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Review Article

Two-Channel Quadrature Mirror Filter Bank: An Overview

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During the last two decades, there has been substantial progress in multirate digital filters and filter banks. This includes the design of quadrature mirror filters (QMF). A two-channel QMF bank is extensively used in many signal processing fields such as subband coding of speech signal, image processing, antenna systems, design of wavelet bases, and biomedical engineering and in digital audio industry. Therefore, new efficient design techniques are being proposed by several authors in this area. This paper presents an overview of analysis and design techniques of the two-channel QMF bank. Application in the area of subband coding and future research trends are also discussed.

1. Introduction

The concept of quadrature mirror filter (QMF) bank was first introduced by Croisier et al. [1] in 1976, and then Esteban and Galand [2] applied this filter bank in a voice coding scheme. QMF have been extensively used for splitting a signal into two or more subbands in the frequency domain, so that each subband signal can be processed in an independent manner and sufficient compression may be achieved. Eventually, at some point in the process, the subband signals are recombined so that the original signal is properly reconstructed [3]. These filters find applications in many signal processing fields, such as design of wavelet bases [4, 5], image compression [6, 7], digital transmultiplexers used in FDM/TDM conversion [8, 9], discrete multitone modulation systems [10], ECG signal compression [11, 12], antenna systems [13], digital audio industry [14], biomedical signal processing [15], equalization of wireless communication channels [16], and analog voice privacy systems [17], due to advancement in QMF bank.

In comparison to earlier band pass filter based subband coding systems, the QMF bank based systems have many advantages as given next.

(a) Aliasing distortion is eliminated in QMF bank based subband coding systems; therefore, the transition width of the filters is not much important. Lower order filters with wider transition band can be used [2].

(b) Computation complexity is reduced in case of subband coding system based on QMF banks [2].

(c) Lower bit rates are possible, without degrading the quality of decoded speech signals.

(d) QMF based subband coders [18, 19] provide more natural sounding, pitch prediction, and wider bandwidth than earlier subband coders.

Two-channel filter banks can be classified into three types: quadrature mirror filter banks, orthogonal filter banks, and biorthogonal filter banks [20]. These filter banks can be designed to have either the perfect reconstruction (PR) or nearly perfect reconstruction (NPR) property [21]. QMF banks and biorthogonal filter banks can be created with the use of either linear-phase or nonlinear-phase filters, whereas for orthogonal filter banks, nonlinear-phase filters are always used [22].

QMF filter sections may be cascaded in a tree structure to generate multichannel filter banks [23, 24]. There are two types of tree structures, namely, uniform and octave filter bank structures. In uniform M-channel filter bank (full-grown tree), at every level, the low-pass and the high-pass channels are divided into two parts, whereas, only the low-pass channel is divided into two parts, in a nonuniform octave filter bank.

A typical two-channel QMF bank shown in Figure 1, splits the discrete input signal $x(n)$ into two subband signals having

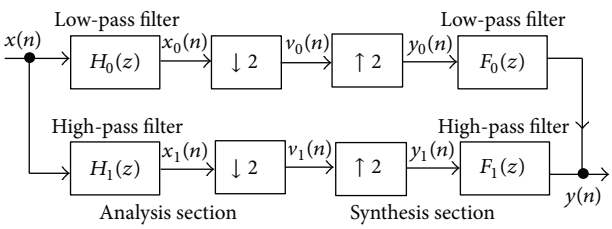

Figure 1: Two-channel quadrature mirror filter bank.

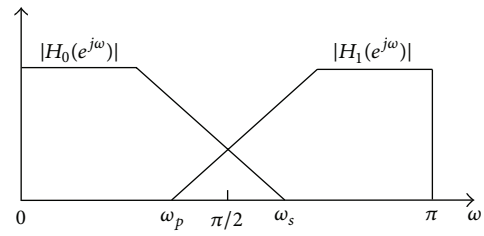

Figure 2: Frequency response of the analysis filters $H_0(z)$ and $H_1(z)$.

equal bandwidth, using the low-pass and high-pass analysis filters $H_0(z)$ and $H_1(z)$, respectively. These subband signals are decimated by a factor of two to achieve signal compression or to reduce processing complexity. The decimated signals are typically coded and transmitted. At the receiver, the two subband signals are decoded and then interpolated by a factor of two and finally passed through low-pass and high-pass synthesis filters, $F_0(z)$ and $F_1(z)$, respectively. The outputs of the synthesis filters are combined to obtain the reconstructed signal $y(n) = \hat{x}(n)$. The reconstructed signal $\hat{x}(n)$ suffers from three types of errors: aliasing distortion (ALD), amplitude distortion (AMD), and phase distortion (PHD), due to the fact that the filters $H_0(z)$, $H_1(z)$, $F_0(z)$, and $F_1(z)$ are not ideal [25]. In most of applications, a common requirement is that reconstructed signal $\hat{x}(n)$ should be "as close" to $x(n)$ as possible. Therefore the main stress of most of the researchers while designing filters for the QMF bank has been on the elimination or minimization of the three distortions to obtain a perfect reconstruction (PR) or nearly perfect reconstruction (NPR) system [22].

