Research Article

Rate of Convergence of Modified Baskakov-Durrmeyer Type Operators for Functions of Bounded Variation

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We study a certain integral modification of well-known Baskakov operators with weight function of beta basis function. We establish rate of convergence for these operators for functions having derivative of bounded variation. Also, we discuss Stancu type generalization of these operators.

1. Introduction

The integral modification of Baskakov operators having weight function of some beta basis function are defined as the following: for $x \in [0, \infty)$, $\gamma > 0$,

$$\begin{aligned} B_{n,\gamma}(f, x) &= \sum_{k=1}^{\infty} p_{n,k,\gamma}(x) \int_0^{\infty} b_{n,k,\gamma}(t) f(t) dt \\ &\quad + (1 + \gamma x)^{-n} f(0) \\ &= \int_0^{\infty} W_{n,\gamma}(x, t) f(t) dt, \end{aligned} \quad (1)$$

where

$$\begin{aligned} p_{n,k,\gamma}(x) &= \frac{\Gamma(n/\gamma + k)}{\Gamma(k+1) \Gamma(n/\gamma)} \cdot \frac{(\gamma x)^k}{(1 + \gamma x)^{(n/\gamma)+k}}, \\ b_{n,k,\gamma}(t) &= \frac{\gamma \Gamma(n/\gamma + k + 1)}{\Gamma(k) \Gamma(n/\gamma + 1)} \cdot \frac{(\gamma t)^{k-1}}{(1 + \gamma t)^{(n/\gamma)+k+1}}, \end{aligned}$$

$$W_{n,\gamma}(x, t) = \sum_{k=1}^{\infty} p_{n,k,\gamma}(x) \int_0^{\infty} b_{n,k,\gamma}(t) dt + (1 + \gamma x)^{-n/\gamma} \delta(t), \quad (2)$$

$\delta(t)$ being the Dirac delta function.

The operators defined by (1) were introduced by Gupta [1]; these operators are different from the usual Baskakov-Durrmeyer operators. Actually these operators satisfy condition $B_{n,\gamma}(at + b, x) = ax + b$, where a and b are constants. In [1], the author estimated some direct results in simultaneous approximation for these operators (1). In particular case $\gamma = 1$, the operators (1) reduce to the operators studied in [2, 3].

In recent years a lot of work has been done on such operators. We refer to some of the important papers on the recent development on similar type of operators [4–9]. The rate of convergence for certain Durrmeyer type operators and the generalizations is one of the important areas of research in recent years. In present article, we extend the studies and here we estimate the rate of convergence for functions having derivative of bounded variation.

We denote $\phi_{n,\gamma}(x, t) = \int_0^t W_{n,\gamma}(x, s) ds$; then, in particular, we have

$$\phi_{n,\gamma}(x, \infty) = \int_0^\infty W_{n,\gamma}(x, s) ds = 1. \quad (3)$$

By $DB_r(0, \infty)$, $r \geq 0$ we denote the class of absolutely continuous functions f defined on the interval $(0, \infty)$ such that,

- (i) $f(t) = O(t^r)$, $t \rightarrow \infty$.
- (ii) having a derivative f' on the interval $(0, \infty)$ coinciding a.e. with a function which is of bounded variation on every finite subinterval of $(0, \infty)$.

It can be observed that all function $f \in BD_r(0, \infty)$ possess for each $c > 0$ a representation

$$f(x) = f(c) + \int_c^x \psi(t) dt, \quad x \geq c. \quad (4)$$

2. Rate of Convergence for $B_{n,\gamma}$

Lemma 1 (see [1]). Let the function $T_{n,m,\gamma}(x)$, $m \in \mathbb{N} \cup \{0\}$, be defined as

$$\begin{aligned} T_{n,m,\gamma}(x) &= B_{n,\gamma}((t-x)^m, x) \\ &= \sum_{k=1}^{\infty} p_{n,k,\gamma}(x) \int_0^\infty b_{n,k,\gamma}(t) (t-x)^m dt \\ &\quad + (1+\gamma x)^{-n/\gamma} (-x)^m. \end{aligned} \quad (5)$$

