Research Article

Flow of an Eyring-Powell Model Fluid between Coaxial Cylinders with Variable Viscosity

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We consider the flow of Eyring-Powell model fluid in the annulus between two cylinders whose viscosity depends upon the temperature. We consider the steady flow in the annulus due to the motion of inner cylinder and constant pressure gradient. In the problem considered the flow is found to be remarkably different from that for the incompressible Navier-Stokes fluid with constant viscosity. An analytical solution of the nonlinear problem is obtained using homotopy analysis method. The behavior of pertinent parameters is analyzed and depicted through graphs.

1. Introduction

The analysis of the behaviour of the fluid motion of the non-Newtonian fluids becomes much complicated and subtle as compared to Newtonian fluids due to the fact that non-Newtonian fluids do not exhibit the linear relationship between stress and strain. Rivlin and Ericksen [1] and Truesdell and Noll [2] classified viscoelastic fluids with the help of constitutive relations for the stress tensor as a function of the symmetric part of the velocity gradient and its higher (total) derivatives. In recent years, there have been several studies [3–12] on flows of non-Newtonian fluids. It is a well-known fact that it is not possible to obtain a single constitutive equation exhibiting all properties of all non-Newtonian fluids from the available literature. That is why several models of non-Newtonian fluids have been proposed in the literature. Eyring-Powell model fluid is one of these models. Eyring-Powell model was first introduced by Powell and Eyring in 1944. However, the literature survey indicates that very low energy has been devoted to the flows of Eyring-Powell model fluid with variable viscosity. Massoudi and Christie [13] have considered the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a uniform pipe. Massoudi and Christie [13] found the numerical solutions with the help of straight forward finite

difference method. They also discussed that the flow of a fluid-solid mixture is very complicated and may depend on many variables such as physical properties of each phase and size and shape of solid particles. Later on, the influence of constant and space dependent viscosity on the flow of a third grade fluid in a pipe has been discussed analytically by Hayat et al. [14]. The approximate and analytical solution of non-Newtonian fluid with variable viscosity has been analyzed by Yürüsoy and Pakdemirli [15] and Pakdemirli and Yilbas [16]. The pipe flow of non-Newtonian fluid with variable viscosity keeping no slip and partial slip has been discussed analytically by Nadeem and Ali [17] and Nadeem et al. [18]. More recently, Nadeem and Akbar [19] studied the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a uniform vertical tube. The main aim of the present study is to venture further in the regime of Eyring-Powell model fluid with variable viscosity. To the best of the authors knowledge no attempt has been made to investigate Eyring-Powell model fluid in the annulus between two cylinders whose viscosity depends upon the temperature. The governing equations for Eyring-Powell model fluid are formulated considering cylindrical coordinates system. The equations are simplified using the assumptions of long wave length and low Reynolds number approximation. The obtained non-linear problem is solved

using homotopy analysis method [20–28]. The effects of the emerging parameters are analyzed and depicted through graphs.

2. Mathematical Model

The constitutive equation for a Cauchy stress in an Eyring-Powell model fluid is given by

$$\bar{\mathbf{S}} = \mu \nabla \bar{\mathbf{V}} + \frac{1}{\beta} \sinh^{-1} \left(\frac{1}{c} \nabla \bar{\mathbf{V}} \right), \quad (1)$$

$$\sinh^{-1} \left(\frac{1}{c} \nabla \bar{\mathbf{V}} \right) \approx \frac{1}{c} \nabla \bar{\mathbf{V}} - \frac{1}{6} \left(\frac{1}{c} \nabla \bar{\mathbf{V}} \right)^3, \quad \left| \frac{1}{c} \nabla \bar{\mathbf{V}} \right| \ll 1,$$

where $\bar{\mathbf{V}}$ is the velocity, $\bar{\mathbf{S}}$ is the Cauchy stress tensor, μ is the coefficient of shear viscosity, and β and c are the material constants. We take the velocity and stress as

$$\mathbf{V}(r) = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}, \quad \mathbf{S}(r) = \begin{bmatrix} S_{rr} & S_{r\theta} & S_{rz} \\ S_{\theta r} & S_{\theta\theta} & S_{\theta z} \\ S_{zr} & S_{z\theta} & S_{zz} \end{bmatrix}. \quad (2)$$

