Research Article

Effect of Initial Stress on the Propagation Characteristics of Waves in Fiber-Reinforced Transversely Isotropic Thermoelastic Material under an Inviscid Liquid Layer

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The present investigation deals with the propagation of waves in fiber-reinforced transversely isotropic thermoelastic solid half space with initial stresses under a layer of inviscid liquid. The secular equation for surface equation in compact form is derived after developing the mathematical model. The phase velocity and attenuation coefficients of plane waves are studied numerically for a particular model. Effects of initial stress and thickness of the layer on the phase velocity, attenuation coefficient, and specific loss of energy are predicted graphically in the certain model. A particular case of Rayleigh wave has been discussed and the dispersion curves of the phase velocity and attenuation coefficients have also been presented graphically. Some other particular cases are also deduced from the present investigation.

1. Introduction

Fiber-reinforced materials are widely used in engineering structures due to their superiority over the structural materials in applications requiring high strength and stiffness in lightweight components. Consequently, characterization of their mechanical behavior is of particular importance for structural design using these materials. Fibers are assumed an inherent material property, rather than some form of inclusion in models as Spencer [1]. In the case of an elastic solid reinforced by a series of parallel fibers it is usual to assume transverse isotropy.

The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. [2]. The characteristic property of reinforced concrete member is that its components, namely, concrete and steel, act together as a single anisotropic unit as main long as they remain in the elastic condition; that is, the two components are bound together so that there can be no relative displacement between them. The dynamical interaction between the thermal and mechanical fields in solids has great practical

applications in modern aeronautics, astronautics, nuclear reactors, and high energy particle accelerators.

The analysis of stress and deformation of fiber-reinforced composite materials has been an important subject of solid mechanics for last three decades. Pipkin [3] did pioneer works on the subject. Sengupta and Nath [4] discussed the problem of surface waves in fiber-reinforced anisotropic elastic media.

Lord and Shulman [5] introduced a theory of generalized thermoelasticity with one relaxation time for an isotropic body. The theory was extended for anisotropic body by Dhaliwal and Sherief [6]. In this theory, a modified law of heat conduction including both the heat flux and its time derivatives replaces the conventional Fourier's Law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both coupled and uncoupled theories of thermoelasticity. Erdem [7] derived heat conduction equation for a composite rigid material containing an arbitrary distribution of fibers. Recently, Kumar and Gupta [8] discussed the Wave motion in an anisotropic fiber-reinforced thermoelastic solid.

Surface waves carry a lot of information about the Earth's crust and dispersion analysis of surface waves is concerned with the phase velocity and wave number. Further the velocity of seismic waves depends upon the elastic parameters of the medium through which they travel. Therefore, many investigators have studied the surface wave propagation by considering different kinds of models. Chadwick and Seet [9] and, Singh and Sharma [10] have discussed the propagation of plane harmonic waves in transversely isotropic thermoelastic materials. Singh [11] studied a problem on wave propagation in an anisotropic generalized thermoelastic solid and obtained a cubic equation, which gives the dimensional velocities of various plane waves.

Chadwick [12] has discussed the propagation of surface waves in homogeneous thermoelastic media. Chadwick and Windle [13] studied the effect of heat conduction on the propagation of Rayleigh waves in the semi-infinite media (i) when the surface is maintained at constant temperature and (ii) when the surface is thermally insulated. Sharma and Singh [14] have studied thermoelastic surface waves in a transversely isotropic half-space.

The study of wave propagation in a generalized thermoelastic media with additional parameters like prestress, porosity, viscosity, microstructure, temperature, and other parameters provide vital information about existence of new or modified waves. Such information may be useful for experimental seismologists in correcting earthquake estimation.

Prestressed materials have various applications, for example, in oil and geophysical industry, NDT in prestressed materials, use of rubber composites in automotive, and aerospace and defense industries (often in prestressed states) and in the study of biological tissues (lung, tendon, etc.) which are all nonlinear prestressed viscoelastic composites.

Montanaro [15] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Wang and Slattery [16] formulated the thermoelastic equations without energy dissipation for a body which has previously received a large deformation and is at nonuniform temperature. Ieşan [17] presented a theory of Cosserat thermoelastic solids with initial stresses. Ames and Straughan [18] derived the continuous dependence results for initially prestressed thermoelastic bodies.

