Research Article

Global Existence and Blowup for a Reaction-Diffusion System with Nonlocal Boundary Condition

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This paper considers the singularity properties of positive solutions for a reaction-diffusion system with nonlocal boundary condition. The conditions on the existence and nonexistence of global positive solutions are given. Moreover, we establish the blow-up rate estimate for the blow-up solution.

1. Introduction

This paper studies the singularity properties of the following reaction-diffusion system with nonlocal boundary condition:

$$\begin{aligned} u_t &= \Delta u + u^\alpha v^p, & v_t &= \Delta v + u^q v^\beta, & x &\in \Omega, & t > 0, \\ u(x, t) &= \int_{\Omega} f(x, y) u(x, y) dy, \\ v(x, t) &= \int_{\Omega} g(x, y) v(x, y) dy, & x &\in \partial\Omega, & t > 0, \\ u(x, 0) &= u_0(x), & v(x, 0) &= v_0(x), & x &\in \bar{\Omega}, \end{aligned} \quad (1)$$

where Ω is a bounded domain of $\mathbb{R}^N$, $N \geq 1$, with smooth boundary $\partial\Omega$ and $\bar{\Omega}$ is the closure of Ω . α , β , p , and q are positive numbers which ensure that the equations in (1) are completely coupled with the nonlinear reaction terms. The functions $f(x, y)$, $g(x, y)$ defined for $x \in \partial\Omega$, $y \in \bar{\Omega}$ are nonnegative and continuous. The initial values $u_0(x)$ and $v_0(x)$ are nonnegative, which are mathematically convenient and currently followed throughout the paper. We also assume that (u_0, v_0) satisfies the compatibility condition on $\partial\Omega$, and that $f(x, \cdot) \not\equiv 0$ and $g(x, \cdot) \not\equiv 0$ for any $x \in \partial\Omega$ for the sake of the meaning of nonlocal boundary.

Denote that $Q_T = \Omega \times (0, T)$, $S_T = \partial\Omega \times (0, T)$, and $\bar{Q}_T = \bar{\Omega} \times [0, T]$, where $0 < T \leq +\infty$. A pair of functions $(u(x, t), v(x, t))$ is called a classical solution of problem (1) if

$(u, v) \in [C^{2,1}(Q_T) \cap C(\bar{Q}_T)]^2$ for some T , $0 < T \leq +\infty$, and satisfies (1). The local existence of classical solution of (1) is standard (see [1, 2]). If $T < +\infty$, it is easy to see $\lim_{t \rightarrow T} [\max_{x \in \bar{\Omega}} u(x, t) + \max_{x \in \bar{\Omega}} v(x, t)] = +\infty$, and we say that the solution $(u(x, t), v(x, t))$ of problem (1) blows up at finite time T . If $T = +\infty$, $(u(x, t), v(x, t))$ is called a global solution of problem (1).

Over the past few years, many physical phenomena were formulated as nonlocal mathematical models (see [3, 4]). It has also been suggested that nonlocal growth terms present a more realistic model in physics for compressible reactive gases. Problem (1) arises in the study of the heat transfer with local source (see [5, 6]) and in the study of population dynamics (see [7, 8]).

In recent years many authors have investigated the following initial boundary value problem of reaction-diffusion system:

$$u_t - \Delta u = f(u, v), \quad v_t - \Delta v = g(u, v), \quad x \in \Omega, \quad t > 0 \quad (2)$$

with Dirichlet, Neumanns or Robin boundary condition, which can be used to describe heat propagation on the boundary of container (see [2, 4, 9–23] and the literatures cited therein). Specially, when $f(u, v)$, $g(u, v)$ have the form

$$f(u, v) = u^\alpha v^p, \quad g(u, v) = u^q v^\beta, \quad (3)$$

a classical result is (see [9, 10, 12, 20]).

Theorem A. *The system (2) (f, g is of the form (3)) with homogenous Dirichlet boundary condition*

$$u(x, t) = v(x, t) = 0, \quad x \in \Omega, \quad t > 0 \quad (4)$$

admits a unique global solution for any nonnegative initial data $u(x, t) = u_0(x) \neq 0$, $v(x, t) = v_0(x) \neq 0$ if and only if $\alpha < 1$, $\beta < 1$, and $pq \leq (1 - \alpha)(1 - \beta)$.

However, there are some important phenomena formulated as parabolic equations which are coupled with nonlocal boundary conditions in mathematical modeling such as thermoelasticity theory (see [24–26]). In this case, the solution could be used to describe the entropy per volume of the material. The problem of nonlocal boundary conditions for linear parabolic equation of the type

$$\begin{aligned} u_t - Au &= 0, \quad x \in \Omega, \quad t > 0, \\ u(x, t) &= \int_{\Omega} f(x, y) u(y, t) dy, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad x \in \bar{\Omega} \end{aligned} \quad (5)$$

with uniformly elliptic operator

$$Au = \sum_{i,j=1}^N a_{i,j}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^N b_i(x) \frac{\partial}{\partial x_i} + c(x) \quad (6)$$

and $c(x) \leq 0$ was studied by Friedman [26]. It was proved that the unique solution of (5) tends to 0 monotonically and exponentially as $t \rightarrow +\infty$ provided $\int_{\Omega} f(x, y) dy \leq \rho < 1$ for any $x \in \partial\Omega$.

As for more general discussions on the dynamic of parabolic problem with nonlocal boundary conditions, one can see Pao [27], where the following problem:

$$\begin{aligned} u_t - Au &= h(x, u), \quad x \in \Omega, \quad t > 0, \\ \alpha_0 \frac{\partial u}{\partial \nu} + u &= \int_{\Omega} f(x, y) u(y, t) dy, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad x \in \bar{\Omega} \end{aligned} \quad (7)$$

was considered, and recently Pao [28] gave the numerical solutions for diffusion equation with nonlocal conditions.