Then it is easily verified that, for each $x \in (0, \infty)$, $T_{n,0,\gamma}(x) = 1$, $T_{n,1,\gamma}(x) = 0$, and $T_{n,2,\gamma}(x) = 2x(1+\gamma x)/(n-\gamma)$, and also the following recurrence relation holds:

$$\begin{aligned} (n-\gamma m) T_{n,m+1,\gamma}(x) &= x(1+\gamma x) \\ &\quad \times [T_{n,m,\gamma}^{(1)}(x) + 2m T_{n,m-1,\gamma}(x)] \\ &\quad + m(1+2\gamma x) T_{n,m,\gamma}(x). \end{aligned} \quad (6)$$

From the recurrence relation, it can be easily be verified that for all $x \in [0, \infty)$, we have $T_{n,m,\gamma}(x) = O(n^{-(m+1)/2})$.

Remark 2. From Lemma 1, using Cauchy-Schwarz inequality, it follows that

$$B_{n,\gamma}(|t-x|, x) \leq [B_{n,\gamma}((t-x)^2, x)]^{1/2} \leq \sqrt{\frac{2x(1+\gamma x)}{n-\gamma}}. \quad (7)$$

Lemma 3. Let $x \in (0, \infty)$ and $W_{n,\gamma}(x, t)$ be the kernel defined in (1). Then for n being sufficiently large, one has

- (a) $\phi_{n,\gamma}(x, y) = \int_0^y W_{n,\gamma}(x, t) dt \leq 2x(1+\gamma x)/(n-\gamma)(x-y)^2$, $0 \leq y < x$.
- (b) $1 - \phi_{n,\gamma}(x, z) = \int_z^\infty W_{n,\gamma}(x, t) dt \leq 2x(1+\gamma x)/(n-\gamma)(z-x)^2$, $x < z < \infty$.

Proof. First we prove (a); by using Lemma 1, we have

$$\begin{aligned} \int_0^y W_{n,\gamma}(x, t) dt &\leq \int_0^y \frac{(x-t)^2}{(x-y)^2} W_{n,\gamma}(x, t) dt \\ &\leq (x-y)^{-2} T_{n,2,\gamma}(x) \\ &\leq \frac{2x(1+\gamma x)}{(n-\gamma)(x-y)^2}. \end{aligned} \quad (8)$$

The proof of (b) is similar; we omit the details. $\square$

Theorem 4. Let $f \in DB_r(0, \infty)$, $r \in \mathbb{N}$, and $x \in (0, \infty)$. Then for n being sufficiently large, we have

$$\begin{aligned} |B_{n,\gamma}(f, x) - f(x)| &\leq \frac{2(1+\gamma x)}{n-\gamma} \sum_{k=1}^{\lfloor \sqrt{n} \rfloor} \bigvee_{x-x/k}^{x+x/k} ((f')_x) \\ &\quad + \frac{x}{\sqrt{n}} \bigvee_{x-x/\sqrt{n}}^{x+x/\sqrt{n}} ((f')_x) + \frac{2(1+\gamma x)}{(n-\gamma)x} \\ &\quad \times [|f(2x) - f(x) - xf'(x^+)| + |f(x)|] \\ &\quad + \sqrt{\frac{2x(1+\gamma x)}{n-\gamma}} [M2^r O(n^{-r/2}) + |f'(x^+)|] \\ &\quad + \sqrt{\frac{x(1+\gamma x)}{2(n-\gamma)}} |f'(x^+) - f'(x^-)|, \end{aligned} \quad (9)$$

where the auxiliary function f_x is given by

$$f_x(t) = \begin{cases} f(t) - f(x^-), & 0 \leq t < x; \\ 0, & t = x; \\ f(t) - f(x^+), & x < t < \infty. \end{cases} \quad (10)$$

$\bigvee_a^b f(x)$ denotes the total variation of f_x on $[a, b]$.