Some theorems in the generalized theory of thermoelasticity for prestressed bodies were studied by Wang et al. [19]. Marin and Marinescu [20] studied the asymptotic partition of total energy for the solutions of the mixed initial boundary value problem within the context of the thermoelasticity of initially stressed bodies. Kalinchuk et al. [21] studied the problem of steady-state harmonic oscillations for a nonhomogeneous thermoelastic prestressed medium.

Othman and Song [22] discussed the reflection of plane waves from a thermoelastic elastic solid half-space under hydrostatic initial stress without energy dissipation. Singh [23] studied wave propagation in an initially stressed transversely isotropic thermoelastic half-space. However, no attempt has been made to discuss the effect of hydrostatic initial stress on propagation of waves in the layer of fiber-reinforced transversely isotropic thermoelastic material. Othman et al. [24] investigated the influences of fractional

order, hydrostatic initial stress, and gravity field on the plane waves in a fiber-reinforced isotropic thermoelastic medium using normal mode analysis. Abd-Alla et al. [25] studied the effects of rotation and gravity field on surface waves in fiber-reinforced thermoelastic media under four theories. Othman and Atwa [26] studied the effect of rotation, gravity, and reinforcement on the total deformation and mutual interaction of the body in a fiber-reinforced thermoelastic solid using normal mode analysis in the context of Green-Naghdi theory of thermoelasticity. Abbas [27] studied the transient phenomena in the magnetothermoelastic model in the context of the Lord and Shulman theory in a perfectly conducting medium. Abbas [28] constructed the equations for generalized thermoelasticity of an unbounded fiber-reinforced anisotropic medium with a circular hole in the context of Green and Naghdi (GN) theory. Recently, Abbas and Zenkour [29] investigated two-temperature generalized thermoelasticity with two relaxation times for an infinite fiber-reinforced anisotropic plate containing a circular cavity using a finite element method.

In the present paper, the governing equations of thermoelasticity of fiber-reinforced transversely isotropic solid half space with initial stresses under a layer of inviscid liquid are formulated and solved analytically in two dimensions at uniform temperature. The phase velocity and attenuation coefficients of plane waves are studied numerically for a particular model. Specific loss of energy is obtained and discussed numerically for a particular model to study the effects of initial stress. The study of present investigation has applications in civil engineering and geophysics.

2. Basic Equations

The basic equations in the dynamic theory of the homogeneous thermally conducting fiber-reinforced medium with an initial hydrostatic stress without body forces and heat sources are given by Lord and Shulman [5] and Abbas and Othman [30] as

$$\sigma_{ij,j} - P\omega_{ij,j} = \rho\ddot{u}_i, \quad i, j = 1, 2, 3 \quad (1)$$

and heat conduction equation is given by

$$K_{ij}T_{,ij} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (T_0 \beta_{ij} \dot{u}_{i,j} + \rho C_e \dot{T}), \quad i, j = 1, 2, 3. \quad (2)$$

The constitutive relations for thermally conducting transversely isotropic, fiber-reinforced linearly elastic media are

$$\begin{aligned} \sigma_{ij} = & \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) \\ & + 2(\mu_L - \mu_T) (a_i a_k e_{kj} + a_j a_k e_{ki}) \\ & + \beta (a_k a_m e_{km} a_i a_j) - \beta_{ij} T \delta_{ij}, \end{aligned} \quad (3)$$

$i, j, k, m = 1, 2, 3,$

where

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad \omega_{ij} = \frac{1}{2} (u_{j,i} - u_{i,j}), \quad (4)$$

$$i, j = 1, 2, 3,$$

and λ, μ_T are elastic parameters, $\alpha, \beta, (\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters, ρ is the mass density, σ_{ij} are components of stress tensor, u_i are the displacement components, e_{ij} are components of infinitesimal strain, T is the temperature change of a material particle, T_o is the reference uniform temperature of the body, k_{ij} are coefficients of thermal conductivity, β_{ij} are thermal elastic coupling tensor, C_e is the specific heat at constant strain, δ_{ij} is the Kronecker delta, and P is the initial Pressure. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation. a_j are components of $\mathbf{a}$, all referred to Cartesian coordinate. The vector $\mathbf{a}$ may be a function of position. We choose $\mathbf{a}$ so that its components are $(1, 0, 0)$.

The equation which governs the motion in terms of velocity potential ψ of the homogeneous inviscid liquid is given by Ewing et al. [31]:

$$\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_2^2} = \frac{1}{(\alpha^\ell)^2} \frac{\partial^2 \psi}{\partial t^2}, \quad (5)$$

where $\alpha^\ell = \sqrt{\lambda^\ell / \rho^\ell}$ is the velocity of sound liquid.