In particular, the following single equation:

$$\begin{aligned} u_t - \Delta u &= u^\alpha, \quad x \in \Omega, \quad t > 0, \\ u &= \int_{\Omega} f(x, y) u(y, t) dy, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad x \in \bar{\Omega} \end{aligned} \quad (8)$$

under the assumption the $\int_{\Omega} f(x, y) dy = 1$ for any $x \in \partial\Omega$ was consider, by Seo [29] and the following blow-up rate estimate is established:

$$(\alpha - 1)^{-1/(\alpha-1)} (T^* - t) \leq \max_{x \in \bar{\Omega}} u(x, t) \leq C(T^* - t)^{-1/(\gamma-1)}, \quad (9)$$

where $T^* < +\infty$ is the blow-up time, and γ is any constant satisfying $1 < \gamma < \alpha$.

Recently, Kong and Wang [30] obtained the blow-up conditions and blow-up profiles of the following system by using some ideas of Souplet [4]:

$$\begin{aligned} u_t &= \Delta u + \int_{\Omega} u^\alpha v^p dx, \\ v_t &= \Delta v + \int_{\Omega} u^q v^\beta dx, \quad x \in \Omega, \quad t > 0, \\ u(x, t) &= \int_{\Omega} f(x, y) u(x, y) dy, \\ v(x, t) &= \int_{\Omega} g(x, y) v(x, y) dy, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad v(x, 0) = v_0(x), \quad x \in \bar{\Omega}. \end{aligned} \quad (10)$$

Furthermore, Zheng and Kong [31] gave the condition for global existence or nonexistence of solution to the following system:

$$\begin{aligned} u_t &= \Delta u + u^\alpha \int_{\Omega} v^p dx, \\ v_t &= \Delta v + v^\beta \int_{\Omega} u^q dx, \quad x \in \Omega, \quad t > 0, \\ u(x, t) &= \int_{\Omega} f(x, y) u(x, y) dy, \\ v(x, t) &= \int_{\Omega} g(x, y) v(x, y) dy, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad v(x, 0) = v_0(x), \quad x \in \bar{\Omega}. \end{aligned} \quad (11)$$

Motivated by the above cited works, in this paper, we deal with singularity analysis of the parabolic system (1) with nonlocal boundary condition and it is seems that there is no work dealing with this type of systems except the single equations case, although this is a very classical model. Our main results read as follows.

Theorem 1. *If $\alpha < 1$, $\beta < 1$, and $pq \leq (1 - \alpha)(1 - \beta)$, then every nonnegative solution of (1) is global.*

Theorem 2. *Suppose $\alpha > 1$ or $\beta > 1$ or $pq > (1 - \alpha)(1 - \beta)$.*

- (i) *For any nonnegative functions $f(x, y)$ and $g(x, y)$, the nonnegative solution of (1) blows up in finite time provided the initial values are large enough.*
- (ii) *If $\int_{\Omega} f(x, y) dy \geq 1$ and $\int_{\Omega} g(x, y) dy \geq 1$ for any $x \in \partial\Omega$, the nonnegative solution of (1) blows up in finite time with any positive initial values.*
- (iii) *If $\int_{\Omega} f(x, y) dy < 1$ and $\int_{\Omega} g(x, y) dy < 1$ for any $x \in \partial\Omega$, the nonnegative solution of (1) is global with small initial values.*

To estimate the blow-up rate of the blow-up solution of (1), we need to add some assumptions for initial data as follows.

(H1) $u_0(x), v_0(x) \in C^{2+\gamma}(\Omega) \cap C(\overline{\Omega})$ for some $0 < \gamma < 1$.

(H2) If $p - \beta \geq q - \alpha \geq 1$, then there exists a sufficient small constant $\varepsilon_0 > 0$ (which will be given in Section 4) such that $\Delta u_0(x) + \varepsilon_0 u_0^\lambda \geq 0$ and $\Delta v_0(x) + \varepsilon_0 v_0^\mu > 0$ on $\overline{\Omega}$, where

$$\lambda = \frac{p(q+1) + \alpha(1-\beta)}{p-\beta+1}, \quad \mu = \frac{q(p+1) + \beta(1-\alpha)}{p-\beta+1}. \quad (12)$$

(H3) If $q - \alpha \geq p - \beta \geq 1$, then there exists a sufficient small constant $\varepsilon_0 > 0$ (which will be given in Section 4) such that $\Delta u_0(x) + \varepsilon_0 v_0^\lambda \geq 0$ and $\Delta v_0(x) + \varepsilon_0 u_0^\mu > 0$ on $\overline{\Omega}$, where

$$l = \frac{p(q+1) + \alpha(1-\beta)}{q-\alpha+1}, \quad m = \frac{q(p+1) + \beta(1-\alpha)}{q-\alpha+1}. \quad (13)$$

Theorem 3. Suppose $p - \beta \geq q - \alpha \geq 1$, and $f(x, y) = g(x, y)$, $\int_{\Omega} f(x, y) dy \leq 1$ for all $x \in \partial\Omega$ and assumptions (H1)-(H2) hold. If $(u(x, t), v(x, t))$ is the smooth solution of (1) and blows up in finite time T^* , then there exist positive constants C_i ($i = 1, 2, 3, 4$) such that

$$C_1 \leq \max_{x \in \overline{\Omega}} u(x, t) (T^* - t)^{(p-\beta+1)/(pq-(1-\alpha)(1-\beta))} \leq C_2, \quad (14)$$

$$C_3 \leq \max_{x \in \overline{\Omega}} v(x, t) (T^* - t)^{(q-\alpha+1)/(pq-(1-\alpha)(1-\beta))} \leq C_4,$$

for $0 < t < T^*$.