The design techniques for QMF bank can be classified as optimization-based or nonoptimization based. Various optimization based techniques [26–51] have been developed for the design of two-channel linear phase NPR QMF bank using constrained or unconstrained optimization. In these design methods, ALD can be cancelled completely by selecting the synthesis filters cleverly in terms of the analysis filters, whereas PHD is eliminated using the linear phase FIR filters. The overall transfer function of such an alias and phase distortion free system turns out to be a function of the filter tap coefficients of the low-pass analysis filter only, as the high-pass and low-pass analysis filters are related to each other by the mirror image symmetry condition around the quadrature frequency $\pi/2$. Then, the AMD can be minimized by optimizing the filter tap weights of the low-pass analysis filter using computer aided techniques.

Several methods [52–57] have been developed using IIR filters to design NPR QMF bank. In these design techniques ALD and AMD eliminated completely, and PHD is minimized. Design techniques for PR QMF can be found in literature [23, 24, 58–68]. In PR QMF bank, all three distortions, namely, ALD, AMD, and PHD, are eliminated simultaneously.

The organization of the paper is as follows. Section 2 gives the analysis of the two-channel QMF bank. Section 3 describes the design techniques for two-channel QMF bank including recent progress in this area. Section 4 presents the application of QMF bank for subband coding of image signal. Finally future research trends are discussed in Section 5, followed by conclusion in Section 6.

2. Analysis of the Two-Channel QMF Bank

The two-channel QMF bank structure is known as critically sampled filter bank as decimation, and interpolation factors are equal to number of bands. The frequency responses of the analysis filters $H_0(z)$ and $H_1(z)$, are shown in Figure 2. $|H_0(e^{j\omega})|$ is a mirror image of $|H_1(e^{j\omega})|$ with respect to the quadrature frequency $\pi/2$; this has given rise to the name quadrature mirror filter bank.

By using input-output relationship of decimator and interpolator, we can write $V_0\ (e^{j\omega}) = [X_0(e^{j\omega/2})+X_0(-e^{j\omega/2})]/2$, which describe the aliasing effect in the top channel of Figure 1, and $Y_0(e^{j\omega}) = V_0(e^{2j\omega})$, which describe the imaging effect [25]. In the similar way, the bottom channel of Figure 1 can be described. Therefore, the z-transform of the signals $y_0(n)$ and $y_1(n)$ can be written as

$$\begin{aligned} Y_0(z) &= \frac{1}{2}\left[X(z)H_0(z) + X(-z)H_0(-z)\right], \\ Y_1(z) &= \frac{1}{2}\left[X(z)H_1(z) + X(-z)H_1(-z)\right]. \end{aligned} \tag{1}$$

By using (1), the relation between $Y(z)$ and $X(z)$ of a two-channel QMF bank, is given by

$$\begin{aligned} Y(z) = {} & \frac{1}{2}\left[H_0(z)F_0(z) + H_1(z)F_1(z)\right]X(z) \\ & + \frac{1}{2}\left[H_0(-z)F_0(z) + H_1(-z)F_1(z)\right]X(-z). \end{aligned} \tag{2}$$

The term which contain $X(-z)$ represents aliasing. It can be completely removed by defining the synthesis filters as given below:

$$F_0(z) = H_1(-z), \qquad F_1(z) = -H_0(-z). \tag{3}$$

Mirror image analysis filters are related to each other as

$$H_1(z) = H_0(-z). \tag{4}$$

This choice ensures that $H_1(z)$ is a good high-pass filter, if $H_0(z)$ is good low-pass filter. The alias cancelation constraint of (3) becomes

$$F_0(z) = H_0(z) \quad \text{and} \quad F_1(z) = -H_1(z) = -H_0(-z). \tag{5}$$

Therefore all the four filters are completely determined by the low-pass analysis filter $H_0(z)$ only. By using (3) and (4), the expression for the alias free reconstructed signal can be written as

$$Y(z) = \frac{1}{2}\left[H_0^2(z) - H_0^2(-z)\right] X(z) \tag{6}$$

or

$$Y(z) = T(z)\, X(z), \tag{7}$$

where $T(z)$ is termed as "distortion transfer function" of the alias free QMF bank and is given by

$$T(z) = \frac{1}{2}\left[H_0^2(z) - H_0^2(-z)\right]. \tag{8}$$

To obtain the perfect reconstruction QMF bank, PHD and AMD should also be eliminated; that can be possible if the reconstructed signal $\hat{x}(n)$ is simply made equal to a scaled and delayed version of the input signal $x(n)$. In this case the overall system function $T(z)$ must be a pure delay; that is,