The displacement components u_1^ℓ, u_2^ℓ and pressure p^ℓ in the medium M_2 are given by

$$u_1^\ell = \frac{\partial \psi}{\partial x_1}, \quad u_2^\ell = \frac{\partial \psi}{\partial x_2}, \quad p^\ell = -\rho^\ell \frac{\partial \psi}{\partial t}. \quad (6)$$

3. Formulation of the Problem

We consider a homogeneous, thermally conducting transversely isotropic fiber-reinforced medium (M_1) with an initial hydrostatic stress lying under a uniform homogeneous inviscid liquid layer medium (M_2) of thickness H . We take the rectangular Cartesian coordinate system $Ox_1x_2x_3$ at any point on the plane horizontal surface and x_1 -axis in the direction of wave propagation and x_2 -axis pointing vertically downward into the half-space so that all particles on a line parallel to x_3 -axis are equally displaced; therefore, all the field quantities will be independent of x_3 -coordinates. Hence the liquid layer medium M_2 occupies the region $-H < x_2 < 0$ and $x_2 > 0$ is occupied by the half-space medium M_1 . The plane $x_2 = -H$ represents the free surface of the liquid and $x_2 = 0$ is taken as interface between M_1 and M_2 medium (Figure 1).

The displacement components for medium M_1 are taken as

$$\vec{u} = (u_1, u_2, 0). \quad (7)$$

FIGURE 1: Geometry of the problem.

With the help of (3), (4), and (7) and (1)-(2), take the form

$$C_{11} \frac{\partial^2 u_1}{\partial x_1^2} + \left(C_{12} + C_0 - \frac{P}{2} \right) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + \left(C_0 + \frac{P}{2} \right) \frac{\partial^2 u_1}{\partial x_2^2} - \beta_{11} \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (8)$$

$$C_{22} \frac{\partial^2 u_2}{\partial x_2^2} + \left(C_{12} + C_0 - \frac{P}{2} \right) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \left(C_0 + \frac{P}{2} \right) \frac{\partial^2 u_2}{\partial x_1^2} - \beta_{22} \frac{\partial T}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (9)$$

$$\begin{aligned} \frac{\partial^2 T}{\partial x_1^2} + \bar{k} \frac{\partial^2 T}{\partial x_2^2} - \frac{\rho C_e}{k_{11}} \left(\frac{\partial T}{\partial t} + \tau_o \frac{\partial^2 T}{\partial t^2} \right) \\ = \varepsilon \left[\left(\frac{\partial^2 u_1}{\partial x_1 \partial t} + \tau_o \frac{\partial^3 u_1}{\partial x_1 \partial t^2} \right) + \bar{\beta} \left(\frac{\partial^2 u_2}{\partial x_2 \partial t} + \tau_o \frac{\partial^3 u_2}{\partial x_2 \partial t^2} \right) \right], \end{aligned} \quad (10)$$

where

$$\begin{aligned} \beta_{11} &= (C_{11} + C_{12}) \alpha_{11} + C_{12} \alpha_{22}, \\ \beta_{22} &= (C_{12} + C_{23} - C_{55}) \alpha_{11} + C_{22} \alpha_{22}, \\ C_{12} &= \lambda + \alpha, \quad C_{11} = \lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta, \\ C_{22} &= C_{33} = \lambda + 2\mu_T, \\ C_{44} &= C_{66} = 2\mu_L, \quad C_{55} = 2\mu_T, \\ C_{23} &= C_{33} - C_{55}, \quad C_o = \frac{C_{44}}{2}, \\ \varepsilon &= \frac{T_o \beta_{11}}{k_{11}}, \quad \bar{k} = \frac{k_{22}}{k_{11}}, \quad \bar{\beta} = \frac{\beta_{22}}{\beta_{11}}, \end{aligned} \quad (11)$$

and $\lambda, \alpha, \beta, \mu_L, \mu_T$ are material constants, α_{11}, α_{22} are components of linear thermal expansion, and τ_o is thermal relaxation time.