Theorem 4. Suppose $q - \alpha \geq p - \beta \geq 1$, and $f(x, y) = g(x, y)$, $\int_{\Omega} f(x, y) dy \leq 1$ for all $x \in \partial\Omega$ and assumptions (H1) and (H3) hold. If $(u(x, t), v(x, t))$ is the smooth solution of (1) and blows up in finite time T^* , then there exist positive constants C_i ($i = 1, 2, 3, 4$) such that

$$C_1 \leq \max_{x \in \overline{\Omega}} u(x, t) (T^* - t)^{(p-\beta+1)/(pq-(1-\alpha)(1-\beta))} \leq C_2, \quad (15)$$

$$C_3 \leq \max_{x \in \overline{\Omega}} v(x, t) (T^* - t)^{(q-\alpha+1)/(pq-(1-\alpha)(1-\beta))} \leq C_4,$$

for $0 < t < T^*$.

The rest of this paper is organized as follows. In the next section, we give some preliminaries, which include the comparison principle related to system (1). In Section 3, we will study the conditions for the solution to blowup and exist globally and prove Theorems 1 and 2. In Section 4, we will establish the precise blowup rate estimate for small weighted nonlocal boundary and prove Theorems 3 and 4.

2. Preliminaries

In this section, we give some basic preliminaries. We begin with the definition of upper and lower solutions of (1).

Definition 5. A pair of nonnegative functions $(\bar{u}(x, t), \bar{v}(x, t))$ is called an upper solution of problem (1) if $(\bar{u}, \bar{v}) \in [C^{2,1}(Q_T) \cap C(\overline{Q_T})]^2$ and satisfies

$$\bar{u}_t \geq \Delta \bar{u} + \bar{u}^\alpha \bar{v}^p, \quad \bar{v}_t \geq \Delta \bar{v} + \bar{u}^q \bar{v}^\beta, \quad (x, t) \in Q_T,$$

$$\bar{u}(x, t) \geq \int_{\Omega} f(x, y) \bar{u}(x, y) dy, \quad (16)$$

$$\bar{v}(x, t) \geq \int_{\Omega} g(x, y) \bar{v}(x, y) dy, \quad (x, t) \in S_T,$$

$$\bar{u}(x, 0) \geq u_0(x), \quad \bar{v}(x, 0) \geq v_0(x), \quad x \in \overline{\Omega}.$$

Similarly, $(\underline{u}, \underline{v}) \in [C^{2,1}(Q_T) \cap C(\overline{Q_T})]^2$ is called a lower solution of (1) if it satisfied all the reversed inequalities in (16).

Lemma 6. Let $\lambda_i(x, t)$, $\Theta_i(x, t)$, $i = 1, 2$, f , and g be continuous and nonnegative functions, and let $(w(x, t), z(x, t)) \in [C^{2,1}(Q_T) \cap C(\overline{Q_T})]^2$ satisfy

$$w_t \geq \Delta w + \lambda_1(x, t) w(x, t) + \Theta_1(x, t) z(x, t), \quad (x, t) \in Q_T,$$

$$z_t \geq \Delta z + \lambda_2(x, t) w(x, t) + \Theta_2(x, t) z(x, t), \quad (x, t) \in Q_T,$$

$$w(x, t) \geq \int_{\Omega} f(x, y) w(x, y) dy,$$

$$z(x, t) \geq \int_{\Omega} g(x, y) z(x, y) dy, \quad (x, t) \in S_T,$$

$$w(x, 0) \geq 0, \quad z(x, 0) \geq 0, \quad x \in \overline{\Omega},$$

(17)

and then $w(x, t) > 0$, $z(x, t) > 0$ on $\overline{Q_T}$.

Proof. Set $t_1 = \sup\{t \in (0, T) : w(x, t) > 0, z(x, t) > 0, x \in \overline{\Omega}\}$. Since $w(x, 0) > 0$, $z(x, 0) > 0$, by continuous, there exists $\delta > 0$ such that $w(x, t) > 0$, $z(x, t) > 0$ for all $(x, t) \in \overline{\Omega} \times [0, \delta]$. Thus, $t_1 \in (\delta, T]$.

We claim that $t_1 < T$ will lead to a contradiction. Indeed, $t_1 < T$ suggests that $w(x_1, t_1) = 0$ or $z(x_1, t_1) = 0$ for some $x_1 \in \overline{\Omega}$. Without loss of generality, we suppose that $w(x_1, t_1) = 0 = \inf_{(x,t) \in \overline{Q_{t_1}}} w(x, t)$.

If $x_1 \in \Omega$, we first notice that

$$w_t - \Delta w \geq \lambda_1(x, t) w(x, t) + \Theta_1(x, t) z(x, t) \geq 0, \quad (x, t) \in Q_{t_1}. \quad (18)$$

In addition, it is clear that $w \geq 0$ on the boundary $\partial\Omega$ and at initial state $t = 0$. Then it follows from the strong maximum principle that $w \equiv 0$ in Q_{t_1} , which contradicts $w(x, 0) > 0$.

If $x_1 \in \partial\Omega$, we will have a contradiction

$$0 = w(x_1, t_1) \geq \int_{\Omega} f(x_1, y) w(y, t_1) dy > 0. \quad (19)$$

In the last inequality, we have used the facts that $f(x, \cdot) \not\equiv 0$ for any $x \in \partial\Omega$ and $w(y, t_1) > 0$ for any $y \in \Omega$, which is a direct result of the previous case.

Therefore, the claim is true and thus $t_1 = T$, which implies that $w > 0$, $z > 0$ on $\overline{Q_T}$. The proof is complete. $\square$

Remark 7. If $\int_{\Omega} f(x, y) dy \leq 1$ and $\int_{\Omega} g(x, y) dy \leq 1$ for any $x \in \partial\Omega$ in Lemma 6, we can obtain $w(x, t) \geq 0$, $z(x, t) \geq 0$ on $\overline{Q_T}$ under the assumption that $w(x, 0) \geq 0$, $z(x, 0) \geq 0$ for any $x \in \overline{\Omega}$. Indeed, for any $\epsilon > 0$, we can conclude that $(w(x, t) + \epsilon e^t, z(x, t) + \epsilon e^t) > (0, 0)$ on $\overline{Q_T}$ as the proof of Lemma 6. Then the desired result follows from the limit procedure $\epsilon \rightarrow 0$.

