Research Article

Remarks on G -Metric Spaces

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In 2005, Mustafa and Sims (2006) introduced and studied a new class of generalized metric spaces, which are called G -metric spaces, as a generalization of metric spaces. We establish some useful propositions to show that many fixed point theorems on (nonsymmetric) G -metric spaces given recently by many authors follow directly from well-known theorems on metric spaces. Our technique can be easily extended to other results as shown in application.

1. Introduction

The literature of the last decades is rich of papers that focus on all matters related to the generalized metric spaces (i.e., $d(x, y) \in \mathbb{R}_+$; $d(x, y) \in K$, K a cone in an ordered Banach space; 2-metric spaces; probabilistic metric spaces, etc, see, e.g., [1–4]).

In 2005, Mustafa and Sims introduced a new class of generalized metric spaces (see [5, 6]), which are called G -metric spaces as a generalization of metric spaces. Subsequently, many fixed point results on such spaces appeared in [6–11].

Here, we present the necessary definitions and results in G -metric spaces, which will be useful for the rest of the paper. However, for more details, we refer to [5, 6].

Definition 1. Let X be a nonempty set. Suppose that $G : X \times X \times X \rightarrow [0, +\infty)$ is a function satisfying the following conditions:

- (a) $G(x, y, z) = 0$ if and only if $x = y = z$;
- (b) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$;
- (c) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$;
- (d) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ (symmetry in all three variables);
- (e) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then G is called a G -metric on X and (X, G) is called a G -metric space.

Definition 2. A G -metric space (X, G) is said to be symmetric if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Definition 3. Let (X, G) be a G -metric space. We say that $\{x_n\}$ is

- (i) a G -Cauchy sequence if, for any $\varepsilon > 0$, there is an $N \in \mathbb{N}$ (the set of all positive integers) such that for all $n, m, l \geq N$, $G(x_n, x_m, x_l) < \varepsilon$;
- (ii) a G -convergent sequence to $x \in X$ if, for any $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that for all $n, m \geq N$, $G(x, x_n, x_m) < \varepsilon$.

A G -metric space (X, G) is said to be complete if every G -Cauchy sequence in X is G -convergent in X .

Proposition 4. Let (X, G) be a G -metric space. The following are equivalent:

- (1) $\{x_n\}$ is G -convergent to x ;
- (2) $G(x_n, x_n, x) \rightarrow 0$ as $n \rightarrow +\infty$;
- (3) $G(x_n, x, x) \rightarrow 0$ as $n \rightarrow +\infty$.

Proposition 5. Let (X, G) be a G -metric space. Then the following are equivalent:

- (i) the sequence $\{x_n\}$ is G -Cauchy;
- (ii) $G(x_n, x_m, x_m) \rightarrow 0$ as $n, m \rightarrow +\infty$.

An interesting observation is that any G -metric space (X, G) induces a metric d_G on X given by

$$d_G(x, y) = G(x, y, y) + G(y, x, x), \quad \forall x, y \in X. \quad (1)$$

Moreover, (X, G) is G -complete if and only if (X, d_G) is complete.

It was observed that in the symmetric case $((X, G)$ is symmetric), many fixed point theorems on G -metric spaces are particular cases of existing fixed point theorems in metric spaces. In this paper, we shall show that also in the nonsymmetric case, many results given recently on such spaces follow directly from existing results on metric spaces. This is done by using as key results some propositions. Our technique can be easily extended to other results as shown in application.

2. Preliminaries: Fixed Point Results on Metric Spaces

In this section, we recall some well-known fixed point theorems on metric spaces.

The following Ćirić's theorem for quasicontractive mappings [12] plays a key role in our paper.

Theorem 6 (see Ćirić [12]). *Let (X, d) be a complete metric space and let $T : X \rightarrow X$ be a mapping with the property:*

$$\begin{aligned} d(Tx, Ty) \\ \leq q \max \{d(x, y), d(x, Tx), d(y, Ty), \\ d(x, Ty), d(y, Tx)\}, \end{aligned} \quad (2)$$

for all $x, y \in X$, where q is a constant such that $q \in [0, 1)$. Then T has a unique fixed point.

The next result is a consequence of Theorem 2.1 of [13]. For details about this refer to Das and Naik [14].

