Research Article

Propagation of Rayleigh Wave in a Two-Temperature Generalized Thermoelastic Solid Half-Space

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The Rayleigh surface wave is studied at a stress-free thermally insulated surface of an isotropic, linear, and homogeneous two-temperature thermoelastic solid half-space in the context of Lord and Shulman theory of generalized thermoelasticity. The governing equations of a two-temperature generalized thermoelastic medium are solved for surface wave solutions. The appropriate particular solutions are applied to the required boundary conditions to obtain the frequency equation of the Rayleigh wave. Some special cases are also derived. The speed of Rayleigh wave is computed numerically and shown graphically to show the dependence on the frequency and two-temperature parameter.

1. Introduction

Lord and Shulman [1] and Green and Lindsay [2] extended the classical dynamical coupled theory of thermoelasticity to generalized thermoelastic theories. These theories treat heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostojă-Starzewski [3] explained in detail the above theories in their book *Thermoelasticity with Finite Wave Speeds*. The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [4]. Wave propagation in thermoelasticity has many applications in various engineering fields. Some problems on wave propagation in coupled or generalized thermoelasticity are studied by various researchers, for example, Deresiewicz [5], Sinha and Sinha [6], Sinha and Elsibai [7, 8], Sharma et al. [9], Othman and Song [10], Singh [11, 12], and many more.

Gurtin and Williams [13, 14] proposed the second law of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another temperature. Based on this law, Chen and Gurtin [15] and Chen et al. [16, 17] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T . The two-temperature

theory involves a material parameter $a^* > 0$. The limit $a^* \rightarrow 0$ implies that $\Phi \rightarrow T$, and the classical theory can be recovered from two-temperature theory. The two-temperature model has been widely used to predict the electron and phonon temperature distributions in ultrashort laser processing of metals. According to Warren and Chen [18], these two temperatures can be equal in time-dependent problems under certain conditions, whereas Φ and T are generally different in particular problems involving wave propagation. Following Boley and Tolins [19], they studied the wave propagation in the two-temperature theory of coupled thermoelasticity. They showed that the two temperatures, T and Φ , and the strain are represented in the form of a traveling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [20] studied the propagation of harmonic plane waves in two-temperature theory. Quintanilla and Jordan [21] derived exact solutions of two initial-boundary value problems in the two-temperature theory with dual-phase-lag delay. Youssef [22] developed a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [23] extended the work of Puri and Jordan [20] in the context of the linear theory of two-temperature generalized thermoelasticity developed by Youssef [22]. Magaña and Quintanilla [24] showed the uniqueness and growth of solutions in two-temperature

generalized thermoelastic theories. Recently, Youssef [25] formulated a theory of two-temperature thermoelasticity without energy dissipation.

In this paper, the Youssef [22] theory is followed for the theoretical study of the Rayleigh wave at the thermally insulated stress-free surface of an isotropic two-temperature thermoelastic solid half-space. The frequency equation of the Rayleigh wave is obtained. The frequency equation is also approximated by assuming small thermal coupling. The dependence of numerical values of the speed of the Rayleigh wave on material parameters, frequency, and two-temperature parameters is shown graphically for a particular material of the model.

2. Basic Equations

We consider an isotropic, linear, and homogeneous two-temperature thermoelastic solid half-space. Following Youssef [22], the governing equations for a two-temperature generalized thermoelastic half-space with one relaxation time are as follows:

(i) the stress-strain-temperature relations

$$\sigma_{ij} = c_{ijkl}e_{kl} - \gamma_{ij}(T - \Phi_0), \quad (1)$$

(ii) the displacement-strain relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (2)$$

(iii) the equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + \rho F_i, \quad (3)$$