Lemma 8. Let $(\bar{u}(x, t), \bar{v}(x, t))$ and $(\underline{u}(x, t), \underline{v}(x, t))$ be a upper and lower solution of (1) in Q_T , respectively. If $(\bar{u}(x, 0), \bar{v}(x, 0)) > (\underline{u}(x, 0), \underline{v}(x, 0))$ for $x \in \overline{\Omega}$, then $(\bar{u}(x, t), \bar{v}(x, t)) > (\underline{u}(x, t), \underline{v}(x, t))$ on $\overline{Q_T}$.

Proof. Let $w(x, t) = \bar{u}(x, t) - \underline{u}(x, t)$ and $z(x, t) = \bar{v}(x, t) - \underline{v}(x, t)$, and then

$$\begin{aligned} w_t - \Delta w &\geq \bar{u}^\alpha \bar{v}^p - \underline{u}^\alpha \underline{v}^p = \bar{v}^p (\bar{u}^\alpha - \underline{u}^\alpha) + \underline{u}^\alpha (\bar{v}^p - \underline{v}^p) \\ &= \bar{v}^p \phi_1 w + \underline{u}^\alpha \phi_2 z := \lambda_1(x, t) w(x, t) \\ &\quad + \Theta_1(x, t) z(x, t), \\ z_t - \Delta z &\geq \bar{u}^q \bar{v}^\beta - \underline{u}^q \underline{v}^\beta \\ &= \bar{v}^\beta (\bar{u}^q - \underline{u}^q) + \underline{u}^q (\bar{v}^\beta - \underline{v}^\beta) \\ &= \bar{v}^\beta \phi_3 w + \underline{u}^q \phi_4 z := \lambda_2(x, t) w(x, t) \\ &\quad + \Theta_2(x, t) z(x, t), \end{aligned} \quad (20)$$

where

$$\begin{aligned} \lambda_1(x, t) &= \bar{v}^p \phi_1 = \bar{v}^p \int_0^1 \alpha(\lambda \bar{u} + (1-\lambda) \underline{u})^{\alpha-1} d\lambda, \\ \lambda_2(x, t) &= \bar{v}^\beta \phi_3 = \bar{v}^\beta \int_0^1 q(\lambda \bar{u} + (1-\lambda) \underline{u})^{q-1} d\lambda, \\ \Theta_1(x, t) &= \underline{u}^\alpha \phi_2 = \underline{u}^\alpha \int_0^1 p(\lambda \bar{v} + (1-\lambda) \underline{v})^{p-1} d\lambda, \\ \Theta_2(x, t) &= \underline{u}^q \phi_4 = \underline{u}^q \int_0^1 \beta(\lambda \bar{v} + (1-\lambda) \underline{v})^{\beta-1} d\lambda. \end{aligned} \quad (21)$$

So, the functions $w(x, t)$ and $z(x, t)$ satisfy

$$\begin{aligned} w_t &\geq \Delta w + \lambda_1(x, t) w(x, t) + \Theta_1(x, t) z(x, t), \quad (x, t) \in Q_T, \\ z_t &\geq \Delta z + \lambda_2(x, t) w(x, t) + \Theta_2(x, t) z(x, t), \quad (x, t) \in Q_T, \\ w(x, t) &\geq \int_{\Omega} f(x, y) w(x, y) dy, \\ z(x, t) &\geq \int_{\Omega} g(x, y) z(x, y) dy, \quad (x, t) \in S_T, \\ w(x, 0) &\geq 0, \quad z(x, 0) \geq 0, \quad x \in \overline{\Omega}. \end{aligned} \quad (22)$$

Lemma 6 ensures that $w(x, t) > 0$, $z(x, t) > 0$ on $\overline{Q_T}$, that is, $(\bar{u}(x, t), \bar{v}(x, t)) > (\underline{u}(x, t), \underline{v}(x, t))$ on $\overline{Q_T}$. The proof is complete. $\square$

3. Global Existence and Blowup

In this section, we will use the upper and lower solutions and their corresponding comparison principle developed in Section 2 to get the global existence or finite blowup of the solution to (1). Let us first give the proof of Theorem 1.

Proof of Theorem 1. Using the condition $\alpha < 1$, $\beta < 1$, $p q \leq (1-\alpha)(1-\beta)$, and $p > 0$, $q > 0$, we have $((1-\alpha)/p) \cdot ((1-\beta)/q) \geq 1$. Thus, we can choose two positive constant and $m, l > 1$ such that

$$\frac{1-\alpha}{p} \geq \frac{l}{m}, \quad \frac{1-\beta}{q} \geq \frac{m}{l}. \quad (23)$$

Then, let $\phi(x, y)$ ($x \in \partial\Omega$, $y \in \overline{\Omega}$) be a continuous function such that $\phi(x, y) \geq \max\{f(x, y), g(x, y)\}$ and set

$$\begin{aligned} a(x) &= \left[\int_{\Omega} \phi(x, y) dy \right]^{(1-m)/m}, \\ b(x) &= \left[\int_{\Omega} \phi(x, y) dy \right]^{(1-l)/l}, \quad x \in \partial\Omega. \end{aligned} \quad (24)$$

We consider the following auxiliary problem:

$$\begin{aligned} w_t &= \Delta w + k w, \quad x \in \Omega, \quad t > 0, \\ w(x, t) &= (a(x) + b(x) + 1) \\ &\quad \times \left[\int_{\Omega} \left(\phi(x, y) + \frac{1}{|\Omega|} \right) w(y, t) dy \right], \quad (25) \\ &\quad x \in \partial\Omega, \quad t > 0, \end{aligned}$$

$$w(x, 0) = 1 + u_0^{1/m}(x) + v_0^{1/l}(x), \quad x \in \overline{\Omega},$$

where $|\Omega|$ is the measure of Ω and $k = (1/m) + (1/l)$. It follows from [32, Theorem 4.2] that $w(x, t)$ exists globally and indeed $w(x, t) > 1$ on $\overline{\Omega} \times [0, +\infty)$ [32, Theorem 2.1].