Theorem 7. *Let C be a subset of a metric space (X, d) , and let $T, S : C \rightarrow C$ be weakly compatible self-mappings of C . Assume that the range of S contains the range of T , SX is a complete subspace of X , and T and S satisfy the condition:*

$$\begin{aligned} d(Tx, Ty) \\ \leq q \max \{d(Sx, Sy), d(Sx, Tx), \\ d(Sy, Ty), d(Sx, Ty), d(Sy, Tx)\}, \end{aligned} \quad (3)$$

for all $x, y \in C$, where q is a constant such that $q \in [0, 1)$. Then T and S have a unique common fixed point.

The following result is a consequence of Theorem 2.3 of [15].

Theorem 8. *Let $T, S : X \rightarrow X$ be self-mappings of a metric space (X, d) such that $TX \subseteq SX$. Assume that there exist $F, \psi : [0, +\infty) \rightarrow [0, +\infty)$ such that*

- (i) F is nondecreasing, continuous, and $F(0) = 0 < F(t)$ for every $t > 0$;
- (ii) ψ is nondecreasing, right continuous, and $\psi(t) < t$ for every $t > 0$;
- (iii) $F(d(Tx, Ty)) \leq \psi(F(M(x, y)))$, for all $x, y \in X$,

where

$$\begin{aligned} M(x, y) = \max \left\{ d(Sx, Sy), d(Sx, Tx), d(Sy, Ty), \right. \\ \left. \frac{d(Sx, Ty) + d(Sy, Tx)}{2} \right\}. \end{aligned} \quad (4)$$

If one of TX and SX is a complete subspace of X , then T and S have a coincidence point. Further, if T and S are weakly compatible, then T and S have a unique common fixed point.

In 1962, Edelstein [16] proved the following version of the Banach Contraction Principle.

Theorem 9. *Let (X, d) be a compact metric space and let $T : X \rightarrow X$ be a given mapping. Assume that*

$$d(Tx, Ty) < d(x, y), \quad (5)$$

for all $x, y \in X$ with $x \neq y$. Then T has a unique fixed point.

3. Main Results

For our purpose we need the following propositions which are key results.

Proposition 10. *Let (X, G) be a G -metric space. Let $\delta_G : X \times X \rightarrow [0, +\infty)$ be defined by*

$$\delta_G(x, y) = \max \{G(x, y, y), G(y, x, x)\}, \quad (6)$$

for all $x, y \in X$. Then (X, δ_G) is a metric space.

Proof. Let $x, y, z \in X$. Clearly, $\delta_G(x, y) = \delta_G(y, x)$. Now, suppose that $\delta_G(x, y) = 0$. This implies that $G(x, y, y) = 0$, thus $x = y$. Finally, we have to prove the triangular inequality. We have

$$\begin{aligned} G(x, y, y) &\leq G(x, z, z) + G(z, y, y) \leq \delta_G(x, z) + \delta_G(z, y), \\ G(y, x, x) &\leq G(y, z, z) + G(z, x, x) \leq \delta_G(y, z) + \delta_G(z, x). \end{aligned} \quad (7)$$

Then we get that

$$\delta_G(x, y) \leq \delta_G(x, z) + \delta_G(z, y). \quad (8)$$

□

Proposition 11. *If (X, G) is a complete G -metric space then (X, δ_G) is a complete metric space.*

Proof. Let $\{x_n\}$ be a Cauchy sequence in (X, δ_G) . For all $n, m \in \mathbb{N}$, we have

$$G(x_n, x_m, x_m) \leq \delta_G(x_n, x_m). \quad (9)$$

Letting $n, m \rightarrow +\infty$ in the above inequality we get that $\lim_{n, m \rightarrow +\infty} G(x_n, x_m, x_m) = 0$, which implies that $\{x_n\}$ is a G -Cauchy sequence. Since (X, G) is a complete G -metric space, there exists some $x \in X$ such that $\{x_n\}$ is G -convergent to x . Then we have

$$\lim_{n \rightarrow +\infty} G(x_n, x, x) = \lim_{n \rightarrow +\infty} G(x, x_n, x_n) = 0, \quad (10)$$

which implies that $\lim_{n \rightarrow +\infty} \delta_G(x_n, x) = 0$. $\square$

Definition 12. Let (X, G) be a G -metric space and let A be a nonempty subset of X . The subset A is called sequentially G -compact if, for any sequence $\{x_n\}$ in A , there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ which G -converges to some $x \in A$.

