Research Article

On the Unstable Solutions to Functional Vector Differential Equations of the Seventh Order

Cemil Tunç

Department of Mathematics, Faculty of Sciences, Yüzüncü Yıl University, 65080 Van, Turkey

Correspondence should be addressed to Cemil Tunç; cemtunc@yahoo.com

Received 25 August 2012; Accepted 16 September 2012

Academic Editor: Yongsheng S. Han

Copyright © 2013 Cemil Tunç. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper studies the instability of the zero solution for a certain nonlinear functional vector differential equation of the seventh order with multiple deviating arguments. Under sufficient conditions, we prove a result on the instability of the zero solution. This work contributes and complements to previously known results in the literature.

1. Introduction

More than 100 years ago, the world famous mathematician Lyapunov established the Lyapunov direct method to study stability problems. From then on, the Lyapunov's direct method was also widely used to study the instability of solutions of ordinary differential equations and functional differential equations, see for example, Bereketoglu [1], Sadek [2], Tejumola [3], Tunç [4–7], C. Tunç and E. Tunç [8], and the references therein. However, a review to date of the literature indicates that the instability of the solutions to the nonlinear functional vector differential equation of the seventh order has not been investigated. This paper is the first work on the subject. It is also known that the expressions of Lyapunov-Krasovskii functional are very complicated and hard to construct. In this paper, we define a Lyapunov-Krasovskii functional [9] and base on the Krasovskii criteria to prove a new theorem on the topic for the nonlinear functional vector differential equation of the seventh order. In this paper, intend to make a contribution to the subject since the functional differential equations have an important place in various fields of science and engineering.

Meanwhile, some respective contributions on the topic can be summarized as follows.

In 2000, Tejumola [3] discussed the instability of the zero solution of the seventh order scalar nonlinear differential equation without a deviating argument

$$x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)}$$

$$\begin{aligned} &+ \psi_4(x, x', \dots, x^{(6)}) x''' + \psi_5(x') x'' \\ &+ \psi_6(x, x', \dots, x^{(6)}) + \psi_7(x) = 0. \end{aligned} \quad (1)$$

Later, in 2003, Sadek [2] proved an instability theorem for the seventh order scalar nonlinear differential equation without a deviating argument

$$\begin{aligned} &x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + a_4 x''' + f(x') x'' \\ &+ g(x) x' + h(x) = 0. \end{aligned} \quad (2)$$

Recently, Tunç [6, 7] discussed the same problem for the seventh order scalar nonlinear delay differential equations of the form:

$$\begin{aligned} &x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} \\ &+ a_4 x''' + f(x') x'' + g(x) x' \\ &+ h(x, x(t-r), x', \dots, x^{(6)}(t-r)) = 0, \end{aligned} \quad (3)$$

$$\begin{aligned}
& x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} \\
& + \psi_4 \left(x, x(t-r), x', \dots, x^{(6)}(t-r) \right) x''' \\
& + \psi_5 \left(x' \right) x'' + \psi_6 \left(x, x(t-r), x', \dots, x^{(6)}(t-r) \right) \\
& + \psi_7 \left(x(t-r) \right) = 0.
\end{aligned} \quad (4)$$

In this paper, instead of (4), we consider the seventh order nonlinear functional vector differential equation with n -deviating arguments, τ_i , ($i = 1, 2, \dots, n$):

$$\begin{aligned}
& X^{(7)} + A_1 X^{(6)} + A_2 X^{(5)} + A_3 X^{(4)} \\
& + \Psi_4 \left(X, X(t-\tau_1), \dots, X(t-\tau_n), \dots, X^{(6)}(t-\tau_n) \right) \\
& \times X''' + \Psi_5 \left(X' \right) X'' + \Psi_6 \\
& \times \left(X, X(t-\tau_1), \dots, X(t-\tau_n), \dots, X^{(6)}(t-\tau_n) \right) X' \\
& + \sum_{i=1}^n H_i \left(X(t-\tau_i) \right) = 0.
\end{aligned} \quad (5)$$