(iv) the energy equation

$$-q_{i,i} = \rho T_0 \dot{S}, \quad (4)$$

(v) the modified Fourier's law

$$-K_{ij}\Phi_{,j} = q_i + \tau_0 \dot{q}_i, \quad (5)$$

(vi) the entropy-strain-temperature relation

$$\rho S = \frac{\rho c_E}{T_0} \theta + \gamma_{ij} e_{ij}. \quad (6)$$

Here, γ_{ij} are the coupling parameters, T is the mechanical temperature, $\Phi_0 = T_0$ is the reference temperature, $\theta = T - T_0$ with $|\theta/T_0| \ll 1$, σ_{ij} is the stress tensor, e_{kl} is the strain tensor, c_{ijkl} is the tensor of elastic constants, ρ is the mass density, q_i is the heat conduction vector, K_{ij} is the thermal conductivity tensor, c_E is the specific heat at constant strain, u_i are the components of the displacement vector, S is the entropy per unit mass, τ_0 is the thermal relaxation time, which will ensure that the heat conduction equation will predict finite speeds of heat propagation, and Φ is the conductive temperature and satisfies the relation

$$\Phi - T = a^* \Phi_{,ii}, \quad (7)$$

where $a^* > 0$ is the two-temperature parameter.

3. Analytical 2D Solution

We consider a homogeneous and isotropic thermoelastic medium of an infinite extent with Cartesian coordinates system (x, y, z) , which is previously at uniform temperature. The origin is taken on the plane surface, and z -axis is taken normally into the medium ($z \geq 0$). The surface $z = 0$ is assumed to be stress-free and thermally insulated. The present study is restricted to the plane strain parallel to x - z plane, with the displacement vector $\mathbf{u} = (u_1, 0, u_3)$. With the help of (1)–(3), we obtain the following two components of the equation of motion:

$$\begin{aligned} (\lambda + 2\mu) u_{1,11} + (\lambda + \mu) u_{3,13} + \mu u_{1,33} - \gamma \theta_{,1} &= \rho \ddot{u}_1, \\ (\lambda + 2\mu) u_{3,33} + (\lambda + \mu) u_{1,13} + \mu u_{3,11} - \gamma \theta_{,3} &= \rho \ddot{u}_3. \end{aligned} \quad (8)$$

Equations (4)–(6) lead to the following heat conduction equation:

$$\begin{aligned} K(\Phi_{,11} + \Phi_{,33}) &= \rho c_E (\dot{\theta} + \tau_0 \ddot{\theta}) \\ &+ \gamma T_0 (\dot{u}_{1,1} + \tau_0 \ddot{u}_{1,1}) + \gamma T_0 (\dot{u}_{3,3} + \tau_0 \ddot{u}_{3,3}), \end{aligned} \quad (9)$$

and (7) becomes

$$\Phi - T = a^* (\Phi_{,11} + \Phi_{,33}). \quad (10)$$

The displacement components u_1 and u_3 are written in terms of potentials q and ψ as

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \quad (11)$$

Using (11) in (8)–(10), we obtain

$$\begin{aligned} (\lambda + 2\mu) \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) - \gamma \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right] \\ = \rho \frac{\partial^2 q}{\partial t^2}, \end{aligned} \quad (12)$$

$$\begin{aligned} K(\Phi_{,11} + \Phi_{,33}) &= \rho c_E \left(\frac{\partial \Phi}{\partial t} + \tau_0 \frac{\partial^2 \Phi}{\partial t^2} \right) \\ &- a^* \rho c_E \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \\ &+ \gamma T_0 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right), \\ \mu \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \right) &= \rho \frac{\partial^2 \psi}{\partial t^2}. \end{aligned} \quad (14)$$

For thermoelastic surface waves in the half-space propagating in x -direction, the potential functions Φ , q , and ψ are taken in the following form:

$$(\Phi, q, \psi) = (\widehat{\Phi}(z), \widehat{q}(z), \widehat{\psi}(z)) \exp i(\eta x - \chi t), \quad (15)$$

where $\chi^2 = \eta^2 c^2$, η is the wave number, and c is the phase velocity.