$$T(z) = cz^{-n_0} \qquad \text{or } \hat{x}(n) = cx(n - n_0). \tag{9}$$

From (8), if the analysis filter $H_0(z)$ is selected to be a linear phase FIR, then $T(z)$ also becomes FIR linear phase and phase distortion of the QMF bank is eliminated. To assure the linear phase FIR constraint, impulse response $h_0(n)$ of the low-pass prototype filter should be symmetric $h_0(n) = h_0(N-1-n)$ [21]. With this selection, the corresponding frequency response [31] can be written as

$$H_0\left(e^{j\omega}\right) = e^{-j\omega(N-1)/2} H_r(\omega), \tag{10}$$

where N is filter length and $H_r(\omega)$ is the amplitude function. For a real impulse response, the magnitude response $|H_0(e^{j\omega})|$ is an even function of ω; hence, by substituting (10) into (8), the overall transfer function of the two-channel QMF bank becomes

$$\begin{aligned} T\left(e^{j\omega}\right) &= \frac{1}{2}\left(e^{-j\omega(N-1)}\right) \\ &\quad \times \left[\left|H_0\left(e^{j\omega}\right)\right|^2 - (-1)^{N-1}\left|H_0\left(e^{j(\pi-\omega)}\right)\right|^2\right] \end{aligned} \tag{11}$$

when N is odd; the above equation gives $T(e^{j\omega}) = 0$ at $\omega = \pi/2$, resulting in severe amplitude distortion. Therefore, N must be chosen to be even to avoid this distortion, so that

$$T\left(e^{j\omega}\right) = \frac{1}{2}\left(e^{-j\omega(N-1)}\right)\left[\left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_0\left(e^{j(\pi-\omega)}\right)\right|^2\right]. \tag{12}$$

Consequently, the condition for perfect reconstruction can be written as

$$\left|T\left(e^{j\omega}\right)\right| = \left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_0\left(e^{j(\pi-\omega)}\right)\right|^2 = c, \tag{13}$$

or

$$\left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_1\left(e^{j\omega}\right)\right|^2 = c. \tag{14}$$

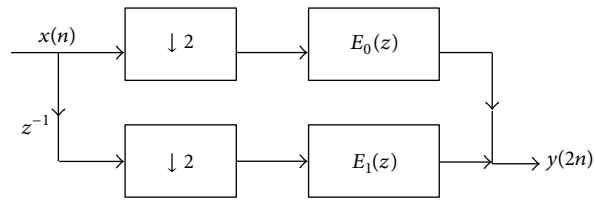

Figure 3: Polyphase implementation of decimation filter.

If $H_0(z)$ is FIR, then due to the constraint $H_1(z) = H_0(-z)$, we cannot achieve the perfect reconstruction [21]. Therefore, in this case after eliminating ALD and PHD completely, we can only minimize amplitude distortion by optimizing the coefficients of $H_0(z)$. So the filters satisfy condition (13) approximately. The peak reconstruction error (PRE) and stop band edge attenuation (A_s) can be calculated as

$$\begin{aligned} \text{PRE} &= \max_{\omega}\left|10\log\left|T\left(e^{j\omega}\right)\right|\right| - \min_{\omega}\left|10\log\left|T\left(e^{j\omega}\right)\right|\right|, \\ A_s &= -20\log\left(H_0(\omega_s)\right), \end{aligned} \tag{15}$$

where ω_s is the stop band edge frequency.

2.1. QMF Bank Using Polyphase Component. QMF bank can be implemented efficiently by using polyphase decomposition [25], which enables us to rearrange the computations of the filtering operation. The prototype filter $H_0(z)$ can be written in its type 1 polyphase form as

$$H_0(z) = E_0\left(z^2\right) + z^{-1}E_1\left(z^2\right). \tag{16}$$

If direct form implementation is used for decimation filter, then only the even numbered output samples are computed and that requires $(N+1)$ multiplications per unit time (MPUs) and N additions per unit time (APUs). However, during the computation of odd numbered output samples, the structure is simply resting. If we use polyphase implementation, then the computation of O/P samples requires only $(N+1)/2$ MPUs and $N/2$ APUs [21]. Thus polyphase representation of decimation filter, as shown in Figure 3, reduces the computational complexity of the multipliers and adders in filter bank.

For complete implementation of two-channel QMF bank using polyphase framework, a total of only about N MPUs and N APUs are required [25], where N is the length of prototype low-pass filter $H_0(z)$.

3. Two-Channel QMF Bank Design Techniques

As we have already discussed in Section 1, QMF bank has been of great interest during the past two decades due to its various applications in different signal processing fields. Nowadays the proper decomposition and perfect reconstruction of a signal have received a lot of attention. Therefore, new algorithms are being continuously proposed by researchers for efficient design of two-channel QMF banks in terms of

low reconstruction error and low delay. In this section the overview of various design techniques for two-channel QMF banks is discussed. Several recent methods are also included.