To facilitate the solution, the following dimensionless quantities are introduced:

$$\begin{aligned} (x'_1, x'_2) &= \frac{w^*}{v_1} (x_1, x_2), & (u'_1, u'_2) &= \frac{\rho v_1 w^*}{\beta_{11} T_o} (u_1, u_2), \\ t'_{ij} &= \frac{t_{ij}}{\beta_{11} T_o}, & T' &= \frac{T}{T_o}, & t' &= w^* t, \\ h' &= \frac{v_1}{w^*} h, & \tau'_o &= w^* \tau_o, & \psi' &= \frac{w^* \psi}{v_1^2}, \\ (u'^{\ell'}_1, u'^{\ell'}_2) &= \frac{w^*}{v_1} (u^\ell_1, u^\ell_2), & p^{\ell'} &= \frac{P^\ell}{\lambda^\ell}, & t'^{\ell'}_{22} &= \frac{t^\ell_{22}}{\lambda^\ell}, \end{aligned} \quad (12)$$

where $w^* = C_e C_{11} / k_{11}$, $v_1^2 = C_{11} / \rho$ and w^* is the characteristic frequency of the medium.

4. Solution of the Problem

We assume the solutions of the form

$$(u_1, u_2, T, \psi) = (1, W, R, S) U \exp \{i\xi (x_1 + m x_2 - ct)\}, \quad (13)$$

where $c = \omega / \xi$ is the nondimensional phase velocity, ω is the frequency, and ξ is the wave number. m is still unknown parameter. $1, W, R, S$ are, respectively, the amplitude ratios of u_1, u_2, T , and u_2^ℓ with respect to u_1 . Substituting the values of u_1, u_2 and T from (13) in (8), (9), and (10), we obtain

$$\begin{aligned} &\left[\frac{m}{C_{11}} \left(C_{12} + C_o - \frac{P}{2} \right) \right] W + \left(\frac{-1}{i\xi} \right) R \\ &+ \left[1 - c^2 + \frac{1}{C_{11}} \left(C_o + \frac{P}{2} \right) m^2 \right] = 0, \\ &\left[\frac{C_{22}}{C_{11}} m^2 + \frac{1}{C_{11}} \left(C_o + \frac{P}{2} \right) - c^2 \right] W \\ &+ \left(\frac{-m\bar{\beta}}{i\xi} \right) R + \left[\frac{1}{C_{11}} \left(C_{12} + C_o - \frac{P}{2} \right) m \right] = 0, \\ &\left[\varepsilon_1 \bar{\beta} (-mc + mc^2 \tau_o i\xi) \right] W + \left[1 + \bar{k} m^2 + \left(\frac{c}{i\xi} - \tau_o c^2 \right) \right] R \\ &+ \varepsilon_1 [c - \tau_o c^2 i\xi] = 0, \end{aligned} \quad (14)$$

where

$$\varepsilon_1 = \frac{T_o \beta_{11}^2}{\rho w^* k_{11}}. \quad (15)$$

The system of (14) has a nontrivial solution if the determinant of the coefficients $[1, W, R]^T$ vanishes, which yields to the following polynomial characteristic equation:

$$m^6 + d_{21} m^4 + d_{31} m^2 + d_{41} = 0, \quad (16)$$

where

$$\begin{aligned} d_1 &= [\bar{k} (L_2 g_1)], \\ d_2 &= \left[-g_1 F L_2 + \bar{k} (L_1 g_1 + L_2 g_2 - A_1^2) \right. \\ &\quad \left. + L_2 g_1 + (\bar{\beta})^2 \varepsilon_1 F L_2 \right], \\ d_3 &= \left[-F g_1 (L_1 + \varepsilon_1) - F g_2 L_2 \right. \\ &\quad \left. + \bar{\beta} F A_1 (1 - \varepsilon_1) + \bar{k} (L_1 g_2) + L_1 g_1 \right. \\ &\quad \left. + L_2 g_2 + (\bar{\beta})^2 \varepsilon_1 F L_1 + A_1^2 (F - 1) \right], \\ d_4 &= [-g_2 F (L_1 + \varepsilon_1) + L_1 g_2], \\ d_{21} &= \frac{d_2}{d_1}, & d_{31} &= \frac{d_3}{d_1}, & d_{41} &= \frac{d_4}{d_1}, \\ F &= \frac{-c + \tau_o c^2 i\xi}{i\xi}, & A &= m A_1, \\ A_1 &= \frac{1}{C_{11}} \left(C_{12} + C_o - \frac{P}{2} \right), \\ G &= g_1 m^2 + g_2, & g_1 &= \frac{C_{22}}{C_{11}}, \\ g_2 &= \frac{1}{C_{11}} \left(C_o + \frac{P}{2} \right) - c^2, \\ L &= L_1 + L_2 m^2, & L_1 &= 1 - c^2, \\ L_2 &= \frac{1}{C_{11}} \left(C_o + \frac{P}{2} \right). \end{aligned} \quad (17)$$

The characteristic equation (16) is cubic in m^2 and hence possesses three roots ($m_p^2, p = 1, 2, 3$). Therefore, there exist three types of quasi-waves in transversely isotropic elastic half-space, namely, quasi-longitudinal waves (QP), quasi-transverse waves (QSV), and quasi-thermal waves (QT).