3. Physical Model

Consider the steady flow of an Eyring-Powell model fluid with variable temperature dependent viscosity between coaxial cylinders. The motion is caused due to a constant pressure gradient and by the motion of the inner cylinder parallel to its length, whereas the outer cylinder is kept stationary. The heat transfer analysis is also taken into account. The dimensionless problem which can describe the flow is

$$\begin{aligned} & \frac{\mu}{r} \frac{dv}{dr} + \frac{M}{r} \frac{dv}{dr} + \mu \frac{d^2 v}{dr^2} + M \frac{d^2 v}{dr^2} \\ & - 3K \left(\frac{dv}{dr} \right)^2 \frac{d^2 v}{dr^2} - \frac{K}{r} \left(\frac{dv}{dr} \right)^3 - B = 0, \\ & \mu \Gamma \left(\frac{dv}{dr} \right)^2 + M \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma K \left(\frac{dv}{dr} \right)^4 - \frac{1}{r} \frac{d\theta}{dr} - \frac{d^2 \theta}{dr^2} = 0, \end{aligned} \quad (3)$$

$$\begin{aligned} v(r) &= 1, \quad \theta(r) = 1, \quad r = 1, \\ v(r) &= 0, \quad \theta(r) = 0, \quad r = b, \end{aligned} \quad (4)$$

whence

$$\begin{aligned} r &= \frac{\bar{r}}{R}, \quad \Gamma = \frac{\mu_* V_0^2}{k(\theta_m - \theta_w)}, \quad \theta = \frac{(\bar{\theta} - \theta_w)}{(\theta_m - \theta_w)}, \\ z &= \frac{V_0^2}{R^2 c^2}, \quad C_1 = \frac{\partial p}{\partial z}, \quad M = \frac{1}{\beta c \mu_*}, \\ K &= \frac{V_0^2}{R^2 c^2}, \quad B = \frac{C_1 R^2}{\mu_* V_0}, \quad v = \frac{\bar{v}}{v_0}, \quad \mu = \frac{\bar{\mu}}{\mu_*}, \end{aligned} \quad (5)$$

where μ_* , θ_m , V_0 and Γ are, respectively, the reference viscosity, a reference temperature (the bulk mean fluid temperature), and reference velocity Γ is related to the Prandtl number and Eckert number.

4. Series Solutions for Reynolds' Model

Here the viscosity is expressed in the form

$$\mu = e^{-P\theta} \quad (6)$$

which by Maclaurin's series can be written as

$$\mu = 1 - P\theta + O(\theta^2). \quad (7)$$

Note that $M = 0$ corresponds to the case of constant viscosity. Invoking the above equation into (3) one has

$$\begin{aligned} & \frac{M}{r} \frac{dv}{dr} + \frac{1}{r} \frac{dv}{dr} - \frac{P\theta}{r} \frac{dv}{dr} + \frac{d^2 v}{dr^2} - P\theta \frac{d^2 v}{dr^2} \\ & + M \frac{d^2 v}{dr^2} - 3K \left(\frac{dv}{dr} \right)^2 \frac{d^2 v}{dr^2} - \frac{K}{r} \left(\frac{dv}{dr} \right)^3 - B = 0, \\ & \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma P\theta \left(\frac{dv}{dr} \right)^2 + M \Gamma \left(\frac{dv}{dr} \right)^2 \\ & - \Gamma K \left(\frac{dv}{dr} \right)^4 - \frac{1}{r} \frac{d\theta}{dr} - \frac{d^2 \theta}{dr^2} = 0. \end{aligned} \quad (8)$$

For HAM solution, we choose the following initial guesses:

$$v_0(r) = \frac{(r-b)}{(1-b)}, \quad \theta_0(r) = \frac{(r-b)}{(1-b)}. \quad (9)$$

The auxiliary linear operators are in the form

$$\mathcal{L}_{vr}(v) = v'', \quad (10)$$

$$\mathcal{L}_{\theta r}(\theta) = \theta'' \quad (11)$$

which satisfy

$$\mathcal{L}_{vr}(A_1 + B_1 r) = 0, \quad \mathcal{L}_{\theta r}(A_2 + B_2 r) = 0, \quad (12)$$

where A_1 , A_2 , B_1 , and B_2 are the constants.