Substituting (15) with (12) and (13) and eliminating $\widehat{q}$, $\widehat{\Phi}$, we obtain the following auxiliary equation:

$$D^4 - AD^2 + B = 0, \quad (16)$$

where $D = d/dz$,

$$\begin{aligned} A &= 2\eta^2 + \eta^2 c^2 \left[\frac{c_1^2 + \epsilon + (\overline{K} - a^* \chi^2)}{a^* \epsilon \chi^2 - c_1^2 (\overline{K} - a^* \chi^2)} \right], \\ B &= \eta^4 + \eta^4 c^2 \left[\frac{c_1^2 + \epsilon + (\overline{K} - a^* \chi^2)}{a^* \epsilon \chi^2 - c_1^2 (\overline{K} - a^* \chi^2)} \right] \\ &\quad - \eta^4 c^4 \left[\frac{1}{a^* \epsilon \chi^2 - c_1^2 (\overline{K} - a^* \chi^2)} \right], \quad (17) \\ \epsilon &= \frac{\overline{\gamma}^2 T_0}{c_E \tau^*}, \quad \overline{K} = \frac{K}{\rho c_E \tau^*}, \quad \tau^* = \tau_0 + \frac{i}{\chi}, \\ \overline{\gamma} &= \frac{\gamma}{\rho}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}. \end{aligned}$$

With the help of (16) and keeping in mind that $\widehat{q}, \widehat{\Phi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, the solutions q, Φ are written as

$$\begin{aligned} q &= [A \exp(-\eta \beta_1 z) + B \exp(-\eta \beta_2 z)] \exp i(\eta x - \chi t), \\ \Phi &= [\zeta_1 A \exp(-\eta \beta_1 z) + \zeta_2 B \exp(-\eta \beta_2 z)] \exp i(\eta x - \chi t), \end{aligned} \quad (18)$$

where

$$\beta_1^2 = \frac{1}{2\eta^2} [A + \sqrt{A^2 - 4B}], \quad (19)$$

$$\beta_2^2 = \frac{1}{2\eta^2} [A - \sqrt{A^2 - 4B}],$$

$$\zeta_i = \frac{\chi^2 - c_1^2 (1 - \beta_i^2) \eta^2}{\overline{\gamma} [1 + a^* \eta^2 (1 - \beta_i^2)]}, \quad (i = 1, 2). \quad (20)$$

Substituting (15) with (14) and keeping in mind that $\widehat{\psi} \rightarrow 0$ as $z \rightarrow \infty$ for surface waves, we obtain the following solution:

$$\psi = C \exp [-\eta \beta_3 z + i(\eta x - \chi t)], \quad (21)$$

where

$$\beta_3^2 = 1 - \frac{c^2}{c_2^2}, \quad c_2^2 = \frac{\mu}{\rho}. \quad (22)$$

4. Derivation of Frequency Equation

The mechanical and thermal conditions at the thermally insulated surface $z = 0$ are as follows:

(i) vanishing of the normal stress component

$$\sigma_{zz} = 0, \quad (23)$$

(ii) vanishing of the tangential stress component

$$\sigma_{zx} = 0, \quad (24)$$

(iii) vanishing of the normal heat flux component

$$\frac{\partial \Phi}{\partial z} = 0, \quad (25)$$

where

$$\begin{aligned} \sigma_{zz} &= \lambda \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2} \right) + 2\mu \left(\frac{\partial^2 \psi}{\partial x \partial z} \right) \\ &\quad + 2\mu \frac{\partial^2 q}{\partial z^2} - \gamma \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} \right) \right], \quad (26) \\ \sigma_{zx} &= \mu \left[2 \frac{\partial^2 q}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} \right]. \end{aligned}$$