In the design of QMF bank, the coefficients of the filters $H_0(z)$, $H_1(z)$, $F_0(z)$, and $F_1(z)$ are obtained in such a way so that the three distortions ALD, AMD, and PHD are minimized or eliminated. There are mainly two research areas in design of QMF banks [22]. Some of design methods lead to nearly perfect reconstruction (NPR) filter banks while others lead to perfect reconstruction (PR) filter banks. Nearly perfect reconstruction QMF banks can be further divided into two types: design of linear phase QMF banks and nonlinear QMF banks.

The general methods for the design of QMF bank can be as follows:

(a) elimination of PHD completely and optimization of the filter coefficients to minimize AMD in an alias free system;

(b) elimination of AMD completely and optimization of the filter coefficients to minimize PHD in an alias free system;

(c) elimination of the three distortions ALD, AMD, and PHD simultaneously.

3.1. Elimination of ALD and PHD Completely and Minimization of AMD. As discussed in the previous section, (8) clearly shows that if the $H_0(z)$ is selected to be linear phase FIR, then PHD is eliminated completely. Thus by constraining $H_0(z)$ to be linear phase and selecting the synthesis filters as in (3), PHD and ALD are eliminated completely. But still we are left with AMD due to the constrained $H_1(z) = H_0(-z)$. Having eliminated aliasing and phase distortion, we can only minimize amplitude distortion systematically [21]. If filter characteristics of $H_0(z)$ are assumed ideal in pass band and stop band regions then the overall amplitude response $|T(e^{jw})|$ of the QMF bank will be constant in the pass bands of $H_0(z)$ and $H_1(z)$. The main difficulty comes in transition band region ($\omega_p < \omega < \omega_s$); therefore, the reconstruction error must be controlled in this region. To extent the overlap of $H_0(z)$ and $H_1(z)$ is very crucial in determining amplitude distortion [25].

Figure 4(a) shows three-linear phase designs of $H_0(z)$, and the corresponding plots of $|T(e^{jw})|$ are shown in Figure 4(b). If the pass band edge is too large means $H_0(z)$ and $H_1(z)$ have too much overlap as in curve 1, then there is peaking effect in $|T(e^{jw})|$ around $\pi/2$. If the pass band edge is too small as in curve 2, then $|T(e^{jw})|$ dips around $\pi/2$. If we choose the pass band edge carefully by trial and error to optimize the overlap as in curve 3, $|T(e^{jw})|$ exhibits much better response [21]. Therefore, by optimizing the coefficients of the low-pass filter $H_0(z)$, the amplitude distortion can be minimized and filters satisfy the condition of (13) approximately.

Systematic computer aided optimization techniques [26–51] have been developed to minimize the amplitude distortion. Johnston [26] minimized AMD, in ALD and PHD free QMF bank, by optimizing the coefficients of the low-pass filter $H_0(z)$, such that the amplitude of the distortion function $|T(\omega)|$ becomes as flat as possible and at the same time the stop band energy of $H_0(z)$ was minimized.

For the design of linear phase FIR low-pass analysis filter $H_0(z)$, the objective function (ϕ), which is to be minimized, may be chosen [21] as follows:

$$\phi = \alpha\phi_1 + (1-\alpha)\,\phi_2, \tag{17}$$

where

$$\phi_1 = \int_0^{\pi} \left|1 - \left|T\left(e^{j\omega}\right)\right|\right|^2 d\omega \tag{18}$$

or

$$\phi_1 = \int_0^{\pi} \left|1 - \left|H_0\left(e^{j\omega}\right)\right|^2 - \left|H_0\left(e^{j(\pi-\omega)}\right)\right|^2\right|^2 d\omega \tag{19}$$

in the stop band; the objective function to be minimized as

$$\phi_2 = \int_{\omega_s}^{\pi} \left|H_0\left(e^{j\omega}\right)\right|^2 d\omega. \tag{20}$$

α is a coefficient used to control the tradeoff between the stop band energy of $H_0(z)$ and the flatness of $|T(e^{jw})|$. The coefficients $h_0(n)$ of $H_0(z)$ are optimized in order to minimize ϕ. Such optimization has been done in [26] using Hooke and Jeeves algorithm. But in this technique a manual intervention is required to select the starting points and increments for minimization of objective function.

Jain and Crochiere [27] proposed a new design technique by time domain formulation of the two-channel QMF bank. This method does not require manual intervention or repeated trials with different start-up guesses as in method [26]. The objective function "ϕ_A" was minimized by optimizing the coefficients of $H_0(z)$, subject to unit energy constraint, and is given as

$$\phi_A = E_r + \alpha \cdot E_s, \tag{21}$$

where E_r is the ripple energy of the overall transfer function $T(z)$ and E_s is the stop band energy of $H_0(z)$. A gradient based method was presented by Swaminathan and Vaidyanathan [38], to design linear phase FIR filter bank that minimizes the same objective function as in the method by Johnston as well as by Jain and Crochiere.