Proof. By the application of mean value theorem, we have

$$\begin{aligned} B_{n,\gamma}(f, x) - f(x) &= \int_0^\infty W_{n,\gamma}(x, t) (f(t) - f(x)) dt \\ &= \int_0^\infty \int_x^t W_{n,\gamma}(x, t) (f'(u) du) dt. \end{aligned} \quad (11)$$

Also, using the identity

$$\begin{aligned} f'(u) &= \frac{f'(x^+) + f'(x^-)}{2} + (f')_x(u) \\ &\quad + \frac{f'(x^+) - f'(x^-)}{2} \operatorname{sgn}(u-x) \\ &\quad + \left[f'(x) - \frac{f'(x^+) + f'(x^-)}{2} \right] \chi_x(u), \end{aligned} \quad (12)$$

where

$$\chi_x(u) = \begin{cases} 1, & u = x; \\ 0, & u \neq x, \end{cases} \quad (13)$$

we can see that

$$\int_0^\infty \left(\int_x^t \left(f'(x) - \frac{f'(x^+) + f'(x^-)}{2} \right) \chi_x(u) du \right) \times W_{n,\gamma}(t, x) dt = 0. \quad (14)$$

Also,

$$\begin{aligned} & \int_0^\infty \left(\int_x^t \left(\frac{f'(x^+) - f'(x^-)}{2} \right) \operatorname{sgn}(u - x) du \right) W_{n,\gamma}(t, x) dt \\ &= \frac{f'(x^+) - f'(x^-)}{2} B_{n,\gamma}(|t - x|, x), \\ & \int_0^\infty \left(\int_x^t \left(\frac{f'(x^+) + f'(x^-)}{2} \right) du \right) W_{n,\gamma}(t, x) dt \\ &= \frac{f'(x^+) + f'(x^-)}{2} B_{n,\gamma}(t - x, x). \end{aligned} \quad (15)$$

Substitute value of $f'(u)$ from (12) in (11) and using (14) and (15), we get

$$\begin{aligned} |B_{n,\gamma}(f, x) - f(x)| &\leq \left| \int_x^\infty \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right| \\ &+ \left| \int_0^x \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right| \\ &+ \frac{|f'(x^+) - f'(x^-)|}{2} B_{n,\gamma}(|t - x|, x) \\ &+ \frac{|f'(x^+) + f'(x^-)|}{2} B_{n,\gamma}(t - x, x). \end{aligned} \quad (16)$$

Using Lemma 1 and Remark 2, we obtain

$$\begin{aligned} |B_{n,\gamma}(f, x) - f(x)| &\leq \left| \int_x^\infty \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right| \\ &+ \left| \int_0^x \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right| \\ &+ \frac{|f'(x^+) - f'(x^-)|}{2} \sqrt{\frac{2x(1 + \gamma x)}{n - \gamma}} \\ &= |P_{n,\gamma}(f, x)| + |Q_{n,\gamma}(f, x)| \\ &+ \frac{|f'(x^+) - f'(x^-)|}{2} \sqrt{\frac{2x(1 + \gamma x)}{n - \gamma}}. \end{aligned} \quad (17)$$

On applying Lemma 3 with $y = x - x/\sqrt{n}$ and integrating by parts, we have

$$\begin{aligned} |Q_{n,\gamma}(f, x)| &= \left| \int_0^x \int_x^t (f')_x(u) du dt (\phi_{n,\gamma}(x, y)) \right| \\ &= \left| \int_0^x \phi_{n,\gamma}(x, y) (f')_x(t) dt \right| \\ &\leq \left(\int_0^y + \int_y^x \right) |(f')_x(t)| |\phi_{n,\gamma}(x, t)| dt \\ &\leq \frac{2x(1 + \gamma x)}{n - \gamma} \int_0^y \bigvee_t ((f')_x) \frac{1}{(x - t)^2} dt \\ &\quad + \int_y^x \bigvee_t ((f')_x) dt \\ &\leq \frac{2x(1 + \gamma x)}{n - \gamma} \int_0^y \bigvee_t ((f')_x) \frac{1}{(x - t)^2} dt \\ &\quad + \frac{x}{\sqrt{n}} \bigvee_{x-x/\sqrt{n}}^x ((f')_x) \\ &= \frac{2x(1 + \gamma x)}{n - \gamma} \int_1^{\sqrt{n}} \bigvee_{x-x/u}^x ((f')_x) du \\ &\quad + \frac{x}{\sqrt{n}} \bigvee_{x-x/\sqrt{n}}^x ((f')_x) \\ &\leq \frac{2x(1 + \gamma x)}{n - \gamma} \sum_{k=1}^{[\sqrt{n}]} \bigvee_{x-x/k}^x ((f')_x) \\ &\quad + \frac{x}{\sqrt{n}} \bigvee_{x-x/\sqrt{n}}^x ((f')_x), \end{aligned} \quad (18)$$