Setting $X = X_1$, $X' = X_2$, $X'' = X_3$, $X''' = X_4$, $X^{(4)} = X_5$, $X^{(5)} = X_6$, $X^{(6)} = X_7$, we can write (5) in system form as follows

$$\begin{aligned}
& X_1' = X_2, \quad X_2' = X_3, \quad X_3' = X_4, \\
& X_4' = X_5, \quad X_5' = X_6, \quad X_6' = X_7, \\
& X_7' = -A_1 X_7 - A_2 X_6 - A_3 X_5 \\
& - \Psi_4 \left(X_1, \dots, X_1(t-\tau_n), \dots, X_7(t-\tau_n) \right) X_4 \\
& - \Psi_5 \left(X_2 \right) X_3 \\
& - \Psi_6 \left(X_1, \dots, X_1(t-\tau_n), \dots, X_7(t-\tau_n) \right) X_2 \\
& - \sum_{i=1}^n H_i \left(X_1 \right) + \sum_{i=1}^n \int_{t-\tau_i}^t J_{H_i} \left(X_1(s) \right) X_2(s) ds,
\end{aligned} \quad (6)$$

where τ_i are certain positive constants, the fixed delays, $t-\tau_i > 0$, A_1, A_2 , and A_3 are constant $n \times n$ -symmetric matrices, the primes in (5) denote differentiation with respect to t , $t \in \mathbb{R}_+$, $\mathbb{R}_+ = [0, \infty)$; Ψ_4, Ψ_5 , and Ψ_6 are continuous $n \times n$ -symmetric matrix functions for the arguments displayed explicitly, $H_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $H_i(0) = 0$, $H_i(X_1) \neq 0$, ($X_1 \neq 0$), and H_i are continuous for all the respective arguments. The Jacobian matrices of $H_i(X)$ are given by

$$\begin{aligned}
J_{H_i}(X) &= \left(\frac{\partial h_{1i}}{\partial x_j} \right), \dots, J_{H_n}(X) \\
&= \left(\frac{\partial h_{ni}}{\partial x_j} \right) \quad (i, j = 1, 2, \dots, n),
\end{aligned} \quad (7)$$

where $(x_1, \dots, x_n)$ and $(h_{1i}), \dots, (h_{ni})$ are the components of X and H_i , respectively. It is also assumed that the Jacobian

matrices $J_{H_i}(X)$ exist and are continuous. The existence and uniqueness of the solutions of (5) are assumed (see El'sgol'ts [10, pages 14, 15]). Throughout what follows that $X_1(t), \dots, X_7(t)$ are abbreviated as $X_1, \dots, X_7$, respectively.

Let $r \geq 0$ be given, and let $C = C([-r, 0], \mathbb{R}^n)$ with

$$\|\phi\| = \max_{-r \leq s \leq 0} |\phi(s)|, \quad \phi \in C. \quad (8)$$

For $H > 0$ define $C_H \subset C$ by

$$C_H = \{\phi \in C : \|\phi\| < H\}. \quad (9)$$

If $x : [-r, A) \rightarrow \mathbb{R}^n$ is continuous, $0 < A \leq \infty$, then, for each t in $[0, A)$, x_t in C is defined by

$$x_t(s) = x(t+s), \quad -r \leq s \leq 0, \quad t \geq 0. \quad (10)$$

Let G be an open subset of C and consider the general autonomous delay differential system

$$\dot{x} = F(x_t), \quad x_t = x(t+\theta), \quad -r \leq \theta \leq 0, \quad t \geq 0, \quad (11)$$

where $F : G \rightarrow \mathbb{R}^n$ is continuous and maps closed and bounded sets into bounded sets. It follows from the conditions on F that each initial value problem

$$\dot{x} = F(x_t), \quad x_0 = \phi \in G \quad (12)$$

has a unique solution defined on some interval $[0, A)$, $0 < A \leq \infty$. This solution will be denoted by $x(\phi)(\cdot)$ so that $x_0(\phi) = \phi$.