Proposition 13. If (X, G) is a sequentially G -compact G -metric space, then (X, δ_G) is a compact metric space.

Proof. Let $\{x_n\}$ be a sequence in (X, δ_G) . Since (X, G) is a sequentially G -compact G -metric space, there exists some $x \in X$ and a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\{x_{n_k}\}$ is G -convergent to x . Then we have

$$\lim_{k \rightarrow +\infty} G(x_{n_k}, x, x) = \lim_{k \rightarrow +\infty} G(x, x_{n_k}, x_{n_k}) = 0, \quad (11)$$

which implies that $\lim_{k \rightarrow +\infty} \delta_G(x_{n_k}, x) = 0$, and hence (X, δ_G) is a compact metric space. $\square$

3.1. Discussion on Fixed Point Results of Mustafa and Obiedat. Recently, Mustafa and Obiedat [7] established the following result.

Theorem 14 (see Mustafa and Obiedat [7]). Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping satisfying the following condition:

$$\begin{aligned} G(Tx, Ty, Tz) \\ \leq k(G(x, Tx, Tx) + G(y, Ty, Ty) \\ + G(z, Tz, Tz)), \end{aligned} \quad (12)$$

for all $x, y, z \in X$, where k is a constant such that $k \in [0, 1/3)$. Then T has a unique fixed point.

Now, we prove the following.

Theorem 15. Theorem 14 is a particular case of Theorem 6.

Proof. Taking $z = y$ in (12), we get

$$G(Tx, Ty, Ty) \leq kG(x, Tx, Tx) + 2kG(y, Ty, Ty), \quad (13)$$

for all $x, y \in X$. Thus, we have

$$G(Tx, Ty, Ty) \leq q \max\{G(x, Tx, Tx), G(y, Ty, Ty)\}, \quad (14)$$

for all $x, y \in X$, where $q = 3k$. Note that $q \in [0, 1)$. Similarly, we can write that

$$G(Ty, Tx, Tx) \leq q \max\{G(x, Tx, Tx), G(y, Ty, Ty)\}, \quad (15)$$

for all $x, y \in X$. It follows from (14), (15), and the definition of δ_G that

$$\delta_G(Tx, Ty) \leq q \max\{\delta_G(x, Tx), \delta_G(y, Ty)\}, \quad (16)$$

for all $x, y \in X$. The existence and uniqueness of the fixed point are followed immediately by Theorem 6, Propositions 10 and 11. $\square$

In the same paper, the authors presented the following fixed point theorem.

Theorem 16 (see Mustafa and Obiedat [7]). Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping satisfying the following condition:

$$\begin{aligned} G(Tx, Ty, Tz) \\ \leq \alpha G(x, y, z) \\ + \beta(G(x, Tx, Tx) + G(y, Ty, Ty) + G(z, Tz, Tz)), \end{aligned} \quad (17)$$

for all $x, y, z \in X$, where α, β are nonnegative constants such that $\alpha + 3\beta < 1$. Then T has a unique fixed point.

Proceeding similarly as above we can prove the following.

Theorem 17. Theorem 16 is a particular case of Theorem 6.

Proof. Taking $z = y$ in (17), we get

$$\begin{aligned} G(Tx, Ty, Ty) \\ \leq \alpha G(x, y, y) + \beta G(x, Tx, Tx) + 2\beta G(y, Ty, Ty), \end{aligned} \quad (18)$$

for all $x, y \in X$. Thus, we have

$$\begin{aligned} G(Tx, Ty, Ty) \\ \leq q \max\{G(x, y, y), G(x, Tx, Tx), \\ G(y, Ty, Ty)\}, \end{aligned} \quad (19)$$

for all $x, y \in X$, where $q = \alpha + 3\beta$. Note that $q \in [0, 1)$. Similarly, we can write that

$$\begin{aligned} G(Ty, Tx, Tx) \\ \leq q \max\{G(y, x, x), G(x, Tx, Tx), \\ G(y, Ty, Ty)\}, \end{aligned} \quad (20)$$

for all $x, y \in X$. It follows from (19), (24), and the definition of δ_G that

$$\delta_G(Tx, Ty) \leq q \max\{\delta_G(x, y), \delta_G(x, Tx), \delta_G(y, Ty)\}, \quad (21)$$

for all $x, y \in X$. The existence and uniqueness of the fixed point are followed immediately by Theorem 6, Propositions 10 and 11. $\square$

Now, we consider another theorem.