Chen and Lee [28] proposed the design of QMF with linear phase in frequency domain with a weighted least squares (WLS) algorithm. Overall reconstruction error was minimized in the minimax sense over the entire frequency band. Lu et al. [30] have proposed a new algebraic method for the design of two-channel quadrature mirror-image filter. This method uses a self-convolution technique to reformulate a fourth-order objective function whose minimization leads to a computational efficient design of QMF banks.

A general purpose approach was proposed by Bregovic and Saramaki [20], for designing two-channel FIR filter banks optimizing the filter bank in alias free case to minimize the maximum of stop band energies of the two analysis filters subject to the given pass band and transition band

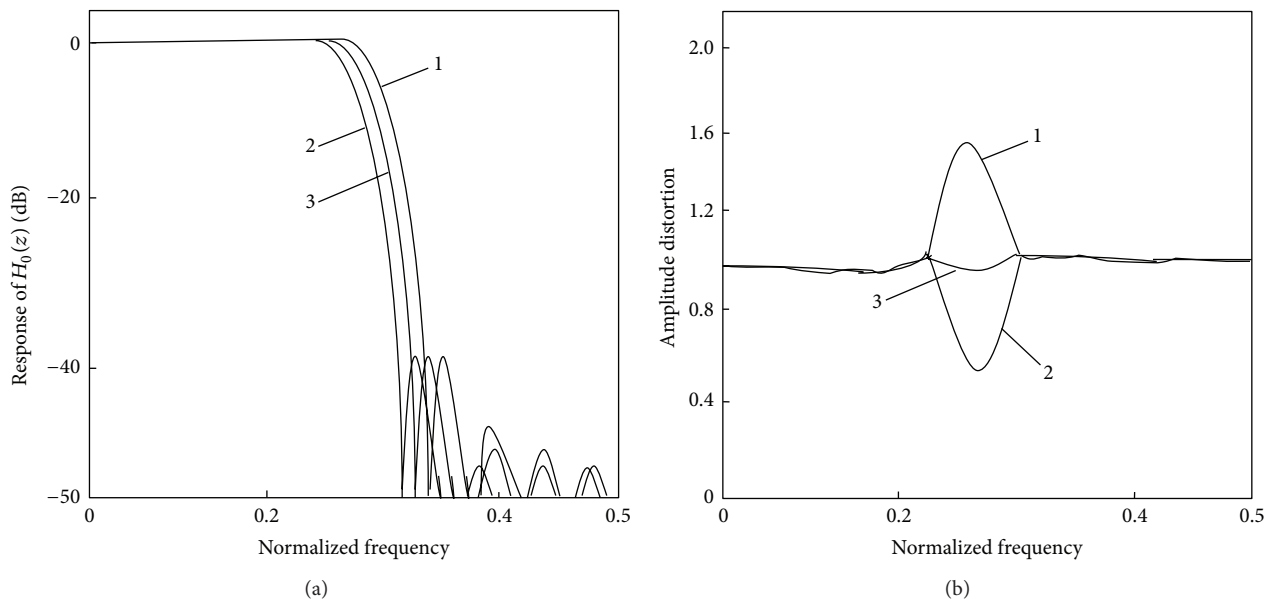

Figure 4: (a) Responses of $H_0(z)$ for three design examples. (b) Corresponding amplitude distortions.

constraints and the given allowable reconstruction error. The optimization was carried out in two steps. The first step generates a good initial starting point using an existing design method and during the second step the optimization of the filter bank using modified Dutta-Vidyasagar algorithm [69] was performed. For QMF bank, other various optimization based design techniques can be found in the literature. Some of the techniques [26–28, 38, 41, 50] are, however, complicated and only applicable to low order two-channel QMF banks.

Sahu et al. [31] presented a new algorithm to design two-channel QMF bank using the Marquardt optimization method. In this algorithm, objective function was formulated as linear combination of pass band error, stop band residual energy of the low-pass analysis filter, and the square error of the overall transfer function of the QMF bank at the quadrature frequency. The objective function "ϕ_B" was minimized by optimizing the coefficients of $H_0(z)$:

$$\phi_B = \alpha_1 E_p + \alpha_2 E_s + \beta E_t, \tag{22}$$

where α_1, α_2, and β are real constants and E_p and E_s are the measure of pass band error and stop-band residual energy, respectively. E_t is square error of the overall transfer function at $\pi/2$. Results of the proposed method were also compared with other methods in terms of significant parameters, that is, E_p, E_s, stop band edge attenuation (A_s) and reconstruction error. Some other design techniques for two-channel QMF based on constrained or unconstrained optimization can be found in [32, 34–37, 51].