where $u = x/(x - t)$.

On the other hand, we have

$$\begin{aligned} |P_{n,\gamma}(f, x)| &= \left| \int_x^\infty \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right| \\ &= \left| \int_{2x}^\infty \left(\int_x^t (f')_x(u) du \right) W_{n,\gamma}(x, t) dt \right. \\ &\quad \left. + \int_x^{2x} \left(\int_x^t (f')_x(u) du \right) dt (1 - \phi_{n,\gamma}(x, t)) \right| \\ &\leq \left| \int_{2x}^\infty (f(t) - f(x)) W_{n,\gamma}(x, t) dt \right| \\ &\quad + |f'(x^+)| \left| \int_{2x}^\infty (t - x) W_{n,\gamma}(x, t) dt \right| \\ &\quad + \left| \int_x^{2x} (f')_x(u) du \right| |1 - \phi_{x,\gamma}(x, 2x)| \end{aligned}$$

$$\begin{aligned}
& + \int_x^{2x} |(f')_x(t)| |1 - \phi_{x,\gamma}(x, t)| dt \\
& \leq \left[\frac{M}{x} \int_{2x}^{\infty} W_{n,\gamma}(x, t) t^r |t - x| dt \right. \\
& \quad + \frac{|f(x)|}{x^2} \int_{2x}^{\infty} W_{n,\gamma}(x, t) (t - x)^2 dt \Big] \\
& \quad + \left[|f'(x^+)| \int_{2x}^{\infty} W_{n,\gamma}(x, t) |t - x| dt \right] \\
& \quad + \frac{2(1 + \gamma x)}{(n - \gamma)x} |f(2x) - f(x) - xf'(x^+)| \\
& \quad + \frac{2(1 + \gamma x)}{n - \gamma} \sum_{k=1}^{[\sqrt{n}]} \bigvee_x^{x+x/k} ((f')_x) \\
& \quad + \frac{x}{\sqrt{n}} \bigvee_x^{x+x/\sqrt{n}} ((f')_x) \\
& = R_{n,\gamma}(f, x) + S_{n,\gamma}(f, x) \\
& \quad + \frac{2(1 + \gamma x)}{(n - \gamma)x} |f(2x) - f(x) - xf'(x^+)| \\
& \quad + \frac{2(1 + \gamma x)}{n - \gamma} \sum_{k=1}^{[\sqrt{n}]} \bigvee_x^{x+x/k} ((f')_x) \\
& \quad + \frac{x}{\sqrt{n}} \bigvee_x^{x+x/\sqrt{n}} ((f')_x).
\end{aligned} \tag{19}$$

Applying Holder's inequality, Remark 2, and Lemma 1, we have

$$\begin{aligned}
R_{n,\gamma}(f, x) & \leq \frac{M}{x} \left(\int_{2x}^{\infty} W_{n,\gamma}(x, t) t^{2r} dt \right)^{1/2} \\
& \quad \times \left(\int_0^{\infty} W_{n,\gamma}(x, t) (t - x)^2 dt \right)^{1/2} \\
& \quad + \frac{|f(x)|}{x^2} \int_{2x}^{\infty} W_{n,\gamma}(x, t) (t - x)^2 dt
\end{aligned} \tag{20}$$

$$\begin{aligned}
& \leq M 2^r O(n^{-r/2}) \sqrt{\frac{2x(1 + \gamma x)}{n - \gamma}} \\
& \quad + |f(x)| \frac{2(1 + \gamma x)}{(n - \gamma)x}.
\end{aligned}$$