The formal expression for displacements and volume fraction field, satisfying the radiation condition that $\text{Re}(m_p) \geq 0$, can be written as

$$(u_1, u_2, T) = \sum_{p=1}^3 G_p (1, n_{1p}, n_{2p}) \exp \{i\xi (x_1 + i m_p x_2 - ct)\}, \quad (18)$$

where

$$\begin{aligned} n_{1p} &= \frac{\Delta_1}{\Delta}, & n_{2p} &= \frac{\Delta_2}{\Delta}, \\ \Delta_1 &= \left(\frac{-\bar{\beta} m \varepsilon_1 c (1 - \tau_o c i\xi)}{i\xi} - A Y \right), \\ \Delta_2 &= (G \varepsilon_1 c (1 - \tau_o c i\xi) - A Z), \end{aligned}$$

$$\begin{aligned}\Delta &= \left(GY + \frac{\bar{\beta}mZ}{i\xi} \right), \\ Y &= 1 + \bar{k}m^2 + \left(\frac{c}{i\xi} - \tau_o c^2 \right), \\ Z &= \varepsilon_1 \bar{\beta}mc (-1 + c\tau_o i\xi)\end{aligned}\quad (19)$$

and G_1, G_2, G_3 are arbitrary constants.

Substituting the values of ψ from (13) in (5) and after simplification, we obtain

$$\psi = [G_4 \sin(\xi m_4 x_2) + G_5 \cos(\xi m_4 x_2)] \exp(i\xi(x_1 - ct)), \quad (20)$$

where

$$m_4 = \pm \left(\left(\frac{v_1 c}{\alpha^\ell} \right)^2 - 1 \right)^{1/2} \quad (21)$$

and G_4, G_5 are arbitrary constants.

The normal displacement and normal stress for medium M_2 , with the help of (5), (6), (12), and (20), are given by

$$\begin{aligned}u_2^\ell &= m_4 \xi [G_4 \cos(\xi m_4 x_2) - G_5 \sin(\xi m_4 x_2)] \\ &\quad \times \exp(i\xi(x_1 - ct)), \\ t_{22}^\ell &= -\xi^2 (1 + m_4^2) [G_4 \sin(\xi m_4 x_2) + G_5 \cos(\xi m_4 x_2)] \\ &\quad \times \exp(i\xi(x_1 - ct)).\end{aligned}\quad (22)$$

Also from (3) we have

$$\begin{aligned}t_{22} &= C_{12}u_{1,1} + C_{22}u_{2,2} - \beta_{22}T, \\ t_{21} &= C_o(u_{1,2} + u_{2,1}).\end{aligned}\quad (23)$$

5. Boundary Conditions

The appropriate boundary conditions are as follows:

- (i) vanishing of normal stress component at the free surface of the liquid layer, $x_2 = -H$; that is,

$$(t_{22}^\ell)_{M_2} = -p^\ell = 0, \quad (24)$$

- (ii) continuity of normal stress components at the interface $x_2 = 0$; that is,

$$(t_{22})_{M_1} = -p^\ell, \quad (25)$$

- (iii) normal displacement components at the interface $x_2 = 0$; that is,

$$(u_2^\ell)_{M_2} = (u_2)_{M_1}, \quad (26)$$

- (iv) vanishing shear stress for medium M_1 at the interface $x_2 = 0$; that is,

$$(t_{21})_{M_1} = 0, \quad (27)$$

- (v) thermal boundary conditions

$$\frac{\partial T}{\partial x_2} + hT = 0, \quad (28)$$

where $h \rightarrow 0$ corresponds Insulated boundary condition, $h \rightarrow \infty$ corresponds Isolated boundary condition.