Due to nonlinearity and nonconvexity of the objective functions, conventional numerical/mathematical methods may find difficulty to achieve an optimal design. In most cases, they are not able to find global optimum solution. Nowadays researchers also started using of nature-inspired optimization techniques to design QMF bank. Popular swarm intelligence approach, known as particle swarm optimization (PSO), has also been applied in [33] to design QMF bank by Upendar et al. The objective function for PSO based technique was taken as weighted sum of four terms as shown below:

$$\phi_C = \alpha_1 E_p + \alpha_2 E_s + \alpha_3 E_t + \alpha_4 \cdot \text{mor}, \tag{23}$$

where α_1–α_4 are the relative weights and E_p, E_s, E_t, and mor are the mean square error in pass band, mean square error in stop band, square error of the overall transfer function at $\pi/2$, and measure of ripple, respectively. To implement the PSO algorithm, a Matlab program was developed and performance of the proposed method had been illustrated by examples [33].

Differential evolution (DE) is one of the most powerful evolutionary algorithm (EAs) and has been used for various signal processing applications. Ghosh et al. [46] proposed a new algorithm based on improved and adaptive variant of the DE for the design of two-channel quadrature mirror filters with liner phase characteristics. The objective function is similar to that of [33]. The DE variant was tested for QMF filter design problem. Design examples were presented to show the effectiveness of the proposed method over the conventional design techniques. Kumar et al. [40] have presented a hybrid method in frequency domain for design of quadrature mirror filter bank. The hybrid method is based on Levenberg-Marquardt (LM) and Quasi-Newton (QN) techniques. The objective function was formulated as weighted sum of errors:

$$\phi_D = \alpha E_p + (1 - \alpha) E_s + E_t. \tag{24}$$

E_p, E_s, and E_t are, respectively, errors in pass band, stop band and transition band. Table 1 summarizes the comparison of various nearly PR algorithms for design of two-channel QMF

bank in terms of significant parameters: PRE (in dB), E_p, E_s, stop band edge attenuation (A_s) in dB, stop band first lobe attenuation (A_I) in dB.

3.2. Elimination of ALD and AMD Completely and Minimization of PHD. It is possible to completely eliminate amplitude distortion, rather than just minimize it. In this method, again analysis filters and synthesis filters are related as in (3) and (4) so that aliasing is completely canceled. However, filter $H_0(z)$ is selected in such a way to eliminate AMD completely, PHD is then minimized. If the low-pass filter $H_0(z)$ is written in the polyphase form of (16), then system function $T(z)$, of the QMF bank can be written as

$$T(z) = \frac{1}{2}\left[H_0^2(z) - H_0^2(-z)\right] = 2z^{-1}E_0\left(z^2\right)E_1\left(z^2\right). \quad (25)$$

If the polyphase components $E_0(z)$ and $E_1(z)$ are selected to be IIR all pass [21, 25], then from (25), $T(z)$ would also be all pass, which means AMD is eliminated. Now the analysis filter $H_0(z)$ is constraining the form:

$$H_0(z) = \left(\frac{1}{2}\right)\left[A_0(z) + A_1(z)\right], \quad (26)$$

where $A_0(z)$ and $A_1(z)$ are all pass filters of the form

$$A_0(z) = a_0\left(z^2\right), \qquad A_1(z) = z^{-1}a_1\left(z^2\right); \quad (27)$$

the polyphase components can be expressed as

$$\begin{aligned} E_0\left(z^2\right) &= \left(\frac{1}{2}\right)A_0(z) = \left(\frac{1}{2}\right)a_0\left(z^2\right), \\ E_1\left(z^2\right) &= \left(\frac{1}{2}\right)A_1(z) = \left(\frac{1}{2}\right)z^{-1}a_1\left(z^2\right). \end{aligned} \quad (28)$$

Digital Butterworth, Chebyshev and Elliptic filters are special cases [21, 53, 54] of the form given by (26). The constraint on $H_0(z)$ by (26), where $A_0(z)$ and $A_1(z)$ are given by (27), is necessary to become $E_0(z^2)$ and $E_1(z^2)$ all pass which in turn makes (25) to be all pass. Therefore, AMD is eliminated and we then have

$$T(z) = \left(\frac{1}{2}\right)a_0\left(z^2\right)\cdot z^{-1}a_1\left(z^2\right). \quad (29)$$

The phase distortion created by the nonlinear phase of $T(e^{jw})$ can be further reduced by all pass equalization. The simple design procedures are given in [53, 54]. Digital elliptic filters, which are optimal in the minimax sense, automatically have the form $H_0(z) = (1/2)[A_0(z) + A_1(z)]$.

Enkanayake and Premaratne et al. [52] presented computationally simple method to obtain IIR analysis and synthesis filters that possess negligible phase distortion. In this method the balanced reduction procedure was applied to obtain nearly all pass IIR polyphase components and then approximating these with perfect all pass IIR polyphase components. Low delay QMF banks can be designed with nonlinear phase QMF banks. In nonlinear QMF banks, analysis/synthesis filters have nonlinear phase IIR filters and delay is not equal to the order of the filters. There are several techniques [21, 52, 57] available to design low delay QMF banks.