If $p \in [0, 1]$ is an embedding parameter and h_v and h_θ are auxiliary parameters, then the problems at the zero and m th order are, respectively, given by

$$(1-p) \mathcal{L}_v [\bar{v}(r, p) - v_0(r)] = p h_v N_v [\bar{v}(r, p), \bar{\theta}(r, p)], \quad (13)$$

$$(1-p) \mathcal{L}_\theta [\bar{\theta}(r, p) - \theta_0(r)] = p h_\theta N_\theta [\bar{v}(r, p), \bar{\theta}(r, p)], \quad (14)$$

$$\mathcal{L}_v [v_m(r) - \chi_m v_{m-1}(r)] = h_v R_v(r), \quad (15)$$

$$\mathcal{L}_\theta [\theta_m(r) - \chi_m \theta_{m-1}(r)] = h_\theta R_\theta(r), \quad (16)$$

$$\bar{v}(r, p) = \bar{\theta}(r, p) = 1, \quad r = 1, \quad (17)$$

$$\bar{v}(r, p) = \bar{\theta}(r, p) = 0, \quad r = b. \quad (18)$$

The boundary conditions at the m th order are

$$\bar{v}_m(r, p) = \bar{\theta}_m(r, p) = 0, \quad r = 1, \quad (19)$$

$$\bar{v}_m(r, p) = \bar{\theta}_m(r, p) = 0, \quad r = b.$$

In (11)–(13)

$$N_v [\bar{v}(r, p), \bar{\theta}(r, p)] = -\frac{P\theta}{r} \frac{dv}{dr} + \frac{1}{r} \frac{dv}{dr} + \frac{M}{r} \frac{dv}{dr} + \frac{d^2v}{dr^2} - P\theta \frac{d^2v}{dr^2} + M \frac{d^2v}{dr^2} - 3K \left(\frac{dv}{dr} \right)^2 \frac{d^2v}{dr^2} - \frac{K}{r} \left(\frac{dv}{dr} \right)^3 - B, \quad (20)$$

$$N_\theta [\bar{v}(r, p), \bar{\theta}(r, p)] = \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma P \theta \left(\frac{dv}{dr} \right)^2 + M \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma K \left(\frac{dv}{dr} \right)^4 - \frac{1}{r} \frac{d\theta}{dr} - \frac{d^2\theta}{dr^2},$$

$$R_v = -\frac{P}{r} \sum_{k=0}^{m-1} v'_{m-1-k} \theta_k + \frac{1}{r} v'_{m-1} + \frac{M}{r} v'_{m-1} + v''_{m-1} - P \sum_{k=0}^{m-1} v''_{m-1-k} \theta_k + M v''_{m-1} - 3K \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} v''_l - \frac{K}{r} \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} v'_l - B, \quad (21)$$

$$R_\theta = \Gamma \sum_{k=0}^{m-1} v'_{m-1-k} v'_k - \Gamma P \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} \theta'_l + M \Gamma \sum_{k=0}^{m-1} v'_{m-1-k} v'_k - \Gamma K \sum_{k=0}^{m-1} \sum_{l=0}^k \sum_{s=0}^l v'_{m-1-k} v'_{k-l} v'_{l-s} v'_s - \frac{1}{r} \theta'_{m-1} - \theta''_{m-1}.$$

By Mathematica the solutions of (21) can be written as

$$v_m(r) = \sum_{n=0}^{3m} a_{m,n} r^n, \quad m \geq 0, \quad (22)$$

$$\theta_m(r) = \sum_{n=0}^{3m+1} d_{m,n} r^n, \quad m \geq 0,$$

where $a_{m,n}$ and $d_{m,n}$ are constants which can be determined on substituting (22) into (15) and (16).