Definition 1. The zero solution, $x = 0$, of $\dot{x} = F(x_t)$ is stable if for each $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon) > 0$ such that $\|\phi\| < \delta$ implies that $|x(\phi)(t)| < \varepsilon$ for all $t \geq 0$. The zero solution is said to be unstable if it is not stable.

Consider the linear constant coefficient differential equation of the seventh order:

$$\begin{aligned}
& x^{(7)} + a_1 x^{(6)} + a_2 x^{(5)} + a_3 x^{(4)} + a_4 \ddot{x} \\
& + a_5 \ddot{x} + a_6 \dot{x} + a_7 x = 0.
\end{aligned} \quad (13)$$

It is known from the qualitative behavior of solutions of linear differential equations that the zero solution of (13) is unstable if and only if, the associated auxiliary equation:

$$\begin{aligned}
& \psi(\lambda) \equiv \lambda^7 + a_1 \lambda^6 + a_2 \lambda^5 + a_3 \lambda^4 + a_4 \lambda^3 \\
& + a_5 \lambda^2 + a_6 \lambda + a_7 = 0
\end{aligned} \quad (14)$$

has at least one root with a positive real part. The existence of such a root naturally depends on (though not always all of) the coefficients $a_1, a_2, \dots, a_7$ in (14). For example, if

$$a_1 < 0 \quad (15)$$

then it is clear from a consideration of the fact that the sum of the roots of (14) equals $-a_1$ and that at the least one root of (14) has a positive real part for arbitrary values of a_2, a_3, a_4, a_5, a_6 , and a_7 . An analogue consideration, combined with the

fact that the product of the roots (14) equals $(-a_7)$ will verify that at least one root of (14) have a positive real part if

$$a_1 = 0, \quad a_7 \neq 0 \quad (16)$$

for arbitrary a_2, a_3, a_4, a_5 and a_6 . The condition $a_1 = 0$ here in (16) is, however, superfluous when

$$a_7 < 0; \quad (17)$$

then for $\psi(0) = a_7 < 0$ and $\psi(R) > 0$ if $R > 0$ is sufficiently large, thus showing that there is a positive real root of (14) subject to (17) and for arbitrary a_1, a_2, a_3, a_4, a_5 and a_6 . Moreover, a necessary and sufficient condition for (14) to have a purely imaginary root $\lambda = i\beta$ (β real) is that the two equations

$$\begin{aligned} -a_1\beta^6 + a_3\beta^4 - a_5\beta^2 + a_7 &= 0, \\ -\beta^6 + a_2\beta^4 - a_4\beta^2 + a_6 &= 0 \end{aligned} \quad (18)$$

are satisfied at the same time. If

$$a_1 \leq 0, \quad a_3 \geq 0, \quad a_5 \leq 0, \quad a_7 > 0 \quad (19)$$

or

$$a_1 \geq 0, \quad a_3 \leq 0, \quad a_5 \geq 0, \quad a_7 < 0, \quad (20)$$

then (14) cannot have any purely imaginary root whatever.

It should be noted that there are no restrictions on the constants a_2, a_4 , and a_6 in (13).

2. Main Result

First, we give the following lemma.

Lemma 2. Let A be a real symmetric $n \times n$ -matrix and

$$a' \geq \lambda_i(A) \geq a > 0, \quad (i = 1, 2, \dots, n), \quad (21)$$

where a' and a are constants, and $\lambda_i(A)$ are the eigenvalues of the matrix A .

Then

$$\begin{aligned} a' \langle X, X \rangle &\geq \langle AX, X \rangle \geq a \langle X, X \rangle, \\ a'^2 \langle X, X \rangle &\geq \langle AX, AX \rangle \geq a^2 \langle X, X \rangle \end{aligned} \quad (22)$$

(Bellman [11]).