6. Derivations of the Secular Equations

Substituting the value of $u_2, T, u_2^\ell, t_{22}^\ell$ from (18) and (22) in (24)–(28) and with the aid of (7) and (12), after simplification, we obtain

$$c^2 = \left(\frac{\alpha^\ell}{v_1} \right)^2 \left[\frac{(\tan^{-1}(T_{11}))^2 + \xi^2 H^2}{\xi^2 H^2} \right], \quad (29)$$

where

$$\begin{aligned}T_{11} &= \frac{C_{11}m_4(E_{11}D_1 - E_{12}D_2 + E_{13}D_3)}{ic\rho^\ell v_1^2(n_{11}D_1 - n_{12}D_2 + n_{13}D_3)}, \\ E_{1p} &= i\xi \left(\frac{C_{12}}{C_{11}} \right) + \left(\frac{C_{22}}{C_{11}} \right) n_{1p} (i^2 \xi m_p) - \left(\frac{\beta_{22}}{\beta_{11}} \right) n_{2p}, \\ E_{2p} &= \xi (n_{1p}i - m_p), \quad E_{3p} = n_{2p} (h - \xi m_p), \\ &\quad (p = 1, 2, 3), \\ D_1 &= E_{22}E_{33} - E_{32}E_{23}, \quad D_2 = E_{21}E_{33} - E_{31}E_{23}, \\ D_3 &= E_{21}E_{32} - E_{31}E_{22}.\end{aligned}\quad (30)$$

Equation (29) is surface waves secular equation of fiber-reinforced transversely isotropic thermoelastic half-space under a uniform homogeneous inviscid liquid layer. The secular equation has complete information about the phase velocity, wave number, and attenuation coefficient of surface waves in such medium.

6.1. Particular Cases

- (i) If we take $h \rightarrow 0$ in (29), we obtain the frequency equation for the thermally insulated boundary.
- (ii) If we take $h \rightarrow \infty$ in (29), we obtain the frequency equation for the thermally isolated boundary.
- (iii) Taking $P \rightarrow 0$ in secular equation (29), yield the frequency equation for fiber-reinforced transversely isotropic thermoelastic half space without initial stress under a uniform homogeneous inviscid liquid layer.
- (iv) For $\mu_T = \mu_L = \mu$, $K_{11} = K_{22} = K$, $\beta_{11} = \beta_{22} = \beta = (3\lambda + 2\mu)\alpha_i$, (29) reduces to the frequency equation in a homogeneous Isotropic half space under inviscid liquid.
- (v) $\tau_o \rightarrow 0$ in (10), we obtained the frequency equation in fiber-reinforced coupled thermoelastic half space under inviscid liquid layer.

7. Rayleigh Wave

(1) Taking $H \rightarrow 0$, the frequency equation (29) reduces to

$$E_{11} (E_{22}E_{33} - E_{23}E_{32}) - E_{12} (E_{21}E_{33} - E_{31}E_{23}) + E_{13} (E_{21}E_{32} - E_{31}E_{22}) = 0. \quad (31)$$

Equation (31) is the frequency equation for Rayleigh surface wave propagation in a fiber-reinforced transversely isotropic thermoelastic half-space with initial stress. In general wave number and hence phase velocity of waves is complex quantity therefore the waves are attenuated in space. If we write

$$c^{-1} = v^{-1} + i\omega^{-1}F \quad (32)$$

so that, $\xi = K + iF$, where $K = \omega/v$, v and F are real. Also the roots of characteristic equation (16) are in general complex and hence we assume that $m_p = p_p + iq_p$, so that the exponent in the plane wave solutions (18) for the half-space becomes

$$iK(x_1 - m_p^I x_2 - vt) - K\left(\frac{F}{K}x_1 + m_p^R x_2\right), \quad (33)$$

where

$$m_p^R = p_p - q_p \frac{F}{K}, \quad m_p^I = q_p + p_p \frac{F}{K}, \quad (p = 1, 2, 3). \quad (34)$$

This shows that v is the propagation velocity and F is the attenuation coefficient of wave.

The equation (18) can be rewritten as

$$\begin{aligned} u_1 &= \sum_{p=1}^3 G_p \exp(-Fx_1 - \lambda_p^R x_2) \\ &\quad \times \exp[i(K(x_1 - vt) - \lambda_p^I x_2)], \\ u_2 &= \sum_{p=1}^3 n_{1p} G_p \exp(-Fx_1 - \lambda_p^R x_2) \\ &\quad \times \exp[i(K(x_1 - vt) - \lambda_p^I x_2)], \\ T &= \sum_{p=1}^3 n_{2p} G_p \exp(-Fx_1 - \lambda_p^R x_2) \\ &\quad \times \exp[i(K(x_1 - vt) - \lambda_p^I x_2)] \end{aligned} \quad (35)$$