Also,

$$\begin{aligned}
S_{n,\gamma}(f, x) & \leq |f(x^+)| \int_0^{\infty} W_n(t, x) |t - x| dt \\
& \leq |f(x^+)| \sqrt{\frac{2x(1 + \gamma x)}{n - \gamma}}.
\end{aligned} \tag{21}$$

Combining the estimates (17)–(21), we get the desired results. This completes the proof of Theorem. $\square$

3. Rate of Convergence for Stancu Type Generalization of $B_{n,\gamma}$

In 1968, Stancu introduces Bernstein-Stancu operators in [10], a sequence of the linear positive operators depending on two parameters α and β satisfying the condition $0 \leq \alpha \leq \beta$. Recently many researchers applied this approach to many operators; for details see [11–17]. For $f \in C[0, \infty)$, Stancu generalization of operators (1) is as follows:

$$\begin{aligned}
B_{n,\gamma}^{\alpha,\beta}(f(t), x) & = \sum_{k=1}^{\infty} p_{n,k,\gamma}(x) \int_0^{\infty} b_{n,k,\gamma}(t) f\left(\frac{nt + \alpha}{n + \beta}\right) dt \\
& \quad + p_{n,0,\gamma}(x) f\left(\frac{\alpha}{n + \beta}\right) \\
& = \int_0^{\infty} W_{n,\gamma}(x, t) f\left(\frac{nt + \alpha}{n + \beta}\right) dt,
\end{aligned} \tag{22}$$

where $p_{n,k,\gamma}(x)$, $b_{n,k,\gamma}(x)$, and $W_{n,\gamma}(x, t)$ are as defined in (1).

Lemma 5 (see [18]). *If we define the central moments, for every $m \in \mathbb{N}$ as*

$$\begin{aligned}
\mu_{n,m,\gamma}^{\alpha,\beta}(x) & = B_{n,\gamma}^{\alpha,\beta}((t - x)^m, x) \\
& = \sum_{k=1}^{\infty} p_{n,k,\gamma}(x) \int_0^{\infty} b_{n,k,\gamma}(t) \left(\frac{nt + \alpha}{n + \beta} - x\right)^m dt \\
& \quad + p_{n,0,\gamma}(x) \left(\frac{\alpha}{n + \beta} - x\right)^m,
\end{aligned} \tag{23}$$

then $\mu_{n,0,\gamma}^{\alpha,\beta}(x) = 1$, $\mu_{n,1,\gamma}^{\alpha,\beta}(x) = (\alpha - \beta x)/(n + \beta)$, and

$$\begin{aligned}
\mu_{n,2,\gamma}^{\alpha,\beta}(x) & = \frac{\alpha^2}{(n + \beta)^2} + x \frac{(2n^2 - 2n\alpha\beta + 2\alpha\beta\gamma)}{(n + \beta)^2(n - \gamma)} \\
& \quad + x^2 \frac{(n\beta^2 + 2n^2\gamma - \beta^2\gamma)}{(n + \beta)^2(n - \gamma)}.
\end{aligned} \tag{24}$$

For $n > m$ we have the following recurrence relation:

$$\begin{aligned}
& (n - \gamma m)(n + \beta) \mu_{n,m+1,\gamma}^{\alpha,\beta}(x) \\
& = nx(1 + \gamma x) \left[(\mu_{n,m,\gamma}^{\alpha,\beta})^{(1)}(x) + m \mu_{n,m-1,\gamma}^{\alpha,\beta}(x) \right] \\
& \quad + [mn + n^2x - (2\gamma m - n)(\alpha - (n + \beta)x)] \mu_{n,m,\gamma}^{\alpha,\beta}(x) \\
& \quad + \left[m\gamma(n + \beta) \left(\frac{\alpha}{n + \beta} - x\right)^2 - mn \left(\frac{\alpha}{n + \beta} - x\right) \right] \\
& \quad \times \mu_{n,m-1,\gamma}^{\alpha,\beta}(x).
\end{aligned} \tag{25}$$

From the recurrence relation, it can be easily verified that for all $x \in [0, \infty)$, we have $\mu_{n,m,\gamma}^{\alpha,\beta}(x) = O(n^{-(m+1)/2})$.