Theorem 18 (see Mustafa and Obiedat [7]). *Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping satisfying the following condition:*

$$\begin{aligned} G(Tx, Ty, Tz) \\ \leq \alpha G(x, y, z) \\ + \beta \max \{G(x, Tx, Tx), G(y, Ty, Ty), \\ G(z, Tz, Tz)\}, \end{aligned} \quad (22)$$

for all $x, y, z \in X$, where α, β are nonnegative constants such that $\alpha + \beta < 1$. Then T has a unique fixed point.

In view of this result we give the following.

Theorem 19. *Theorem 18 is a particular case of Theorem 6.*

Proof. Taking $z = y$ in (22), we get

$$\begin{aligned} G(Tx, Ty, Ty) \\ \leq \alpha G(x, y, y) + \beta \max \{G(x, Tx, Tx), G(y, Ty, Ty)\}, \end{aligned} \quad (23)$$

for all $x, y \in X$. Thus, we have

$$\begin{aligned} G(Tx, Ty, Ty) \\ \leq q \max \{G(x, y, y), G(x, Tx, Tx), G(y, Ty, Ty)\}, \end{aligned} \quad (24)$$

for all $x, y \in X$, where $q = \alpha + \beta$. Note that $q \in [0, 1)$. Similarly, we can write that

$$\begin{aligned} G(Ty, Tx, Tx) \\ \leq q \max \{G(y, x, x), G(x, Tx, Tx), G(y, Ty, Ty)\}, \end{aligned} \quad (25)$$

for all $x, y \in X$. It follows from (24), (25), and the definition of δ_G that

$$\delta_G(Tx, Ty) \leq q \max \{\delta_G(x, y), \delta_G(x, Tx), \delta_G(y, Ty)\}, \quad (26)$$

for all $x, y \in X$. The existence and uniqueness of the fixed point are followed immediately by Theorem 6, Propositions 10 and 11. $\square$

3.2. Discussion on Fixed Point Results of Mustafa and Sims. Recently, Mustafa and Sims [9] established the following result.

Theorem 20 (see Mustafa and Sims [9]). *Let (X, G) be a complete G -metric space and let $T : X \rightarrow X$ be a mapping satisfying the following condition:*

$$\begin{aligned} G(Tx, Ty, Tz) \\ \leq k \max \{G(x, y, z), G(x, Tx, Tx), G(y, Ty, Ty), \\ G(z, Tz, Tz), G(x, Ty, Ty), \\ G(y, Tz, Tz), G(z, Tx, Tx)\}, \end{aligned} \quad (27)$$

for all $x, y, z \in X$, where k is a constant such that $k \in [0, 1/2)$. Then T has a unique fixed point.

We give the following.

Theorem 21. *Theorem 20 is a particular case of Theorem 6.*

Proof. Taking $z = y$ in (27), we get

$$\begin{aligned} G(Tx, Ty, Ty) \\ \leq k \max \{G(x, y, y), G(x, Tx, Tx), G(y, Ty, Ty), \\ G(x, Ty, Ty), G(y, Tx, Tx)\}, \end{aligned} \quad (28)$$

for all $x, y \in X$. Similarly, we can write that

$$\begin{aligned} G(Ty, Tx, Tx) \\ \leq k \max \{G(x, y, y), G(x, Tx, Tx), G(y, Ty, Ty), \\ G(x, Ty, Ty), G(y, Tx, Tx)\}, \end{aligned} \quad (29)$$

for all $x, y \in X$.

It follows from (28), (29), and the definition of δ_G that

$$\begin{aligned} \delta_G(Tx, Ty) \\ \leq k \max \{\delta_G(x, y), \delta_G(x, Tx), \\ \delta_G(y, Ty), \delta_G(x, Ty), \delta_G(y, Tx)\}, \end{aligned} \quad (30)$$

for all $x, y \in X$. The existence and uniqueness of the fixed point are followed immediately by Theorem 6, Propositions 10 and 11. $\square$

Remark 22. From the proof of Theorem 21, it follows that Theorem 20 holds also if $k \in [1/2, 1)$.

4. Additional Results

In this section, using our previous technique, we establish very easily some fixed point theorems on G -metric spaces.