5. Series Solutions for Vogel's Model

Here

$$\mu = \mu_* \exp \left[\frac{A}{(t + \theta)} - \theta_w \right] \quad (23)$$

FIGURE 1: h -curve for Reynolds' model for velocity profile.

FIGURE 2: h -curve for Reynolds' model for temperature profile.

FIGURE 3: h -curve for Vogel's model for velocity profile.

which by Maclaurin's series reduces to

$$\mu = \frac{B}{s} \left(1 - \frac{A\theta}{t^2} \right). \quad (24)$$

FIGURE 4: h -curve for Vogel's model for temperature profile. FIGURE 5: Temperature profile for Reynolds' model for different values of B . FIGURE 6: Velocity profile for Reynolds' model for different values of B . FIGURE 7: Velocity profile for Reynolds' model for different values of P . FIGURE 8: Temperature profile for Reynolds' model for different values of Γ .

Invoking the above expressions, (1) become

$$\begin{aligned}
 & -\frac{AB\theta}{rst^2} \frac{dv}{dr} + \frac{1}{r} \frac{dv}{dr} + \frac{M}{r} \frac{dv}{dr} + \frac{B}{s} \frac{d^2v}{dr^2} - \frac{AB}{st^2} \theta \frac{d^2v}{dr^2} \\
 & + M \frac{d^2v}{dr^2} - 3K \left(\frac{dv}{dr} \right)^2 \frac{d^2v}{dr^2} - \frac{K}{r} \left(\frac{dv}{dr} \right)^3 - B = 0, \\
 & \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma \frac{AB}{st^2} \theta \left(\frac{dv}{dr} \right)^2 + M \Gamma \left(\frac{dv}{dr} \right)^2 \\
 & - \Gamma K \left(\frac{dv}{dr} \right)^4 - \frac{1}{r} \frac{d\theta}{dr} - \frac{d^2\theta}{dr^2} = 0.
 \end{aligned} \tag{25}$$

FIGURE 9: Temperature profile for Vogel's model for different values of t . FIGURE 11: Temperature profile for Vogel's model for different values of Γ . FIGURE 10: Velocity profile for Vogel's model for different values of t .

With the following initial guesses and auxiliary linear operators

$$\begin{aligned} v_{0v}(r) &= \frac{(r-b)}{(1-b)}, & \theta_{0v}(r) &= \frac{(r-b)}{(1-b)}, \\ \mathcal{L}_{vv}(v) &= v'', & \mathcal{L}_{\theta v}(\theta) &= \theta'', \end{aligned} \quad (26)$$

the m th-order deformation problems are

$$\begin{aligned} \mathcal{L}_{vr} [v_m(r) - \chi_m v_{m-1}(r)] &= \hbar_v R_{vr}(r), \\ \mathcal{L}_{\theta r} [\theta_m(r) - \chi_m \theta_{m-1}(r)] &= \hbar_\theta R_{\theta r}(r), \\ N_v [\bar{v}(r, p), \bar{\theta}(r, p)] \\ &= -\frac{AB\theta}{rst^2} \frac{dv}{dr} + \frac{1}{r} \frac{dv}{dr} + \frac{M}{r} \frac{dv}{dr} + \frac{B}{s} \frac{d^2v}{dr^2} + M \frac{d^2v}{dr^2} \\ &\quad - \frac{AB}{st^2} \theta \frac{d^2v}{dr^2} - 3K \left(\frac{dv}{dr} \right)^2 \frac{d^2v}{dr^2} - \frac{K}{r} \left(\frac{dv}{dr} \right)^3 - B, \\ N_\theta [\bar{v}(r, p), \bar{\theta}(r, p)] \\ &= \Gamma \left(\frac{dv}{dr} \right)^2 - \Gamma \frac{AB}{st^2} \theta \left(\frac{dv}{dr} \right)^2 + M \Gamma \left(\frac{dv}{dr} \right)^2 \\ &\quad - \Gamma K \left(\frac{dv}{dr} \right)^4 - \frac{1}{r} \frac{d\theta}{dr} - \frac{d^2\theta}{dr^2}, \\ N_v [\bar{v}(r, p), \bar{\theta}(r, p)] \\ &= -\frac{AB}{rst^2} \sum_{k=0}^{m-1} v'_{m-1-k} \theta_k + \frac{1}{r} v'_{m-1} \\ &\quad + \frac{M}{r} v'_{m-1} + \frac{B}{s} v''_{m-1} \\ &\quad - \frac{AB}{st^2} \sum_{k=0}^{m-1} v''_{m-1-k} \theta_k + M v''_{m-1} \\ &\quad - 3K \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} v''_l \end{aligned}$$