with $\lambda_p = K(m_p^R + im_p^I) = \lambda_p^R + i\lambda_p^I$ (say), ($p = 1, 2, 3$). Moreover it is clear that

$$|\lambda_p^R|^2 - |\lambda_p^I|^2 = K^2 [(m_p^R)^2 - (m_p^I)^2]. \quad (36)$$

7.1. *Normal Stress.* Normal stress at the interface $x_2 = 0$ is given by

$$\begin{aligned} t_{22} &= \left(C_{12} i\xi \sum_{p=1}^3 G_p - C_{22} \xi \sum_{p=1}^3 m_p G_p n_{1p} - \beta_{22} \sum_{p=1}^3 G_p n_{2p} \right) \\ &\quad \times \exp(i\xi(x_1 - vt)), \quad (p = 1, 2, 3). \end{aligned} \quad (37)$$

7.2. *Specific Loss.* The Specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through a stress cycle, to the elastic energy (W) stored in the specimen when the strain is maximum. The Specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, Kolsky [32] shows that the specific loss $\Delta W/W$ equals 4π times the absolute value of imaginary part of ξ to the real part of ξ ; that is,

$$\frac{\Delta W}{W} = 4\pi \left| \frac{\text{Im}(\xi)}{\text{Re}(\xi)} \right| = 4\pi \left| \frac{vG}{\omega} \right| = 4\pi \left| \frac{G}{F} \right|. \quad (38)$$

8. Numerical Results and Discussion

For numerical computations, we take the following values of the relevant parameters for generalized fiber-reinforced transversely isotropic thermoelastic solid:

$$\begin{aligned} \lambda &= 5.66 \times 10^{10} \text{ N/m}^2, & \rho &= 2660 \text{ Kg/m}^3, \\ \mu_T &= 2.46 \times 10^{10} \text{ N/m}^2, & \mu_L &= 5.66 \times 10^{10} \text{ N/m}^2, \\ \alpha &= -1.28 \times 10^{10} \text{ N/m}^2, & \beta &= 220.90 \times 10^{10} \text{ N/m}^2, \\ K_{11} &= 0.0921 \times 10^3 \text{ Jm}^{-1} \text{ deg}^{-1} \text{ s}^{-1}, & P &= 100 \\ K_{22} &= 0.0963 \times 10^3 \text{ Jm}^{-1} \text{ deg}^{-1} \text{ s}^{-1}, \\ \alpha_1 &= 0.017 \times 10^4 \text{ deg}^{-1}, \\ \alpha_2 &= 0.015 \times 10^4 \text{ deg}^{-1}, & T_o &= 293 \text{ K}, \\ \tau_o &= 0.05 \text{ s}, & h &= 1, & H &= 1, \\ C_e &= 0.787 \times 10^3 \text{ JKg}^{-1} \text{ deg}^{-1}, & \omega &= 2 \text{ s}^{-1}. \end{aligned} \quad (39)$$

The physical constants for water are given by Ewing et al. [31]:

$$\rho^\ell = 1.0 \times 10^3 \text{ Kg/m}^3, \quad \lambda^\ell = 2.14 \times 10^3 \text{ N/m}^2. \quad (40)$$

For comparison with the generalized fiber-reinforced isotropic thermoelastic solid following Ezzat [33], we take

FIGURE 2: Variation of phase velocity w.r.t. wave number (with and without initial stress).

the following values of relevant parameters for the case of copper material as

$$\begin{aligned}
 \lambda &= 7.76 \times 10^{10} \text{ N/m}^2, \\
 \mu_T = \mu_L = \mu &= 3.86 \times 10^{10} \text{ N/m}^2, \\
 K_{11} = K_{22} = K &= 386 \text{ NK}^{-1} \text{ s}^{-1}, \\
 \beta_{11} = \beta_{22} = \beta &= (3\lambda + 2\mu) \alpha_t, \\
 \alpha_t &= 1.78 \times 10^{-5} \text{ K}^{-1} \\
 \alpha_1 &= 1.005 \text{ deg}^{-1}, \quad \alpha_2 = 1.001 \text{ deg}^{-1}, \\
 C_e &= 383.1 \text{ m}^2 \text{ K}^{-1} \text{ s}^{-1}, \quad P = 0, \\
 H &= 1, \quad \rho = 8954 \text{ Kg/m}^3, \\
 \tau_o &= 0.1 \text{ s}, \quad h = 1.
 \end{aligned} \tag{41}$$

Using the above values of parameters, the dispersion curves of nondimensional phase velocity, attenuation coefficient and specific heat are shown graphically with respect to nondimensional wave number. Here, we consider abbreviations FTTIWIS, FTTIIS, FTIWIS, and FTIIS for fiber-reinforced transversely isotropic thermoelastic material without initial stress, fiber-reinforced transversely isotropic thermoelastic material with initial stress, fiber-reinforced isotropic thermoelastic material without initial stress, and fiber-reinforced isotropic thermoelastic material with initial stress respectively and H refers to the thickness of the layer.