4.1. Common Fixed Point Theorem for Quasicontractive Condition. We have the following result.

Theorem 23. Let (X, G) be a G -metric space and let $T, S : X \rightarrow X$ be weakly compatible self-mappings of X . Suppose that the mappings T and S satisfy one of the following conditions:

$$G(Tx, Ty, Ty) \leq k \max \{G(Sx, Sy, Sy), G(Sx, Tx, Tx), G(Sy, Ty, Ty)\} \quad (31)$$

or

$$G(Tx, Ty, Ty) \leq k \max \{G(Sx, Sy, Sy), G(Sx, Tx, Tx), G(Sy, Ty, Ty)\} \quad (32)$$

for all $x, y \in X$, where $k \in [0, 1)$ is a constant. If the range of S contains the range of T and SX is a G -complete subspace of X , then T and S have a unique common fixed point.

Proof. Suppose that T and S satisfy inequality (31). Then we have

$$G(Ty, Tx, Tx) \leq k \max \{G(Sy, Sx, Sx), G(Sy, Ty, Ty), G(Sx, Tx, Tx)\}. \quad (33)$$

It follows from (31), (33), and the definition of δ_G that

$$\delta_G(Tx, Ty) \leq k \max \{\delta_G(Sx, Sy), \delta_G(Sx, Tx), \delta_G(Sy, Ty)\}. \quad (34)$$

The existence and uniqueness of the common fixed point are followed immediately by Theorem 7, Propositions 10 and 11. $\square$

4.2. Edelstein Type Theorem. Here, we give a fixed point result of Edelstein type in the setting of G -metric spaces.

Theorem 24. Let (X, G) be a sequentially G -compact G -metric space and let $T : X \rightarrow X$ be a mapping such that

$$G(Tx, Ty, Ty) < G(x, y, y), \quad (35)$$

for all $x, y \in X$ with $x \neq y$. Then T has a unique fixed point.

Proof. From (35), we have

$$G(Ty, Tx, Tx) < G(y, x, x), \quad \forall x \neq y. \quad (36)$$

It follows from (35), (36), and the definition of δ_G that

$$\delta_G(Tx, Ty) < \delta_G(x, y), \quad \forall x \neq y. \quad (37)$$

By Proposition 13, (X, δ_G) is compact and by Theorem 9, the mapping T has a unique fixed point. $\square$

4.3. ψ -Contractive Condition. To conclude this paper, we study the existence and uniqueness of common fixed points for two self-mappings satisfying a ψ -contractive condition. The following result is a consequence of Theorem 8.

Theorem 25. Let $T, S : X \rightarrow X$ be self-mappings of a G -metric space (X, G) such that $TX \subseteq SX$. Assume that there exist $F, \psi : [0, +\infty) \rightarrow [0, +\infty)$ such that

- (i) F is nondecreasing, continuous, and $F(0) = 0 < F(t)$ for every $t > 0$;
- (ii) ψ is nondecreasing, right continuous, and $\psi(t) < t$ for every $t > 0$;
- (iii) $F(G(Tx, Ty, Ty)) \leq \psi(F(\max\{G(Sx, Sy, Sy), G(Sx, Tx, Tx), G(Sy, Ty, Ty)\}))$, for all $x, y \in X$.

If one of TX and SX is a G -complete subspace of X , then T and S have a coincidence point. Further, if T and S are weakly compatible, then T and S have a unique common fixed point.

Proof. From condition (iii), we have

$$F(G(Ty, Tx, Tx)) \leq \psi(F(\max\{G(Sy, Sx, Sx), G(Sy, Ty, Ty), G(Sx, Tx, Tx)\})). \quad (38)$$

Combining (ii) with the above inequality, using the monotone property of ψ and F , we get

$$F(\delta_G(Tx, Ty)) \leq \psi(F(\max\{\delta_G(Sx, Sy), \delta_G(Sx, Tx), \delta_G(Sy, Ty)\})), \quad (39)$$

for all $x, y \in X$. Now, the result follows from Theorem 8, Propositions 10 and 11. $\square$

5. Conclusions

In this paper, we showed that some fixed point generalizations in fixed point theory are not real generalizations as they could easily be obtained from the corresponding fixed point theorems in metric spaces. Here, we reviewed only some theorems which appeared recently in the literature. It follows that researchers in fixed point theory should take care in obtaining real generalizations.

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