Our aim is to show that $(\bar{u}, \bar{v}) = (w^m, w^l)$ is a global upper solution of (1). Indeed, a direct computation yields

$$\begin{aligned} \bar{u}_t - \Delta \bar{u} &\geq w^m = (w^m)^\alpha w^{m(1-\alpha)} \\ &= \bar{u}^\alpha (\bar{v}^l)^{m(1-\alpha)/l} \geq \bar{u}^\alpha \bar{v}^p, \quad x \in \Omega, \quad t > 0. \end{aligned} \quad (26)$$

Here, we have used that conclusion $w > 1$ and the inequality (23). We still have to consider the boundary and initial conditions. When $x \in \partial\Omega$, we have

$$\begin{aligned} \bar{u}(x, t) &= (a(x) + b(x) + 1)^m \\ &\quad \times \left[\int_{\Omega} \left(\phi(x, y) + \frac{1}{|\Omega|} \right) w(y, t) dy \right]^m \end{aligned}$$

$$\begin{aligned}
&\geq (a(x))^m \left[\int_{\Omega} \phi(x, y) w(y, t) dy \right]^m \\
&= \left[\int_{\Omega} \phi(x, y) dy \right]^{1-m} \left[\int_{\Omega} \phi(x, y) w(y, t) dy \right]^m \\
&\geq \left[\int_{\Omega} f(x, y) dy \right]^{1-m} \left[\int_{\Omega} f(x, y) w(y, t) dy \right]^m \\
&= \left[\int_{\Omega} (f^{1-m}(x, y))^{1/(1-m)} dy \right]^{1-m} \\
&\quad \times \left[\int_{\Omega} (f^m(x, y) w^m(y, t))^{1/m} dy \right]^m \\
&\geq \int_{\Omega} f^{1-m}(x, y) (f(x, y) w(x, y))^m dy \\
&= \int_{\Omega} f(x, y) w^m(x, y) dy \\
&= \int_{\Omega} f(x, y) \bar{u}(x, y) dy, \quad x \in \partial\Omega, \quad t > 0.
\end{aligned} \tag{27}$$

Similarly, we have

$$\begin{aligned}
\bar{v}_t - \Delta \bar{v} &\geq \bar{u}^q v^\beta, \quad x \in \Omega, \quad t > 0, \\
\bar{v} &\geq \int_{\Omega} g(x, y) \bar{v}(y, t) dy, \quad x \in \partial\Omega, \quad t > 0.
\end{aligned} \tag{28}$$

It is clear that $u_0(x) < \bar{u}(x, 0)$ and $v_0(x) < \bar{v}(x, 0)$. Therefore, we get that $(\bar{u}, \bar{v})$ is a global upper solution of (1) and hence the solution of (1) exists globally by Lemma 8. The proof is complete. $\square$

Proof of Theorem 2. (i) Let $(\underline{u}, \underline{v})$ be the solution of (2) and (3) with homogeneous Dirichlet boundary. Then it is well known for sufficiently large initial data that the solution $(\underline{u}, \underline{v})$ blows up in finite time when $\alpha > 1$ or $\beta > 1$ or $pq > (1 - \alpha)(1 - \beta)$ (Theorem A). On the other hand, it is obvious that $(\underline{u}, \underline{v})$ is a lower solution of (1). Hence, the solution of (1) with large initial data blows up in finite time.

(ii) We consider the following ODE system:

$$\begin{aligned}
s'(t) &= s^\alpha h^p, \quad h'(t) = s^q h^\beta, \quad t > 0, \\
s(0) &= a = \frac{1}{2} \min_{x \in \Omega} u_0(x), \quad h(0) = b = \frac{1}{2} \min_{x \in \Omega} v_0(x).
\end{aligned} \tag{29}$$

If $\alpha > 1$ or $\beta > 1$, it is clear that the solution $(s(t), h(t))$ of (29) blows up in finite time. For the case $0 < \alpha, \beta < 1$ and $pq > (1 - \alpha)(1 - \beta)$, it follows that

$$\begin{aligned}
&\frac{1}{q - \alpha + 1} [s^{q-\alpha+1}(t) - a^{q-\alpha+1}] \\
&= \frac{1}{p - \beta + 1} [h^{p-\beta+1}(t) - b^{p-\beta+1}].
\end{aligned} \tag{30}$$

Thus, we get

$$\begin{aligned}
h'(t) &= \left[\frac{q - \alpha + 1}{p - \beta + 1} h^{p-\beta+1} \right. \\
&\quad \left. + \left(a^{q-\alpha+1} - \frac{q - \alpha + 1}{p - \beta + 1} b^{p-\beta+1} \right) \right]^{q/(p-\beta+1)} h^\beta \\
&\quad t > 0,
\end{aligned} \tag{31}$$

$$h(0) = b > 0.$$

Then $pq > (1 - \alpha)(1 - \beta)$ implies that $h(t)$ blows up in finite time, and so does $f(t)$. From the above analysis, we see that $\alpha > 1$ or $\beta > 1$ or $pq > (1 - \alpha)(1 - \beta)$ implies that (s, h) blows up in finite time. Under the assumption $\int_{\Omega} f(x, y) dy \geq 1$ and $\int_{\Omega} g(x, y) dy \geq 1$ for any $x \in \partial\Omega$, (s, h) is a lower solution of problem (1). Therefore, by Lemma 8, we see that the solution (u, v) of (1) satisfies $(u, v) \geq (s, h)$ and then (u, v) blows up in finite time.