FIGURE 12: Velocity profile for Vogel's model for different values of s .

FIGURE 13: Velocity profile for Vogel's model for different values of B .

$$\begin{aligned}
 & -\frac{K}{r} \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} v'_l - B, \\
 N_\theta [\bar{v}(r, p), \bar{\theta}(r, p)] \\
 & = \Gamma \sum_{k=0}^{m-1} v''_{m-1-k} v'_k \\
 & - \Gamma \frac{AB}{st^2} \sum_{k=0}^{m-1} \sum_{l=0}^k v'_{m-1-k} v'_{k-l} \theta'_l \\
 & + M\Gamma \sum_{k=0}^{m-1} v'_{m-1-k} v'_k \\
 & - \Gamma K \sum_{k=0}^{m-1} \sum_{l=0}^k \sum_{s=0}^l v'_{m-1-k} v'_{k-l} v'_{l-s} v'_s \\
 & - \frac{1}{r} \theta'_{m-1} - \theta''_{m-1}.
 \end{aligned} \tag{27}$$

The expressions of v_m and θ_m are finally given by

$$\begin{aligned}
 v_m(r) &= \sum_{n=0}^{3m} a'_{m,n} r^n, \quad m \geq 0, \\
 \theta_m(r) &= \sum_{n=0}^{3m+1} d'_{m,n} r^n, \quad m \geq 0.
 \end{aligned} \tag{28}$$

6. Graphical Results and Discussion

In order to report the convergence of the obtained series solutions and the effects of sundry parameters in the present

investigation we plotted Figures 1–13. Figures 1–4 are prepared to see the convergence region. Figures 1 and 2 correspond to Reynolds' model whereas Figures 3 and 4 relate to Vogel's model. Figure 5 shows the temperature variation for different values of B for Reynolds' model. It can be seen that temperature decreases as B increases. Figure 6 depicts the velocity variation for Reynolds' model for different values of B . Velocity also decreases as B increases. Figure 7 shows the velocity variation for different values of P for Reynolds' model. It can be seen that velocity increases as P increases. Figure 8 is plotted in order to see the temperature variation for Reynolds' model for different values of Γ ; it is depicted that temperature increases as Γ increases. Figures 9–13 correspond to Vogel's model. Figure 9 depicts temperature variation for Vogel's model for different values of t . It is seen that temperature increases as t increases. Figure 10 shows the velocity variation for Vogel's model for different values of t . It is observed that velocity decreases as t increases. Figure 11 is prepared to observe the temperature variation for Vogel's model for different values of Γ . It is observed that temperature decreases as Γ increases. Figure 12 is plotted to see the the velocity variation for Vogel's model for different values of s . It is observed that velocity decreases as s increases. Figure 13 depicts the velocity variation for Vogel's model for different values of B . It is observed that velocity decreases as B increases.

7. Conclusions

In this paper, we consider the flow of Eyring-Powell model fluid in the annulus between two cylinders whose viscosity depends upon the temperature. We discussed the steady flow in the annulus due to the motion of inner cylinder and constant pressure gradient. In the problem considered the flow is found to be remarkably different from that for the incompressible Navier-Stokes fluid with constant viscosity. The behavior of pertinent parameters is analyzed

and depicted through graphs. Using usual similarity transformations the governing equations have been transformed into non-linear ordinary differential equations. The highly non-linear problem is then solved by homotopy analysis method. Effects of the various parameters on velocity and temperature profiles are examined.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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