Making use of solutions (18) and (21) for q, Φ , and Ψ in (23) to (25) and eliminating A, B , and C , we obtain the following equation:

$$\begin{aligned} &4\mu\eta^2\beta_1\beta_2\beta_3(\zeta_2 - \zeta_1) - (1 + \beta_3^2) \\ &\quad \times [\eta^2 \{-\lambda(\zeta_2\beta_2 - \zeta_1\beta_1) + (\lambda + 2\mu)\beta_1\beta_2(\zeta_2\beta_1 - \zeta_1\beta_2)\} \\ &\quad - \rho c_1^2 \zeta_1 \zeta_2 (\beta_2 - \beta_1) \{1 + a^* \eta^2 (1 + \beta_1\beta_2)\}] = 0, \end{aligned} \quad (27)$$

which is the frequency equation of thermoelastic Rayleigh wave in a two-temperature generalized thermoelastic medium in the context of the Lord and Shulman [1] theory.

5. Special Cases

5.1. Small Thermal Coupling ($\epsilon \ll 1$). For most of materials, ϵ is small at a normal temperature. For $\epsilon \ll 1$ and using (19), we obtain the following approximated relations as

$$\begin{aligned} \beta_1^2 + \beta_2^2 &\equiv 2 - c^2 \left\{ \frac{c_1^2 + (\bar{K} - a^* \chi^2)}{c_1^2 (\bar{K} - a^* \chi^2)} \right\} \\ &\times \left[1 + \epsilon \left\{ \frac{1}{c_1^2 + (\bar{K} - a^* \chi^2)} + \frac{a^* \chi^2}{c_1^2 (\bar{K} - a^* \chi^2)} \right\} \right], \\ \beta_1^2 \beta_2^2 &\equiv 1 - c^2 \left\{ \frac{c_1^2 + (\bar{K} - a^* \chi^2)}{c_1^2 (\bar{K} - a^* \chi^2)} \right\} \\ &\times \left[1 + \epsilon \left\{ \frac{1}{c_1^2 + (\bar{K} - a^* \chi^2)} + \frac{a^* \chi^2}{c_1^2 (\bar{K} - a^* \chi^2)} \right\} \right] \\ &+ \frac{c^4}{c_1^2 (\bar{K} - a^* \chi^2)} \left[1 + \epsilon \frac{a^* \chi^2}{c_1^2 (\bar{K} - a^* \chi^2)} \right]. \end{aligned} \quad (28)$$

With the help of these approximations, we can approximate β_1 , β_2 , and the coupling coefficients ζ_1 and ζ_2 . Finally, the frequency equation (27) can be approximated. Further, if we consider $\epsilon \rightarrow 0$, $a^* \rightarrow 0$, we obtain $\beta_1 \equiv \sqrt{1 - c^2/c_1^2}$ and $\beta_2 \equiv \sqrt{1 - c^2/\bar{K}}$.

For numerical purpose, we put $c^2 = c^{*2} + \epsilon(\xi_1 + i\xi_2)$, where c^* is the classical Rayleigh wave velocity, and ξ_1 and ξ_2 are two reals, then

$$\eta = \frac{\chi}{c^*} \left(1 - \frac{\epsilon \xi_1}{2c^{*2}} - i \frac{\epsilon \xi_2}{2c^{*2}} \right). \quad (29)$$

The velocity of propagation is equal to $(c^* + \epsilon \xi_1 / 2c^*)$, and the amplitude attenuation factor is equal to $\exp[\epsilon \chi \xi_2 x / 2c^{*3}]$ with $\xi_2 < 0$.

5.2. Small Frequency ($\chi \ll 1$). For $\chi \ll 1$ and using (19), we obtain the following approximated relations as

$$\begin{aligned} \beta_1^2 + \beta_2^2 &\equiv 2 - c^2 \left\{ \frac{c_1^2 + \epsilon + \bar{K}}{c_1^2 \bar{K}} \right\} \\ &\times \left[1 + \chi^2 \left\{ \frac{(1 + \epsilon) a^*}{c_1^2 \bar{K}} - \frac{a^*}{c_1^2 + \epsilon + \bar{K}} \right\} \right], \end{aligned}$$

$$\begin{aligned} \beta_1^2 \beta_2^2 &\equiv 1 - c^2 \left\{ \frac{c_1^2 + \epsilon + \bar{K}}{c_1^2 \bar{K}} \right\} \\ &\times \left[1 + \chi^2 \left\{ \frac{(1 + \epsilon) a^*}{c_1^2 \bar{K}} - \frac{a^*}{c_1^2 + \epsilon + \bar{K}} \right\} \right] \\ &+ \frac{c^4}{c_1^2 \bar{K}} \left[1 + \chi^2 \frac{(1 + \epsilon) a^*}{c_1^2 \bar{K}} \right]. \end{aligned} \quad (30)$$

