t-2u) du \right] \\
&= \frac{1}{2} \left[t \cos t - \frac{1}{2} \sin(t-2u) \right]_0^t \\
&= \frac{1}{2}t \cos t + \frac{1}{2} \sin t \\
&\implies x(t) = \alpha \cos t + \beta \sin t + \frac{1}{2}t \cos t + \frac{1}{2} \sin t \\
&\implies x(t) = \alpha \cos t + \left(\beta + \frac{1}{2} \right) \sin t + \frac{1}{2}t \cos t.
\end{aligned} \tag{16}$$

Similarly,

$$y(t) = \beta \cos t - \alpha \sin t - \frac{1}{2}t \sin t. \tag{17}$$

References

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