Remark 6. Observe that $B_{n,\gamma}^{\alpha,\beta}$ preserve constant functions but not linear functions. If $\alpha = \beta = 0$ these operators reduce to the operators defined in (1). Notice that

$$\mu_{n,2,\gamma}^{\alpha,\beta}(x) = \left[\frac{\beta^2}{\gamma(n+\beta)^2} + \frac{2n^2}{(n+\beta)^2(n-\gamma)} \right] x(1+\gamma x) + \frac{\beta[(2\alpha+\beta)\gamma - n(\beta+2\alpha\gamma)]}{\gamma(n+\beta)^2(n-\gamma)} x + \frac{\alpha^2}{(n+\beta)^2}. \quad (26)$$

Remark 7. From Lemma 3, taking n to be sufficiently large and $x \in (0, \infty)$, we observe that

$$\mu_{n,2,\gamma}^{\alpha,\beta}(x) \leq \frac{Cx(1+\gamma x)}{n-\gamma}, \quad (27)$$

where C is positive constant.

Remark 8. Applying the Cauchy-Schwarz inequality and keeping the same condition as in Remark 7 for x , n , and C , we derive from Lemma 5 that

$$B_{n,\gamma}^{\alpha,\beta}(|t-x|, x) \leq [B_{n,\gamma}^{\alpha,\beta}((t-x)^2, x)]^{1/2} \leq \sqrt{\frac{Cx(1+\gamma x)}{n-\gamma}}. \quad (28)$$

Lemma 9. Let $x \in (0, \infty)$ and $W_{n,\gamma}(x, t)$ be the kernel defined in (1). Then for n being sufficiently large, we have

$$(c) \phi_{n,\gamma}(x, y) = \int_0^y W_{n,\gamma}(x, t) dt = Cx(1+\gamma x)/(n-\gamma)(x-y)^2, \quad 0 \leq y < x.$$

$$(d) 1 - \phi_{n,\gamma}(x, z) = \int_z^\infty W_{n,\gamma}(x, t) dt = Cx(1+\gamma x)/(n-\gamma)(z-x)^2, \quad x < z < \infty.$$

The proof is the same as Lemma 3, thus we omit the details.

Theorem 10. Let $f \in DB_r(0, \infty)$, $r \in \mathbb{N}$, and $x \in (0, \infty)$. Then for n being sufficiently large, we have

$$\begin{aligned} & |B_{n,\gamma}^{\alpha,\beta}(f, x) - f(x)| \\ & \leq \frac{C(1+\gamma x)}{n-\gamma} \sum_{k=1}^{[\sqrt{n}]} \bigvee_{x-x/k}^{x+x/k} ((f')_x) \\ & \quad + \frac{x}{\sqrt{n}} \bigvee_{x-x/\sqrt{n}}^{x+x/\sqrt{n}} ((f')_x) + \frac{C(1+\gamma x)}{(n-\gamma)x} \end{aligned}$$

$$\begin{aligned} & \times [|f(2x) - f(x) - xf'(x^+)| + |f(x)|] \\ & + \sqrt{\frac{Cx(1+\gamma x)}{n-\gamma}} \\ & \times [M_1 2^r O(n^{-r/2}) + |f'(x^+)|] \\ & + \frac{1}{2} \sqrt{\frac{Cx(1+\gamma x)}{(n-\gamma)}} |f'(x^+) - f'(x^-)| \\ & + \frac{1}{2} |f'(x^+) + f'(x^-)| \frac{|\alpha - \beta x|}{n+\beta}, \quad (29) \end{aligned}$$

where the auxiliary functions f_x and $\bigvee_a^b f(x)$ were defined in Theorem 4.

The proof of the above theorem follows along the lines of Theorem 4; thus we omit the details.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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