Research Article

Effects of Chemical Reaction on Dissipative Radiative MHD Flow through a Porous Medium over a Nonisothermal Stretching Sheet

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The steady two-dimensional radiative MHD boundary layer flow of an incompressible, viscous, electrically conducting fluid caused by a nonisothermal linearly stretching sheet placed at the bottom of fluid saturated porous medium in the presence of viscous dissipation and chemical reaction is studied. The governing system of partial differential equations is converted to ordinary differential equations by using the similarity transformations, which are then solved by shooting method. The dimensionless velocity, temperature, and concentration are computed for different thermophysical parameters, namely, the magnetic parameter, permeability parameter, radiation parameter, wall temperature parameter, Prandtl number, Eckert number, Schmidt number, and chemical reaction.

1. Introduction

The fluid flows with chemical reaction have attracted the attention of engineers and scientists in the recent times. Such flows have key importance in many processes including drying evaporation at the surface of a water body, energy transfer in a wet cooling tower, flow in a desert cooler, generating electric power, food processing, groves of fruit trees, and crops damage because of freezing. There is always a molecular diffusion of species in the presence of chemical reaction within or at the boundary during several practical diffusive operations. There are two types of reactions, namely, homogeneous and heterogeneous. A homogeneous reaction takes place uniformly in the entire given phase whereas a heterogeneous reaction exists in a restricted region or within the boundary of a phase. The smog formation is an important example representing a first-order homogeneous chemical reaction. Several researchers in view of such facts are engaged in the discussion of flows with chemical reactions. For instance Seddeek and Almushigeh [1] investigated the effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Kandasamy et al. [2] presented group analysis for Soret and Dufour effects on free convective heat and mass transfer with thermophoresis and chemical reaction over a porous stretching surface in the presence of heat source/sink. Pal and Talukdar [3] presented the combined effects of Joule heating and chemical reaction on unsteady magneto-hydrodynamic mixed convection with viscous dissipation over a vertical plate in the presence of porous media and thermal radiation. Joneidi et al. [4] presented analytical treatment of MHD free convection flow over a stretching sheet with chemical reaction. Anjalidevi and Kandasamy [5] studied effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. There are intensive studies which have been carried out to investigate effects of chemical reaction on different flow types (see Seddeek et al. [6]; Salem and Abd El-Aziz [7]).

The study of convection heat and mass transfer phenomenon in porous media has gained attention due to its interesting applications. Processes involving heat and mass transfer in porous media are often encountered in the chemical industry, in reservoir engineering in connection with thermal recovery process, and in the study of dynamics of hot
and salty springs of a sea. Underground spreading of chemical wastes and other pollutants, grain storage, evaporation cooling, and solidification are the few other application areas where the combined thermosolutal convection in porous media is observed. Detailed reviews of flow through and past porous media can be found in Nield and Bejan [8]. Abel et al. [9] studied the two-dimensional boundary layer problem on mixed convection of an incompressible viscoelastic fluid immersed in porous medium over a stretching sheet. Vyas and Srivastava [10] investigated the radiation effect on MHD flow over a nonisothermal stretching sheet in a porous medium.

Magnetohydrodynamic (MHD) boundary layers with heat and mass transfer over nonisothermal stretching sheet are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, and cooling of nuclear reactors. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched into a cooling system. The use of magnetic field that influences heat generation/absorption process in electrically conducting fluid flows has many engineering applications, for example, many metallurgical processes which involve cooling of continuous strips or filaments, which are drawn through a quiescent fluid. The properties of the final product depend to a great extent on the rate of cooling. The rate of cooling and, therefore, the desired properties of the end product can be controlled by the use of electrically conducting fluids and the applications of the magnetic fields (Vajravelu and Hadjinicolaou [11]). Many works have been reported on flow, heat and mass transfer of electrically conducting fluids over semi-infinite/infinite plates/stretching surfaces in the presence of magnetic field (see, e.g., Mohammed Ibrahim and Bhaskar Reddy [12]; Shateyi et al. [13]; Makinde and Sibanda [14]; Pal and Talukdar [15]; and Makinde and Aziz [16]).

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, and gas turbines and various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering applications. The effect of radiation on MHD flow, heat and mass transfer becomes more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. The quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system can lead to a desired product with sought qualities. Different researches have been forwarded to analyze the effects of thermal radiation from different flows (Cortell [17]; Ibrahim et al. [18]; Shateyi [19]; Shateyi and Motsa [20]; and Aliakbar et al. [21]).

Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. Gebhart [22] came out with observations that devices which operate at high rotational speeds or which are subject to large decelerations experience significant viscous dissipation effect. The effect is felt prominently in strong gravitational fields and in processes wherein the scale of the process is very large, for example, on larger planets, in large masses of gas in space, and in geological processes in fluids contained in various bodies. It is pertinent to record that even if viscous dissipation effect is quantitatively negligible in some cases, its qualitative effect is significantly observed. Vyas and Srivastava [23] studied the radiation effects on MHD boundary layer flow in a porous medium over a nonisothermal stretching sheet in presence of dissipation. The works of Gebhart and Mollendorf [24], Nield [25], and Rees et al. [26] shed light on the importance of viscous heating.

The objective of this paper was to explore the effects of thermal radiation, viscous dissipation, and chemical reaction on MHD boundary layer flow over a nonisothermal stretching sheet embedded in porous medium. Using a similarity approach, the governing equations are transformed into nonlinear ordinary equations and solved numerically using shooting technique along with fourth-order Runge-Kutta method. The pertinent results are displayed graphically and discussed quantitatively.

2. Mathematical Model

Consider the steady two-dimensional forced convection boundary-layer flow of viscous, incompressible, electrically conducting fluid in a fluid saturated horizontal porous medium caused by linearly stretching sheet placed at the bottom of the porous medium. A Cartesian coordinate system is used. The $x$-axis is along the sheet and $y$-axis is normal to the $x$-axis (see Figure 1). Two equal and opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The stretching velocity varies linearly with the distance from the origin. A uniform magnetic field of strength $B_0$ is applied normal to sheet. We assume that the wall temperature $T_w > T_\infty$, where $T_\infty$ is the uniform temperature of the ambient temperature. We also assume that the fluid is optically dense, Newtonian, and without phase change. Further it is assumed that both the fluid and the porous medium are in local thermal
equilibrium. We also consider that both the surroundings and the fluid are maintained at a constant temperature $T_{co}$ far away from the sheet. We further assume that there exists a homogeneous chemical reaction between the fluid and species concentration. The foreign mass present in the flow is assumed to be at low level and hence Soret and Dufour effects are negligible.

Under these assumptions and considering the viscous dissipation, the governing boundary layer equations for the momentum, heat and mass transfer in the presence of dissipation, the governing boundary layer equations for are negligible.

The boundary conditions for the velocity, temperature, and concentration fields are

\[ u = cx, \quad v = 0, \quad T = T_w = T_{co} + dx^i, \]
\[ C = C_w = C_{co} + dx^o \quad \text{at} \quad y = 0 \]
\[ u \rightarrow 0, \quad T \rightarrow T_{co}, \quad C \rightarrow C_{co} \quad \text{as} \quad y \rightarrow \infty, \]

where $x$ and $y$ represent the coordinate axes along the continuous stretching surface in the direction of motion and normal to it, respectively, $u$ and $v$ are the velocity components along the $x$- and $y$-axes respectively, $v$ is the kinematics viscosity, $\mu$ is the fluid viscosity, $\nu$ is electric conductivity, $B_0$ is the uniform magnetic field, $\rho$ is the density, $k$ is the permeability of the porous medium, $c_p$ is the specific heat at constant pressure, $q_r$ is the radiation heat flux, $T$ is the temperature inside the boundary layer, $T_{co}$ is the temperature far away from the plate, $C$ is the species concentration in the boundary layer, $C_{co}$ is the concentration far away from the plate, $\beta$ is the coefficient of thermal expansion, $\beta^*$ is the coefficient of concentration expansion, $f$ is the gravitational acceleration, $\alpha$ is the wall temperature parameter, $D$ is the molecular diffusivity of the species concentration, $c > 0, d$ are constants, and $Kr^*$ is the first-order homogeneous constant reaction rate.

The radiative heat flux $q_r$ is described by Roseland approximation for radiation (Brewster [27]) such that

\[ q_r = -\frac{4\sigma^*}{3K^*} \frac{\partial T^4}{\partial y}, \]  

where $\sigma^*$ and $K^*$ are Stefan-Boltzmann constant and mean absorption coefficient, respectively. Following Chamkha [28], we assume that the temperature differences within the flow are sufficiently small so that $T^4$ can be expressed as a linear function after using Taylor series to expand $T^4$ over the free stream temperature $T_{co}$ and neglecting higher-order terms. This result is the following approximation:

\[ T^4 \approx 4T_{co}^3 T - 3T_{co}^4. \]  

Using (6) and (7) in (3), we obtain

\[ \frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3K} \frac{\partial^2 T^4}{\partial y^2}. \]  