(iii) Let $\psi_1(x)$ be the positive solution of the linear elliptic problem

$$\begin{aligned}
-\Delta \psi_1(x) &= \varepsilon_0, \quad x \in \Omega, \\
\psi_1(x) &= \int_{\Omega} f(x, y) dy, \quad x \in \partial\Omega,
\end{aligned} \tag{32}$$

and $\psi_2(x)$ be the positive solution of the linear elliptic problem

$$\begin{aligned}
-\Delta \psi_2(x) &= \varepsilon_0, \quad x \in \Omega, \\
\psi_2(x) &= \int_{\Omega} g(x, y) dy, \quad x \in \partial\Omega.
\end{aligned} \tag{33}$$

Since $\int_{\Omega} f(x, y) dy < 1$ and $\int_{\Omega} g(x, y) dy < 1$ for any $x \in \partial\Omega$, we can choose $\varepsilon_0 > 0$ such that $0 \leq \psi_i(x) \leq 1, i = 1, 2$.

Let $\bar{u}(x) = a\psi_1(x)$ and $\bar{v}(x) = b\psi_2(x)$, where a, b are positive constants which satisfy $a\varepsilon_0 \geq a^\alpha b^p, b\varepsilon_0 \geq a^q b^\beta$. We remark that under the assumption $\alpha > 1$ or $\beta > 1$ or $pq > (1 - \alpha)(1 - \beta)$, we can choose such a, b easily. We now show that $(\bar{u}, \bar{v})$ is an upper solution of (1) for small initial data (u_0, v_0) . Indeed, for any $x \in \Omega$, we have

$$\begin{aligned}
\bar{u}_t - \Delta \bar{u} &= a\varepsilon_0 \geq a^\alpha b^p \geq \bar{u}^\alpha \bar{v}^p, \\
\bar{v}_t - \Delta \bar{v} &= b\varepsilon_0 \geq a^q b^\beta \geq \bar{u}^q \bar{v}^\beta.
\end{aligned} \tag{34}$$

When $x \in \partial\Omega$,

$$\begin{aligned}
\bar{u}(x) &= a \int_{\Omega} f(x, y) dy \geq \int_{\Omega} f(x, y) a\psi_1(y) dy \\
&= \int_{\Omega} f(x, y) \bar{u}(y) dy, \\
\bar{v}(x) &= b \int_{\Omega} g(x, y) dy \geq \int_{\Omega} g(x, y) b\psi_2(y) dy \\
&= \int_{\Omega} g(x, y) \bar{v}(y) dy.
\end{aligned} \tag{35}$$

Here, we have used $\psi_i \leq 1, i = 1, 2$. The above inequalities show that $(\bar{u}, \bar{v})$ is an upper solution of (1) whenever $u_0(x) < a\psi_1(x)$ and $v_0(x) < b\psi_2(x)$. The proof is complete. $\square$

4. Blowup Rate

In this section, we will estimate the blow-up rate of (1). By the standard methods (see [1, 2, 6]), we can show that system (1) has a smooth solution (u, v) provided that u_0, v_0 satisfy the hypotheses (H1). We thus assume that the smooth solution (u, v) of (1) blows up at finite time T^* and set $M_1(t) = \max_{x \in \bar{\Omega}} u(x, t)$, $M_2(t) = \max_{x \in \bar{\Omega}} v(x, t)$. We can obtain the blow-up rate from the following lemmas.

Lemma 9. Suppose that $u_0(x), v_0(x)$ satisfy (H1), and then there exists a positive constant K_1 such that

$$\begin{aligned} M_1^{q-\alpha+1}(t) + M_2^{p-\beta+1}(t) \\ \geq K_1(T^* - t)^{-(q-\alpha+1)(p-\beta+1)/(pq-(1-\alpha)(1-\beta))}. \end{aligned} \quad (36)$$

Proof. By the equations in (1), we have [33, Theorem 4.5]

$$M_1' \leq M_1^\alpha M_2^p, \quad M_2' \leq M_1^q M_2^\beta, \quad \text{a.e.} \quad (37)$$

Noticing that $q - \alpha + 1 > 0$ and $p - \beta + 1 > 0$, we have

$$\begin{aligned} & (M_1^{q-\alpha+1}(t) + M_2^{p-\beta+1}(t))' \\ & \leq (p + q - \alpha - \beta + 2) M_1^\beta(t) M_2^\alpha(t) \\ & \leq K_2 (M_1^{q-\alpha+1}(t) \\ & \quad + M_2^{p-\beta+1}(t))^{((q-\alpha+1)q + (p-\beta+1)p)/((q-\alpha+1)(p-\beta+1))} \end{aligned} \quad (38)$$

by virtue of Young's inequality. Integrating (38) from t to T^* , we can get (36). The proof is complete. $\square$

Lemma 10. If $p - \beta \geq q - \alpha \geq 1$, $f(x, y) = g(x, y)$, and $\int_{\Omega} f(x, y) dy \leq 1$ for any $x \in \partial\Omega$, then there exists a positive constant c_0 such that the solution (u, v) of (1) with positive initial value (u_0, v_0) satisfies

$$u(x, t) \geq c_0 v^{(p-\beta+1)/(q-\alpha+1)}(x, t), \quad (x, t) \in \Omega \times [0, T^*). \quad (39)$$