With the help of these approximations, we can approximate β_1 , β_2 , and the coupling coefficients ζ_1 and ζ_2 . Finally, the frequency equation (27) can be approximated. Also, if we consider $\chi \rightarrow 0$, $\epsilon \rightarrow 0$, we obtain $\beta_1 \equiv \sqrt{1 - c^2/c_1^2}$ and $\beta_2 \equiv \sqrt{1 - c^2/\bar{K}}$.

5.3. Isotropic Elastic Case. If we neglect thermal parameters, then the frequency equation (27) reduces to

$$\left(2 - \frac{c^2}{c_2^2} \right)^2 = 4 \sqrt{1 - \frac{c^2}{c_1^2}} \sqrt{1 - \frac{c^2}{c_2^2}}, \quad (31)$$

which is the frequency equation of Rayleigh wave for an isotropic elastic case.

6. Numerical Example

The speed of propagation is computed for the following material parameters of aluminium metal: $\lambda = 5.775 \times 10^{11}$ Dyn·cm⁻², $\mu = 2.646 \times 10^{11}$ Dyn·cm⁻², $\rho = 2.7$ g·cm⁻³, $c_E = 0.236$ Cal·g⁻¹ · °C⁻¹, $K = 0.492$ Cal·cm⁻¹·s⁻¹ · °C⁻¹, $\gamma = 0.05(3\lambda + 2\mu)$, $T_0 = 27^\circ\text{C}$, and $c^* = 0.9554$.

The nondimensional speed of Rayleigh wave is shown graphically against the frequency (2–20 Hz) in Figure 1, when the two-temperature parameter $a^* = 0.5$. With the increase in frequency, it increases very sharply to its maximum value near $\chi = 4.3$ Hz, and then it decreases slowly for the higher frequency range. The nondimensional speed of Rayleigh wave is also shown graphically against the two-temperature parameter a^* in Figure 2 for $\chi = 10$ Hz. With the increase in the value of two-temperature parameter, it increases very sharply to its maximum value at $a^* = 0.47$. Thereafter, it decreases sharply to its minimum value.

7. Conclusion

The appropriate solutions of all the governing equations of a two-temperature generalized thermoelastic medium are applied at the boundary conditions at a thermally insulated free surface of a half-space to obtain the frequency equation of Rayleigh wave in the context of the Lord and Shulman [1] theory. The frequency equation is approximated for the case of small thermal coupling and small frequency and reduced for isotropic elastic case. From the frequency equation of Rayleigh wave, it is observed that the phase speed of Rayleigh wave depends on various material parameters including the two-temperature parameter. The dependence of numerical

FIGURE 1: Variations of the nondimensional speed of propagation of Rayleigh wave against the frequency (χ) when $a^* = 0.5$.

FIGURE 2: Variations of the nondimensional speed of propagation of Rayleigh wave against the two-temperature parameter (a^*) when $\chi = 10$ Hz.

values of speed of propagation on the frequency and two-temperature parameter is shown graphically for a particular material representing the model. The present problem is of geophysical interest, particularly in investigations concerned with earthquakes and other phenomena in seismology. The applications range from geophysical problems to quantitative nondestructive evaluation of mechanical structures and acoustic tomography for medical purposes. The problem

though is theoretical, but it can provide useful information for experimental researchers working in the field of geophysics and earthquake engineering and seismologist working in the field of mining tremors and drilling into the crust of the earth.

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