Figure 2 depicts the variations of phase velocity with respect to the wave number. The behavior of phase velocity

FIGURE 3: Variation of attenuation coefficient w.r.t. wave number (with and without initial stress).

for FTTIWIS, FTTIIS, FTIWIS, and FTIIS is similar but the corresponding values are different in magnitude. The values of phase velocity decrease for lower wave number and become stable for higher wave number in all the cases. Figure 3 shows the variation of attenuation coefficient with respect to the wave number. For FTTIWIS and FTTIIS the values of attenuation coefficient increase monotonically up to wave number (ξ) = 2 and after that it becomes constant for whole range of wave number. On the other hand, the values of attenuation coefficients are consistent within the whole range of wave number for FTIWIS and FTIIS.

Figure 4 indicates the variation of phase velocity with respect to the wave number for different values of thickness of layer. Here, we consider the particular case of fiber-reinforced transversely isotropic thermoelastic material and analyses of the behavior of curves with respect to initial stress. It is observed that for small value of wave number the graph of phase velocity shows opposite behavior w.r.t. thickness, that is, on increasing the values of thickness of layer, the values of corresponding phase velocity decreases, and it became steady for large values of wave number.

Figure 5 shows the variation of attenuation coefficient with respect to the wave number for different value of thickness of the layer. Here also we considered the particular case of fiber-reinforced transversely isotropic thermoelastic material and analyses of the behavior of curves with respect to initial stress. It is perceived that on decreasing the value of H the curve shows consistency for small value of wave number and on increasing the value of H the curve shows uniformity for large value of ξ .

Figure 6 depicts the variations of specific loss verses wave number. For FTIWIS and FTIIS the curve shows sharp decrease in the value of specific loss for the range $0 \leq \xi \leq 1$ and become parallel for $\xi > 1$. It is observed that graphs

FIGURE 4: Variation of phase velocity w.r.t. wave number (with and without initial stress for different value of H).

FIGURE 5: Variation of attenuation coeff. w.r.t. wave number (with and without initial stress for different value of H).

corresponding to FTTIWIS and FTTIIS follow the similar pattern for whole range of ξ but the values are different in magnitude.

8.1. Rayleigh Wave. Figure 7 shows the variations of phase velocity of Rayleigh wave versus wave number for FTTIWIS, FTTIIS, FTIWIS, and FTIIS. The value of phase velocity decreases for small values of ξ and become stable for higher value of ξ . It is analyzed that in all the cases behavior of curves are almost the same within the range $2 \leq \xi \leq 5$. Figure 8 shows the variations of attenuation coefficient of Rayleigh wave versus wave number for FTTIWIS, FTTIIS, FTIWIS, and FTIIS. For FTIWIS and FTIIS the graph shows a sudden fall for $\xi = 0.5$ and then increases monotonically up to $\xi = 2$ and becomes constant for the remaining range of ξ . A large difference between the behavior of curves for the transversely isotropic and isotropic case is also perceived.

9. Conclusions

The propagation of surface waves in an initially stressed fiber-reinforced transversely isotropic thermoelastic material under an inviscid liquid layer has been studied. After developing the formal solutions, the secular equation for surface wave propagation is derived. Some special cases of frequency equation are also discussed. The attenuation coefficients and specific loss (liquid layer and half-space) have also been studied and computed for FTTIWIS, FTTIIS, FTIWIS, and FTIIS. The trend of phase velocity is the same in all these four cases for surface wave but varies for attenuation

FIGURE 6: Variation of specific loss w.r.t. wave number (with and without initial stress).

coefficient. From numerical analysis, it is shown that the phase speeds and attenuation coefficients for Rayleigh wave are also affected significantly due to the presence of initial stress parameters.

FIGURE 7: Variation of phase velocity w.r.t. wave number for Rayleigh wave (with and without initial stress).

FIGURE 8: Variation of attenuation coeff. w.r.t. wave number for Rayleigh wave (with and without initial stress).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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