Proof. Let $J(x, t) = u(x, t) - c_0 v^{(p-\beta+1)/(q-\alpha+1)}(x, t)$, where c_0 is a positive constant to be chosen. For $(x, t) \in \Omega \times (0, T^*)$, a series of calculations shows that

$$\begin{aligned} J_t - \Delta J \\ = u_t - c_0 \frac{p-\beta+1}{q-\alpha+1} v^{(p-q+\alpha-\beta)/(q-\alpha+1)} v_t - \Delta u \\ + c_0 \frac{(p-\beta+1)(p-q+\alpha-\beta)}{q-\alpha+1} v^{(p-2\alpha-2q-\beta-1)/(q-\alpha+1)} \end{aligned}$$

$$\begin{aligned} & \times |\nabla u|^2 + c_0 \frac{p-\beta+1}{q-\alpha+1} v^{(p-q+\alpha-\beta)/(q-\alpha+1)} \Delta v \\ & \geq u^\alpha v^p - c_0 \frac{p-\beta+1}{q-\alpha+1} v^{(p-q+\alpha-\beta)/(q-\alpha+1)} u^q v^\beta \\ & = u^\alpha v^{\beta+(p-q+\alpha-\beta)/(q-\alpha+1)} \\ & \quad \times \left[v^{(p-\beta+1)(q-\alpha)/(q-\alpha+1)} - c_0 \frac{p-\beta+1}{q-\alpha+1} u^{q-\alpha} \right] \\ & = u^\alpha v^{\beta+(p-q+\alpha-\beta)/(q-\alpha+1)} \\ & \quad \times \left[c_0^{\alpha-q} (u-J)^{q-\alpha} - c_0 \frac{p-\beta+1}{q-\alpha+1} u^{q-\alpha} \right]. \end{aligned} \quad (40)$$

If we choose $c_0 \leq [(q-\alpha+1)/(p-\beta+1)]^{1/(q-\alpha+1)}$, then $c_0^{\alpha-q} \geq c_0((p-\beta+1)/(q-\alpha+1))$. So, we have

$$\begin{aligned} J_t - \Delta J + c_0(q-\alpha) \frac{p-\beta+1}{q-\alpha+1} u^\alpha \\ v^{\beta+(p-q+\alpha-\beta)/(q-\alpha+1)} \theta^{q-\alpha+1}(u, v) J \geq 0, \end{aligned} \quad (41)$$

where $\theta(u, v)$ is a function of u and v , which lines between $u - J$ and u .

When $(x, t) \in \partial\Omega \times (0, T^*)$, we have

$$\begin{aligned} J(x, t) &= \int_{\Omega} f(x, y) u(y, t) dy \\ &\quad - c_0 \left[\int_{\Omega} f(x, y) v(y, t) dy \right]^{(p-\beta+1)/(q-\alpha+1)}. \end{aligned} \quad (42)$$

Denote that $H(x) = \int_{\Omega} f(x, y) dy \geq 0$ for any $x \in \partial\Omega$. Since $f(x, \cdot) \not\equiv 0$ for any $x \in \partial\Omega$, $H(x) > 0$. It follows from Jensen's inequality $H(x) \leq 1$ and $(p-\beta+1)/(q-\alpha+1) \geq 1$ that

$$\begin{aligned} & \int_{\Omega} f(x, y) v^{(p-\beta+1)/(q-\alpha+1)}(y, t) dy \\ & \quad - \left[\int_{\Omega} f(x, y) v(y, t) dy \right]^{(p-\beta+1)/(q-\alpha+1)} \\ & \geq H(x) \left[\int_{\Omega} f(x, y) \frac{v(y, t)}{H(x)} dy \right]^{(p-\beta+1)/(q-\alpha+1)} \\ & \quad - \left[\int_{\Omega} f(x, y) v(y, t) dy \right]^{(p-\beta+1)/(q-\alpha+1)} \geq 0. \end{aligned} \quad (43)$$

Combining the above inequality with (42), we obtain

$$\begin{aligned} J(x, t) &\geq \int_{\Omega} f(x, y) u(y, t) dy \\ &\quad - c_0 \int_{\Omega} f(x, y) v^{(p-\beta+1)/(q-\alpha+1)}(y, t) dy \\ &= \int_{\Omega} f(x, y) J(y, t) dy. \end{aligned} \quad (44)$$

For the initial condition, we have

$$J(x, 0) = u_0(x) - c_0 v_0^{(p-\beta+1)/(q-\alpha+1)}(x) \geq 0 \quad (45)$$

on $\bar{\Omega}$ provided that $c_0 \leq \inf_{x \in \bar{\Omega}} u_0(x) v_0^{-(p-\beta+1)/(q-\alpha+1)}(x)$.

Summarily, if we take c_0 small enough such that

$$c_0 \leq \min \left\{ \inf_{x \in \bar{\Omega}} u_0(x) v_0^{-(p-\beta+1)/(q-\alpha+1)}(x), \left[\frac{q-\alpha+1}{p-\beta+1} \right]^{1/(q-\alpha+1)} \right\}, \quad (46)$$

it follows from (41), (44), (45), and [32, Theorem 2.1] that $J(x, t) \geq 0$, which implies (39). The proof is complete. $\square$

Combining (39) with (1), we know that the solution (u, v) of (1) satisfies

$$\begin{aligned} u_t &\leq \Delta u + a u^\lambda, \quad v_t \leq \Delta v + b v^\mu, \quad x \in \Omega, \quad 0 < t < T^*, \\ u(x, t) &= \int_{\Omega} f(x, y) u(x, y) dy, \\ v(x, t) &= \int_{\Omega} g(x, y) v(x, y) dy, \quad x \in \partial\Omega, \quad 0 < t < T^*, \\ u(x, 0) &= u_0(x), \quad v(x, 0) = v_0(x), \quad x \in \bar{\Omega} \end{aligned} \quad (47)$$

if $f(x, y) = g(x, y)$, where $a = [1/c_0]^{(p(q-\alpha+1))/(p-\beta+1)}$, $b = [1/c_0]^{(\beta(q-\alpha+1))/(p-\beta+1)}$, $\lambda = (p(q+1) + \alpha(1-\beta))/(p-\beta+1)$, and $\mu = (q(p+1) + \beta(1-\alpha))/(p-\beta+1)$. It is easy to see that $\lambda > 1$ and $\mu > 1$ if $p-\beta \geq q-\alpha \geq 1$.