We introduce the following nondimensional variables:

\[ \eta = \sqrt{\frac{c}{v} y}, \quad u = \frac{\partial \psi}{\partial y} = xe f', \]
\[ v = -\frac{\partial \psi}{\partial x} = -\sqrt{cy} f, \]
\[ M = \frac{\sigma B_0^2}{\rho c}, \quad K = \frac{\nu}{k} \]
\[ \theta = \frac{T - T_{co}}{T_{w} - T_{co}}, \quad \phi = \frac{C - C_{co}}{C_w - C_{co}}, \]
\[ R = \frac{KK^*}{4\sigma T_{co}^3}, \]
\[ Pr = \frac{\rho c_w}{k}, \quad Gr = \frac{\beta^* (C_w - C_{co})}{c^2 x}, \]
\[ Gc = \frac{\alpha (c^2 - c_{co})}{c^2 x}, \]
\[ Ec = \frac{c^2 x^2}{c_p (T_{w} - T_{co})}, \]
\[ Sc = \frac{\nu}{D}, \quad Kr = \frac{Kr^*}{c}. \]

In view of (6), (2)–(4) take the form:

\[ f''' + f'f'' - \left(f' \right)^2 + Gr \theta + Gc \phi - (M + K) f' = 0, \]
\[ \theta'' - \left(\frac{3PrR}{3R + 4}\right) \left[ a \theta f' - f' \theta \right] + \left(\frac{3PrR}{3R + 4}\right) Ec (f'')^2 = 0, \]
\[ \phi'' + Sc f \phi' - Kr Sc \phi = 0, \]  

where the primes denote the differentiation with respect to $\eta, M$ is the magnetic parameter, $K$ is the permeability parameter, $Pr$ is the Prandtl number, $R$ is the radiation parameter, $\alpha$ is wall temperature parameter, $Gr$ is the Grashof number, $Gc$ is the modified Grashof number, $Ec$ is the Eckert number, $Sc$ is the Schmidt number, and $Kr$ is the chemical reaction parameter.
The corresponding boundary conditions are

\[ f' = 1, \quad f = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad \eta = 0 \]
\[ f' = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad \eta \to \infty. \]  

(11)

3. Numerical Computation

The numerical solutions of the nonlinear differential equations (10) under the boundary conditions (11) have been performed by applying a shooting method along with the fourth-order Runge-Kutta method. First of all higher-order nonlinear differential equations (10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique. From this process of numerical computation, the skin-friction coefficient, the Nusselt number, and Sherwood number which are, respectively, proportional to \( f''(0) \), \(-\theta'(0)\), and \(-\phi'(0)\) are also sorted out and their numerical values are presented in a tabular form.

4. Results and Discussion

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various governing parameters, namely, thermal Grashof number \( Gr \), modified Grashof number \( Gc \), the magnetic field parameter \( M \), permeability parameter \( K \), Prandtl number \( Pr \), wall temperature parameter \( \alpha \), thermal radiation parameter \( R \), Eckert number \( Ec \), Schmidt number \( Sc \), and chemical reaction parameter \( Kr \). In the present study the following default parameter values are adopted for computations: \( Gr = 2.0, \ Gc = 2.0, \ M = 1.0, \ K = 0.5, \ Pr = 0.72, \ R = 1.0, \ \alpha = 1.0, \ Ec = 0.5, \ Sc = 0.6, \) and \( Kr = 0.5 \). All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

The influence of the thermal Grashof number \( Gr \) on the velocity is presented in Figure 2. It is observed that there is a rise in the velocity due to enhancement of thermal buoyancy force. Figure 3 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number \( Gc \), while all other parameters are kept at some fixed values. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force.

The effects of the magnetic field parameter \( M \) on the dimensionless velocity, temperature, and concentration fields are shown in Figures 4, 5, and 6; it is obvious that an increase in the magnetic field parameter \( M \) results in a decrease in the velocity. It is observed that the temperature and concentration profiles increase with increasing of magnetic field parameter \( M \).

Figures 7, 8, and 9 show the dimensionless velocity, temperature, and concentration profiles for different values of permeability parameter \( K \). It can be seen that the velocity profile decreases with the increase of permeability parameter \( K \). It is noticed that the temperature and concentration profiles increase permeability parameter \( K \).
The effects of the thermal radiation parameter $R$ on velocity and temperature profiles are shown in Figures 10 and 11. It is obvious that both velocity and temperature decrease as radiation parameter $R$ increases.

The influence of the Prandtl number $Pr$ on velocity and temperature field is shown in Figures 12 and 13. It is obvious that both velocity and temperature decrease as Prandtl parameter $Pr$ increases.

Figures 14 and 15 show the dimensionless velocity and temperature profiles for different values of wall temperature parameter $\alpha$. It can be seen that both velocity and the temperature profiles decrease with the increase of wall temperature parameter $\alpha$.

Figures 16 and 17 depict the effect of Eckert number $Ec$ on the dimensionless velocity and temperature profile. It is revealed that velocity and temperature profiles’ scores grow with the increasing of the Eckert number $Ec$. Eckert number, physically, is a measure of frictional heat in the
Figure 10: Velocity profiles for different values of $R$.

Figure 11: Temperature profiles for different values of $R$.

Figure 12: Velocity profiles for different values of $Pr$.

Figure 13: Temperature profiles for different values of $Pr$.