Let $\tau = \max \tau_i$, ($i = 1, 2, \dots, n$).

The following theorem is our main result.

Theorem 3. In addition to all the assumptions imposed to $A_1, A_2, A_3, \Psi_4, \Psi_5, \Psi_6$, and H_i that appear in (5), we assume that there exist constants $a_2 < 0$, $\alpha_i > 0$, $\beta_i > 0$, and $\delta > 0$ such that the conditions

$$\begin{aligned} \lambda_i(A_2) &\leq a_2, \beta_1 \leq \lambda_i(J_{H_1}(X_1)) \\ &\leq \alpha_1, \dots, \beta_n \leq \lambda_i(J_{H_n}(X_1)) \leq \alpha_n, \end{aligned} \quad (23)$$

$$\frac{1}{4a_2} [\lambda_i(\Psi_4(\cdot))]^2 - \lambda_i(\Psi_6(\cdot)) \geq \delta > 0$$

hold. If $\tau < \delta/\sqrt{n}(\alpha_1 + \dots + \alpha_n)$, then the zero solution of (5) is unstable.

Remark 4. For the proof of the theorem, under the conditions sated in the theorem, it suffices to find that there exists a continuous Lyapunov-Krasovskii functional $V(\cdot) = V(X_{1t}, \dots, X_{7t})$, which has the Krasovskii properties [9], say (K_1) , (K_2) , and (K_3) .

(K_1) In every neighborhood of $(0, 0, 0, 0, 0, 0, 0)$ there exists a point $(\xi_1, \dots, \xi_7)$ such that $V(\xi_1, \dots, \xi_7) > 0$.

(K_2) The time derivative $\dot{V}(\cdot) = (d/dt)V(X_{1t}, \dots, X_{7t})$ along solution paths of (6) is positive semidefinite.

(K_3) The only solution $(X_1, \dots, X_7) = (X_1(t), \dots, X_7(t))$ of (6) which satisfies $(d/dt)V(X_{1t}, \dots, X_{7t}) = 0, (t \geq 0)$, is the trivial solution $(0, 0, 0, 0, 0, 0, 0)$.

Proof. Consider the Lyapunov-Krasovskii functional $V(\cdot) = V(X_{1t}, \dots, X_{7t})$ defined by

$$\begin{aligned} V(\cdot) &= \langle X_7, X_2 \rangle + \langle A_1 X_2, X_6 \rangle \\ &+ \langle A_2 X_2, X_5 \rangle + \langle A_3 X_2, X_4 \rangle \\ &- \langle X_3, X_6 \rangle - \langle A_1 X_3, X_5 \rangle \\ &- \langle A_2 X_3, X_4 \rangle + \langle X_4, X_5 \rangle \\ &- \frac{1}{2} \langle A_3 X_3, X_3 \rangle + \frac{1}{2} \langle A_1 X_4, X_4 \rangle \\ &+ \int_0^1 \langle H_1(\sigma X_1), X_1 \rangle d\sigma \\ &+ \dots + \int_0^1 \langle H_n(\sigma X_1), X_1 \rangle d\sigma \\ &+ \int_0^1 \langle \Psi_5(\sigma X_2), X_2, X_2 \rangle d\sigma \\ &- \sum_{i=1}^n \lambda_i \int_{-\tau_i}^0 \int_{t+s}^t \|X_2(\theta)\|^2 d\theta ds, \end{aligned} \quad (24)$$

where s is a real variable such that the integrals $\int_{-\tau_i}^0 \int_{t+s}^t \|X_2(\theta)\|^2 d\theta ds$ are nonnegative, and λ_i are certain positive constants to be determined later in the proof.