Let (w, z) be the solution of the following system:

$$\begin{aligned} w_t &= \Delta w + a w^\lambda, \quad z_t = \Delta z + b w^\mu, \quad x \in \Omega, \quad 0 < t < T^*, \\ w(x, t) &= \int_{\Omega} f(x, y) w(x, y) dy, \\ z(x, t) &= \int_{\Omega} f(x, y) z(x, y) dy, \quad x \in \partial\Omega, \quad 0 < t < T^*, \\ w(x, 0) &= u_0(x), \quad z(x, 0) = v_0(x), \quad x \in \bar{\Omega}. \end{aligned} \quad (48)$$

It is easy to see that $(w, z) \geq (u, v)$ by Remark 7 if $\int_{\Omega} f(x, y) dy \leq 1$ for any $x \in \partial\Omega$.

Lemma 11. Suppose that $u_0(x)$ and $v_0(x)$ satisfy (H1)-(H2) and that the assumptions in Lemma 10 hold; then the solution (w, z) of (48) satisfies

$$w_t - a \delta w^\lambda \geq 0, \quad z_t - b \delta w^\mu \geq 0, \quad (x, t) \in \Omega \times (0, T^*) \quad (49)$$

if ε_0 is small enough such that $0 < \varepsilon_0 < \min\{a, b\}$ and $\delta \leq \min\{1 - \varepsilon_0/a, 1 - \varepsilon_0/b\}$.

Proof. Let $J_1(x, t) = w_t - a \delta w^\lambda$ and $J_2(x, t) = z_t - b \delta w^\mu$. For $(x, t) \in \Omega \times (0, T^*)$, a series of calculations shows that

$$J_{1t} - \Delta J_1 \geq a \lambda w^{\lambda-1} J_1, \quad J_{2t} - \Delta J_2 \geq b \mu w^{\mu-1} J_1. \quad (50)$$

For $(x, t) \in \partial\Omega \times (0, T^*)$, using the boundary conditions we have

$$\begin{aligned} J_1(x, t) &= w_t - a \delta w^\lambda \\ &= \int_{\Omega} f(x, y) w_t(y, t) dy \\ &\quad - a \delta \left[\int_{\Omega} f(x, y) w(y, t) dy \right]^\lambda \\ &= \int_{\Omega} f(x, y) (J_1 + a \delta w^\lambda)(y, t) dy \\ &\quad - a \delta \left[\int_{\Omega} f(x, y) w(y, t) dy \right]^\lambda \\ &= \int_{\Omega} f(x, y) J_1(x, y) dy \\ &\quad + a \delta \left[\int_{\Omega} f(x, y) w^\lambda(y, t) dy \right. \\ &\quad \left. - \left[\int_{\Omega} f(x, y) w(y, t) dy \right]^\lambda \right]. \end{aligned} \quad (51)$$

It follows from $\int_{\Omega} f(x, y) dy \leq 1$ for any $x \in \partial\Omega$ and Jensen's inequality that

$$\int_{\Omega} f(x, y) w^\lambda(y, t) dy \geq \left[\int_{\Omega} f(x, y) w(y, t) dy \right]^\lambda. \quad (52)$$

Thus,

$$J_1(x, t) \geq \int_{\Omega} f(x, y) J_1(x, y) dy, \quad (x, t) \in \partial\Omega \times (0, T^*). \quad (53)$$

Similarly, we have

$$J_2(x, t) \geq \int_{\Omega} f(x, y) J_2(x, y) dy, \quad (x, t) \in \partial\Omega \times (0, T^*). \quad (54)$$

For the initial condition, under assumption (H2), we have

$$J_1(x, 0) \geq 0, \quad J_2(x, 0) \geq 0, \quad x \in \bar{\Omega} \quad (55)$$

if ε_0 is small enough such that $0 < \varepsilon_0 < \min\{a, b\}$ and $\delta \leq \min\{1 - \varepsilon_0/a, 1 - \varepsilon_0/b\}$. Then (49) follows from (50)–(55). The proof is complete. $\square$

Proof of Theorem 3. Integrating the inequality for w in (49) on $[t, T^*)$ yields

$$w(x, t) \leq c_1 (T^* - t)^{1/(1-\lambda)}, \quad (x, t) \in \bar{\Omega} \times [0, T^*), \quad (56)$$

where $c_1 = (ad(\lambda - 1))^{1/(1-\lambda)}$. Since $u(x, t) \leq w(x, t)$, we obtain

$$M_1(t) \leq c_1(T^* - t)^{1/(1-\lambda)} = c_1(T^* - t)^{-(p-\beta+1)/(pq-(1-\alpha)(1-\beta))}. \quad (57)$$

Combining (36) and (57), we get

$$C_1 \leq \max_{x \in \Omega} u(x, t) (T^* - t)^{(p-\beta+1)/(pq-(1-\alpha)(1-\beta))} \leq C_2, \quad (58)$$

where C_1 and C_2 are two positive constants.

Since $M_2' \leq M_1^q M_2^\beta$, it follows from (57) that

$$M_2^{-\beta} M_2' \leq c_1^q (T^* - t)^{-q(p-\beta+1)/(pq-(1-\alpha)(1-\beta))}. \quad (59)$$

Integrating this inequality from 0 to t , we get

$$M_2(t) \leq c_2(T^* - t)^{-(q-\alpha+1)/(pq-(1-\alpha)(1-\beta))}, \quad (60)$$

where c_2 is a positive constant. Combining (36) and (60), we get

$$C_3 \leq \max_{x \in \Omega} v(x, t) (T^* - t)^{(q-\alpha+1)/(pq-(1-\alpha)(1-\beta))} \leq C_4, \quad (61)$$

where C_3 and C_4 are two positive constants. We completed the proof of Theorem 3. $\square$

Proof of Theorem 4. The proof is similar to the proof of Theorem 3, and so we omit it. $\square$

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