Figure 14: Velocity profiles for different values of $\alpha$.

Figure 15: Temperature profiles for different values of $\alpha$. 
system. Hence the thermal regime with larger Ec values is subjected to rather more frictional heating causing a source of rise in the temperature. To be specific, the Eckert number Ec signifies the relative importance of viscous heating to thermal diffusion. Viscous heating may serve as energy source to modify the temperature regime qualitatively.

The influence of the Schmidt number Sc on the dimensionless velocity and concentration profiles is plotted in Figures 18 and 19. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease, yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figures 18 and 19. From the definition of the Schmidt number Sc given in (9), it is clearly observed that Sc is inversely proportional to the diffusion coefficient $D$. Therefore larger values of Sc correspond to a decrease in the concentration field.

The effects of the chemical reaction parameter Kr on dimensionless velocity component profiles are plotted in Figure 20. As the chemical reaction parameter increases, velocity profile decreases. These behaviors are clear from Figure 20.

Figure 21 explains the variation of generative ($Kr < 0$) chemical reaction on the concentration field $\phi(\eta)$. It is noted that generative ($Kr < 0$) chemical reaction causes an increase in the concentration. Figure 22 predicts the effects of destructive ($Kr > 0$) chemical reaction on the concentration field $\phi(\eta)$. It is observed that $\phi(\eta)$ decreases in case of destructive ($Kr > 0$) chemical reaction. It is also noticed that the magnitude of $\phi(\eta)$ is large in case of generative chemical reaction ($Kr < 0$) in comparison to the case of destructive chemical reaction ($Kr > 0$). Physically for generative case, chemical reaction Kr takes place without creating much disturbance whereas in the case of destructive chemical reaction is much larger. Due to this fact molecular
motion in the case of $Kr > 0$ is quite larger which finally results in an increase in the mass transport phenomenon.

From Table 1, it is observed that the skin friction coefficient $f''(0)$, Nusselt number $-\theta'(0)$, and Sherwood number $-\phi'(0)$ decrease with the increase of magnetic field number $M$ or permeability parameter $K$. The skin friction coefficient, Nusselt number, and Sherwood number increase with the increase of Grashof number $Gr$ or modified Grashof number $Gc$. From Table 2 it is found that the magnitude of the wall temperature gradient $-\theta'(0)$ increases as Prandtl number $Pr$ or radiation parameter $R$ or wall temperature parameter $\alpha$ increases, while it decreases as the Eckert number $Ec$ increases. From Table 3, it is noticed that the magnitude of the wall concentration gradient $-\phi'(0)$ decreases as the magnetic field parameter $M$ increases, while it increases with an increase in the Schmidt number $Sc$ or chemical reaction parameter $Kr$.

### 5. Conclusion

In this paper we study the chemical reaction effects on radiative MHD flow over a nonisothermal stretching sheet embedded in porous medium in the presence of viscous dissipation. The expressions for the velocity, temperature, and concentration distributions which are the equations governing the flow are numerically solved by the fourth-order Runge-Kutta method along with shooting technique. The
Table 3: Numerical values of skin friction coefficient, Nusselt number, and Sherwood number for $Gr = 2.0,$ $Gc = 2.0,$ $M = 1.0,$ $K = 1.0,$ $Pr = 0.72,$ $R = 1.0,$ $\alpha = 1.0,$ and $Ec = 0.5.$

<table>
<thead>
<tr>
<th>Sc</th>
<th>$Kr$</th>
<th>$f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\psi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.5</td>
<td>0.150762</td>
<td>0.442613</td>
<td>0.777892</td>
</tr>
<tr>
<td>0.78</td>
<td>0.5</td>
<td>0.116005</td>
<td>0.433268</td>
<td>0.89045</td>
</tr>
<tr>
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<td>0.0816344</td>
<td>0.425098</td>
<td>1.01176</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0</td>
<td>0.110634</td>
<td>0.433838</td>
<td>0.94824</td>
</tr>
<tr>
<td>0.6</td>
<td>1.5</td>
<td>0.0794772</td>
<td>0.427467</td>
<td>1.09362</td>
</tr>
</tbody>
</table>

effects of various governing parameters on the skin friction, Nusselt number, and Sherwood number are shown in tables. It is observed that the skin friction, Nusselt number, and Sherwood number decrease with the increase of magnetic field parameter or permeability parameter. It can be seen that the velocity profiles increase with the increase of Grashof number or modified Grashof number. It is observed that both velocity and the temperature profiles decrease with the increase of wall temperature parameter $\alpha$. As thermal radiation parameter increases, velocity and temperature profiles decrease. The concentration field $\phi(\eta)$ is a decreasing function of Schmidt number $Sc$. The concentration field $\phi(\eta)$ has opposite results for destructive ($Kr > 0$) and generative ($Kr < 0$) chemical reactions.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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