We see that

$$V(0, 0, 0, 0, 0, 0, 0) = 0. \quad (25)$$

Since $(\partial/\partial\sigma)H_i(\sigma X_1) = J_{H_i}(\sigma X_1)X_1$, $H_i(0) = 0$, then

$$\begin{aligned}
 & \int_0^1 \langle H_1(\sigma X_1), X_1 \rangle d\sigma \\
 &= \iint_0^1 \langle \sigma_1 J_{H_1}(\sigma_1 \sigma_2 X_1) X_1, X_1 \rangle d\sigma_2 d\sigma_1 \\
 &\geq \beta_1 \|X_1\|^2, \\
 & \int_0^1 \langle H_2(\sigma X_1), X_1 \rangle d\sigma \\
 &= \iint_0^1 \langle \sigma_1 J_{H_2}(\sigma_1 \sigma_2 X_1) X_1, X_1 \rangle d\sigma_2 d\sigma_1 \quad (26) \\
 &\geq \beta_2 \|X_1\|^2, \\
 & \vdots \\
 & \int_0^1 \langle H_n(\sigma X_1), X_1 \rangle d\sigma \\
 &= \iint_0^1 \langle \sigma_1 J_{H_n}(\sigma_1 \sigma_2 X_1) X_1, X_1 \rangle d\sigma_2 d\sigma_1 \\
 &\geq \beta_n \|X_1\|^2.
 \end{aligned}$$

Let

$$\bar{\varepsilon} = (\varepsilon_{11}, \dots, \varepsilon_{1n}). \quad (27)$$

Then, we get

$$V(\bar{\varepsilon}, 0, 0, 0, 0, 0) = \left(\sum_{i=1}^n \beta_i \right) \|\bar{\varepsilon}\|^2 > 0 \quad (28)$$

for all $\bar{\varepsilon}, \bar{\varepsilon} \in \mathbb{R}$, so that every neighborhood of the origin in the $(X_1, \dots, X_7)$ -space contains points $(\xi_1, \dots, \xi_7)$ such that $V(\xi_1, \dots, \xi_7) > 0$.

Let

$$(X_1, \dots, X_7) = (X_1(t), \dots, X_7(t)) \quad (29)$$

be an arbitrary solution of (6).

Calculating the time derivative of the Lyapunov-Krasovskii functional V along this solution, we obtain

$$\begin{aligned}
 \dot{V}(\cdot) &= \langle X_5, X_5 \rangle - \langle A_2 X_4, X_4 \rangle - \langle \Psi_4(\cdot) X_2, X_4 \rangle \\
 &\quad - \langle \Psi_6(\cdot) X_2, X_2 \rangle \\
 &\quad + \left\langle X_2, \int_{t-\tau_1}^t J_{H_1}(X_1(s)) X_2(s) ds \right\rangle \\
 &\quad + \dots + \left\langle X_2, \int_{t-\tau_n}^t J_{H_n}(X_1(s)) X_2(s) ds \right\rangle \quad (30) \\
 &\quad - \langle \lambda_1 \tau_1 X_2, X_2 \rangle - \dots - \langle \lambda_n \tau_n X_2, X_2 \rangle \\
 &\quad + \lambda_1 \int_{t-\tau_1}^t \|X_2(\theta)\|^2 d\theta \\
 &\quad + \dots + \lambda_n \int_{t-\tau_n}^t \|X_2(\theta)\|^2 d\theta.
 \end{aligned}$$

Using the assumption $0 < \lambda_i (J_{H_i}(X_1)) \leq \alpha_i$ and the Schwarz inequality, we have