3.3. Elimination of ALD, PHD, and AMD Simultaneously. We can obtain a perfect reconstruction QMF bank by eliminating all three distortions, namely, ALD, AMD, and PHD, simultaneously. The reconstructed signal is therefore just a time delayed version of the input signal $x(n)$; that is, $y(n) = cx(n - n_0)$ for some nonzero constant c and some positive integer n_0. Example of a trivial FIR perfect reconstruction (PR) system can be obtained as follows:

$$\begin{aligned} H_0(z) &= 1, \qquad H_1(z) = z^{-1}, \\ F_0(z) &= z^{-1}, \qquad F_1(z) = 1. \end{aligned} \quad (30)$$

By substituting in (2), the distortion function simplifies to $T(z) = z^{-1}$ so that $y(n) = x(n-1)$. In this system the analysis bank just partitions the input samples into even and odd numbered subsets, and the synthesis bank interlaces these samples back in their original places except for one unit of delay. This example shows the existence of FIR PR QMF banks. With the help of polyphase decomposition more useful nontrivial FIR PR QMF bank can be developed [25], where the filters $H_0(z)$ and $H_1(z)$ have good attenuation. To obtain FIR PR QMF banks, it is necessary to give up the constraints $H_1(z) = H_0(-z)$ of earlier design. Smith and Branwell III [23] designed an FIR PR system based on spectral factorization of half-band filters. Vaidyanathan and Hoang [58] presented a lattice structure and an algorithm for the design of two-channel QMF banks, satisfying a sufficient condition for PR. The algorithm ensures good stop band attenuation for each of the analysis filters. The lattice structure has the hierarchical property that a higher order PR QMF bank can be obtained from a lower order PR QMF bank, simply by adding more lattice sections.

In some PR systems, it is desirable that the analysis filters are constrained to have linear phase. To obtain FIR linear phase PR QMF banks, it is necessary to give up [21] the power complementary condition of (13) as well as the constraints $H_1(z) = H_0(-z)$. Nguyen and Vaidyanathan [61] derived lattice type structures enforce the perfect reconstruction and linear phase properties simultaneously. Table 2 shows the comparison among three types of QMF banks.

4. Application to Subband Coding of Image Signals

Subband coding of signals is an effective method to achieve bandwidth compression when the signal energy is dominantly concentrated in a particular region of frequency. In subband coding, the signal is subdivided into several frequency bands and each band is digitally encoded separately. Vetterli [70] had extended the concept of subband coding of one-dimensional signal to multidimensional signals. Subband coding of image is possible in two ways: using 1-D filter and 2-D filter for different applications, that is, image compression, texture classification, noise reduction, and so forth. Tree structure QMF bank can be used for 1-D filter. In single level decomposition, as shown in Figure 5, image signal is first filtered along row using the analysis filters of two-channel QMF bank into two subbands. Further, each

Table 1: Comparison of various nearly PR algorithms for design of two-channel QMF bank based on significant parameters for $N = 32$.

Methods	PRE (dB)	E_p	E_s	A_s (dB)	A_I (dB)
Jain-Crochiere [27]	0.015	2.30×10^{-8}	1.50×10^{-6}	33.00	44.25
Chen-Lee [28]	0.016	2.11×10^{-8}	1.55×10^{-6}	34.00	44.40
Gradient method [38]	0.016	2.64×10^{-8}	3.30×10^{-6}	33.60	35.00
Sahu [31]	0.027	1.45×10^{-8}	2.76×10^{-6}	33.93	44.25
Upender et al. [33]	0.015	2.35×10^{-8}	5.79×10^{-6}	36.87	44.75
Lu-Xu-Antoniou [30]	0.015	1.50×10^{-8}	1.54×10^{-6}	35.00	44.30
Xu-Lu-Antoniou [29]	0.031	3.50×10^{-8}	5.71×10^{-6}	35.00	43.60
Steepest Descent [36]	0.050	9.84×10^{-7}	8.36×10^{-6}	34.56	40.00
Kumar et al. [32]	0.010	7.42×10^{-9}	1.27×10^{-6}	36.59	43.50
Ghosh [46]	0.0085	2.48×10^{-8}	3.16×10^{-6}	36.91	44.88
Bergovic [20]	0.009	0.155	6.54×10^{-8}	49.20	61.00
Algorithm in [37]	0.023	3.05×10^{-8}	5.08×10^{-6}	35.40	44.10

Table 2: Comparison of three types of two-channel QMF banks.