$$\begin{aligned}
 & \left\langle X_2, \int_{t-\tau_1}^t J_{H_1}(X_1(s)) X_2(s) ds \right\rangle \\
 &\geq -\|X_2\| \left\| \int_{t-\tau_1}^t J_{H_1}(X_1(s)) X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_1 \|X_2\| \left\| \int_{t-\tau_1}^t X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_1 \|X_2\| \int_{t-\tau_1}^t \|X_2(s)\| ds \\
 &\geq -\frac{1}{2} \sqrt{n}\alpha_1 \tau_1 \|X_2\|^2 - \frac{1}{2} \sqrt{n}\alpha_1 \int_{t-\tau_1}^t \|X_2(s)\|^2 ds, \\
 & \left\langle X_2, \int_{t-\tau_2}^t J_{H_2}(X_1(s)) X_2(s) ds \right\rangle \\
 &\geq -\|X_2\| \left\| \int_{t-\tau_2}^t J_{H_2}(X_1(s)) X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_2 \|X_2\| \left\| \int_{t-\tau_2}^t X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_2 \|X_2\| \int_{t-\tau_2}^t \|X_2(s)\| ds \\
 &\geq -\frac{1}{2} \sqrt{n}\alpha_2 \tau_2 \|X_2\|^2 - \frac{1}{2} \sqrt{n}\alpha_2 \int_{t-\tau_2}^t \|X_2(s)\|^2 ds, \\
 & \vdots \\
 & \left\langle X_2, \int_{t-\tau_n}^t J_{H_n}(X_1(s)) X_2(s) ds \right\rangle \\
 &\geq -\|X_2\| \left\| \int_{t-\tau_n}^t J_{H_n}(X_1(s)) X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_n \|X_2\| \left\| \int_{t-\tau_n}^t X_2(s) ds \right\| \\
 &\geq -\sqrt{n}\alpha_n \|X_2\| \int_{t-\tau_n}^t \|X_2(s)\| ds \\
 &\geq -\frac{1}{2} \sqrt{n}\alpha_n \tau_n \|X_2\|^2 - \frac{1}{2} \sqrt{n}\alpha_n \int_{t-\tau_n}^t \|X_2(s)\|^2 ds. \quad (31)
 \end{aligned}$$

Hence

$$\begin{aligned}
 \dot{V}(\cdot) &\geq \|X_5\|^2 - a_2 \|X_4 + 2^{-1} a_2^{-1} \Psi_4(\cdot) X_2\|^2 \\
 &\quad + \frac{1}{4a_2} \langle \Psi_4(\cdot) X_2, \Psi_4(\cdot) X_2 \rangle - \langle \Psi_6(\cdot) X_2, X_2 \rangle \\
 &\quad - \left(\lambda_1 + \frac{1}{2} \sqrt{n}\alpha_1 \right) \tau_1 \|X_2\|^2
 \end{aligned}$$

$$\begin{aligned}
& - \cdots - \left(\lambda_n + \frac{1}{2} \sqrt{n} \alpha_n \right) \tau_n \|X_2\|^2 \\
& + \left(\lambda_1 - \frac{1}{2} \sqrt{n} \alpha_1 \right) \int_{t-\tau_1}^t \|X_2(s)\|^2 ds \\
& + \cdots + \left(\lambda_n - \frac{1}{2} \sqrt{n} \alpha_n \right) \int_{t-\tau_n}^t \|X_2(s)\|^2 ds.
\end{aligned} \tag{32}$$

Let $\lambda_i = (1/2)\sqrt{n}\alpha_i$. Then, using the assumptions of the theorem and the estimate $\tau = \max \tau_i$, we have

$$\begin{aligned}
\dot{V}(\cdot) & \geq \|X_5\|^2 - a_2 \|X_4 + 2^{-1} a_2^{-1} \Psi_4(\cdot) X_2\|^2 \\
& + \frac{1}{4a_2} \langle \Psi_4(\cdot) X_2, \Psi_4(\cdot) X_2 \rangle - \langle \Psi_6(\cdot) X_2, X_2 \rangle \\
& - \sqrt{n} \alpha_1 \tau_1 \|X_2\|^2 - \cdots - \sqrt{n} \alpha_n \tau_n \|X_2\|^2 \\
& \geq \{\delta - \sqrt{n}(\alpha_1 + \cdots + \alpha_n) \tau\} \|X_2\|^2.
\end{aligned} \tag{33}$$

If $\tau < \delta/\sqrt{n}(\alpha_1 + \cdots + \alpha_n)$, then, for some positive constant k , we have

$$\dot{V}(\cdot) \geq k \|X_2\|^2 > 0. \tag{34}$$

Finally, $\dot{V}(\cdot) = 0$ for all $t \geq 0$ necessarily implies that $X_2 = 0$. Hence, it follows that

$$\begin{aligned}
X_1 &= \xi_1 \text{ (constant vector)}, & X_2 &= X' = 0, \\
X_3 &= X'' = 0, & X_4 &= X''' = 0, & X_5 &= X^{(4)} = 0, \\
X_6 &= X^{(5)} = 0, & X_7 &= X^{(6)} = 0,
\end{aligned} \tag{35}$$

for all $t \geq 0$ so that

$$\begin{aligned}
X_1 &= \xi_1 \text{ (constant vector)}, \\
X_2 &= X_3 = X_4 = X_5 = X_6 = X_7 = 0, \quad (t \geq 0).
\end{aligned} \tag{36}$$

Therefore, the estimates $(d/dt)V(\cdot) = 0$ and (6) imply $X_1 = X_2 = X_3 = X_4 = X_5 = X_6 = X_7 = 0$ since $H_i(\xi_1) = 0$ if and only if $\xi_1 = 0$. It now follows that functional V thus has all the requisite Krasovskii [9] properties subject to the conditions of the theorem. Hence, we can conclude that the zero solution of (5) is unstable.

The proof of the theorem is completed. $\square$

References

[1] H. Bereketoglu, "On the periodic solutions of certain class of seventh-order differential equations," *Periodica Mathematica Hungarica*, vol. 24, no. 1, pp. 13–22, 1992.

[2] A. I. Sadek, "An instability theorem for a certain seventh-order differential equation," *Annals of Differential Equations*, vol. 19, no. 1, pp. 1–5, 2003.

[3] H. O. Tejumola, "Instability and periodic solutions of certain nonlinear differential equations of orders six and seven," in *Proceedings of the National Mathematical Centre on Ordinary Differential Equations*, vol. 1, pp. 56–65, Abuja, Nigeria, 2000.

[4] C. Tunç, "A result on the instability of solutions to a class of non-autonomous differential equations of seventh-order," *Mathematical Sciences Research Journal*, vol. 11, no. 2, pp. 360–366, 2007.

[5] C. Tunç, "New results about instability of nonlinear ordinary vector differential equations of sixth and seventh orders," *Dynamics of Continuous, Discrete and Impulsive Systems Series A*, vol. 14, no. 1, pp. 123–136, 2007.

[6] C. Tunç, "On the instability of solutions of seventh order nonlinear delay differential equations," *Buletinul Academiei de Stiinte a Republicii Moldova, Matematica*, vol. 65, no. 1, pp. 60–65, 2011.

[7] C. Tunç, "An instability theorem for a kind of seventh order nonlinear delay differential equations," *Annals of Differential Equations*, vol. 28, no. 1, pp. 11–14, 2012.

[8] C. Tunç and E. Tunç, "Instability of solutions of certain non-linear vector differential equations of order seven," *Iranian Journal of Science and Technology, Transaction A*, vol. 29, no. 3, pp. 515–521, 2005.

[9] N. N. Krasovskii, *Stability of Motion: ApplicationS of Lyapunov's Second Method to Differential Systems and equations with Delay*, Stanford University Press, Stanford, Calif, USA, 1963, Translated by J. L. Brenner.

[10] L. È. Èl'sgol'ts, *Introduction to the Theory of Differential Equations With Deviating Arguments*, Holden-Day, San Francisco, Calif, USA, 1966, Translated from the Russian by Robert J. McLaughlin.

[11] R. Bellman, *Introduction to Matrix Analysis*, vol. 19 of *Classics in Applied Mathematics*, Society for Industrial and Applied Mathematics, Philadelphia, Pa, USA, 2nd edition, 1997, With a foreword by Gene Golub.