Feature	NPR FIR Based	Perfect-reconstruction system	NPR IIR based
Phase response	Linear	Linear/nonlinear	Nonlinear
Aliasing	Canceled	Canceled	Canceled
Phase distortion	Eliminated	Eliminated	Minimized
Amplitude distortion	Minimized	Eliminated	Eliminated
Overall group delay	$N - 1$	$N - 1$	Complicated
Relation between analysis filters	$H_1(z) = H_0(-z)$	Not explicit. Implicitly	$H_1(z) = H_0(-z)$
Power complementarity	Approximately holds	Does not hold	Approximately holds

decimated subband is filtered again along column using analysis filters of next stage, which results in four subbands. Finally, the image signal is decomposed into four bands labeled by XX, YX, XY, and YY.

The individual subbands can be processed according the required applications. XX subband can be further decomposed for multilevel decomposition. The reconstruction of the full band signal is done using interpolators and synthesis filters.

Following fidelity assessment parameters can be used to analyze the satisfactory reconstruction of original image [32].

(i) Mean square error (MSE):

$$\mathrm{MSE} = \frac{1}{\mathrm{MN}} \sum_{v=1}^{M} \sum_{u=1}^{N} \left[I(u,v) - I'(u,v) \right]^2; \tag{31}$$

(ii) Peak signal to noise ratio (PSNR):

$$\mathrm{PSNR} = 20\log_{10}\left(255/\mathrm{sqrt}\left(\mathrm{MSE}\right)\right), \tag{32}$$

where $I(u,v)$ is the original image, $I'(u,v)$ is the reconstructed image, and M, N are the dimensions of the image.

5. Further Research Trends in QMF Banks

Two-channel QMF was first used in subband coding and then find applications in various signal processing fields. Since 1976, various design techniques for QMF banks have been proposed by researchers. Due to different applications, the QMF banks are growing field in digital signal processing. The research areas in QMF banks are design of two-channel nearly PR QMF banks, PR QMF banks, designing M-channel QMF banks, and applications of QMF banks. There are few points that may need to be further researched.

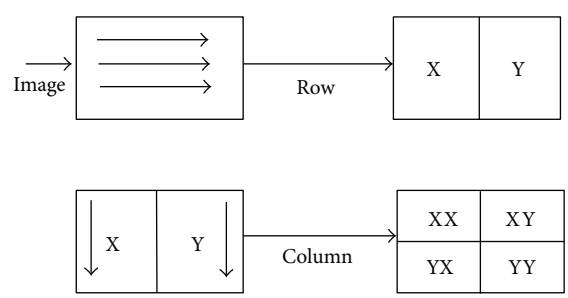

Figure 5: Single level decomposition.

(a) While designing QMF banks, the performance and effectiveness of a method are measured in terms of peak reconstruction error (PRE), mean square error in pass band, and stop band regions, error in transition band, stop band attenuation, group delay, phase response and computational complexity.

There is a need for suitable algorithms for finding optimum solutions for these significant parameters.

(b) If the characteristics of prototype filter are assumed to be ideal in its pass band and stop band regions, consequently, reconstruction error lies only in transition band. Hence, a suitable objective function is required to minimize this error. In earlier proposed design techniques, suitable modifications can be made in such a way that the reconstruction error and computational complexity will reduce by using new algorithms.

(c) Classical optimization methods may fail to achieve optimal design as they do not guarantee for convergence on the global optimum. Therefore, genetic algorithms (based on law of nature) and population-based optimization algorithms [71] may be used to design QMF bank for better results.

(d) Hybrid optimization methods may be used to design QMF banks for improved performance. The combination of classical optimization method and genetic algorithm can be used.

(e) QMF banks are used extensively in designing efficient subband coders for speech, image, and video signals. Tree structure QMF bank can be used in wireless communication as interference canceler [72, 73] for coexistence problem of TDMA/CDMA systems, which can jointly cancel both wideband interference (CDMA signals in terms of TDMA signal) and narrowband interference (TDMA signals in terms of CDMA signal). Efficient receiver design for OFDMA systems [74] can be obtain using filter banks and multirate signal processing. In biomedical signal processing data, compression can be done with the help of QMF bank. They can also be used for efficient subband coding of ECG and ultrasonic signals [75, 76]. Wavelets are related to nonuniform filter banks that decompose a signal into unequal subbands. Nonuniform filter banks are also related to the so-called dyadic wavelets [77], especially useful in signal compression. Orthonormal wavelet representation with finite duration basis functions is related to a tree-structured two-channel filter bank. By designing analysis filters in two-channel module, we can make the wavelet basis as smooth as possible [78].

6. Conclusions

In this paper, a comprehensive overview of two-channel QMF banks has been presented. Different design techniques for nearly perfect reconstruction and perfect reconstruction QMF banks have been discussed. New algorithms are being proposed for design of QMF bank with improved performance in terms of reconstruction error and computational time. Advances in QMF banks provide efficient subband coders for speech, image, and video signal. Application of two-channel QMF bank for subband coding of image signal is also presented. Some future research trends in designing of QMF banks and its applications have also been discussed which may be helpful for the researchers.

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