Research Article
Sliding Mode Control for the Synchronous Generator

Yaote Chang and Chih-Chin Wen

Department of Electrical Engineering, Kao Yuan University, Kaohsiung 821, Taiwan

Correspondence should be addressed to Yaote Chang; t20052@cc.kyu.edu.tw

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Based on the Lyapunov stability theorem and sliding mode control technique, a design of the nonlinear controller is proposed for the dual-excited and steam-valving control of the synchronous generators with matched and mismatched perturbations in this paper. By using some constant gains designed in the sliding surface function, the perturbations in the power system can be suppressed, and the property of asymptotical stability of the rotor angle and the voltage can be achieved at the same time.

1. Introduction

To achieve a high degree of reliability in the power systems, many works [1–11] have studied the stability of generators. In general, there are two ways to stabilize the generators: the excited control [1–3] and the steam-valving control [4–7]. Using the excited control, Xie et al. [1] designed a linear matrix inequality (LMI) controller for a class of multimachine power systems with uncertain parameters to achieve the property of asymptotical stability. Galaz et al. [2] proposed a passivity-based controller and discussed the domain of attraction of the equilibria in power systems. Huang et al. [3] utilized a physical exact linearization method to design a controller for a dual-excited synchronous generators. For the steam-valving control, Zhang and Sun [4], Fu [5], Li et al. [6], and Li et al. [7] designed the adaptive backstepping controller for single machine infinite bus system in the presence of internal and external disturbances to achieve the property of asymptotical stability.

As for the systems with both steam turbine dynamics and the excited generator, Xi et al. [8] and Ma et al. [9] presented a novel nonlinear controller based on Hamiltonian energy theory steam for the turbine dynamics and single excited generator to achieve the property of asymptotical stability. The dual-excitation means the system has $d$-axis and $q$-axis field winding simultaneously. Each field voltage can be adjusted separately and hence the control objectives can be achieved more flexibly. Based on the passive lemma, Wang and Lin [10] designed the bounded passivity controller for the synchronous generators to achieve the property of asymptotical stability. Using the coordinated passivation technique, Chen et al. [11] designed backstepping controller for steam-valving and dual-excited synchronous generators to achieve the property of asymptotical stability. However, the perturbations were not considered in the works [10, 11].

Sliding mode control (SMC) is well known to possess several advantages, for example, fast response, good transient performance, robustness of stability, and insensitivity to matched parameter variations and external disturbances [12, 13]. However, the property of asymptotical stability is in general hard to achieve by using the traditional SMC technique if the mismatched perturbations are presented in the systems [14]. A lot of researchers applied SMC techniques to solve the tracking problems with mismatched perturbations [15–17]. For example, Shieh and Shyu [15], Chen and Dunnigan [16], and Kwan [17] employed SMC techniques for an induction machine with an uncertain load torque; however, the mismatched perturbations considered in these works [15–17] belong to the unknown constants.

In this paper, we have proposed the nonlinear sliding mode controller for the dual-excited and steam-valving control of the synchronous generators with matched and
mismatched perturbations to achieve the property of asymptotical stability. Our proposed control scheme can be thought of as the extension work of [10, 11], where no perturbations are considered in the works [10, 11]. Furthermore, the mismatched perturbations considered in this paper can be time varying.

2. System Model

Consider a machine power system with dynamic equations [11] and model uncertainties given by

\[
\dot{\delta} = \omega - \omega_0, \\
\dot{\omega} = -\frac{D}{H} (\omega - \omega_0) + \frac{\omega_0}{H} (P_m - P_c), \\
P_m = \frac{P_{m0}}{T_H} + \frac{C}{T_H} \mu, \\
E_q = -\frac{X_{d} \Sigma}{T_{do} X_{d}^2} E_q' + \frac{X_q \Sigma - X_{q} X_{q}'}{T_{do} X_{q}^2} V_q \cos \delta + \frac{1}{T_{d0}} E_{fd}, \\
E_d = -\frac{X_{q} \Sigma}{T_{do} X_{q}^2} E_d' + \frac{X_q \Sigma - X_{q} X_{q}'}{T_{do} X_{q}^2} V_q \sin \delta + \frac{1}{T_{d0}} E_{fd}',
\]

(1)

where

\[
P_e \equiv \frac{E_d'}{X_{d} \Sigma} V_q \sin \delta - \frac{E_q'}{X_{q} \Sigma} V_q \cos \delta - \frac{X_{d} X_{d}'}{X_{q} X_{q}'} \Sigma \cos 2\delta, \\
\]

(2)

\(\delta, \omega, P_m, P_c, D, H, V_c, T_H, \Sigma,\) and \(C\) are the power angle, relative speed, mechanical input power, electromagnetic power, per-unit damping constant, inertia constant, infinite bus voltage, steam-valving control time constant, and power coefficient, respectively. \(\mu, E_{fd},\) and \(E_{fd}'\) are steam-valving controller, \(d\)-axis field voltage, and \(q\)-axis field voltage, respectively. \(T_{d0}^d\) and \(T_{q0}^q\) are the \(d\)-axis and \(q\)-axis transient short-circuit time constants, respectively. \(X_{d} \Sigma = X_{d} + X_T + X_L, X_{d} = X_{d} + X_T + X_L, X_{d}^2 = X_{d} X_{d}, X_{d} \Sigma = X_{q} X_{d} + X_T X_L, X_{d} X_{d} = X_{q} X_{d} + X_T X_L, X_{d} X_{d} = X_{q} X_{d} + X_T X_L, X_{d} X_{d} = X_{q} X_{d} + X_T X_L,\) where \(X_{d} X_{d} = X_{d} + X_T + X_L, X_{d} \Sigma = X_{d} \Sigma + X_T \Sigma + X_L \Sigma, \Sigma = X_{d} + X_T + X_L,\) and \(X_{d} \Sigma = X_{d} \Sigma + X_T \Sigma + X_L \Sigma, \Sigma = X_{d} + X_T + X_L,\) are the \(q\)-axis transient reactance, \(q\)-axis reactance, \(d\)-axis transient reactance, \(d\)-axis reactance, reactance of transmission line, and reactance of transformer, respectively. \(E_{d}^{'}\) and \(E_{d}^{'}\) are the \(q\)-axis internal transient voltage and \(q\)-axis internal transient voltage, respectively. Let \((\delta_0, \omega_0, P_{m0}, P_{c0}, E_{d0}', E_{d0})\) be an operation point, and define the state variable by \(x_1 = \delta - \delta_0, x_2 = \omega - \omega_0, x_3 = P_m - P_{m0}, x_4 = E_{q} - E_{q0}, x_5 = E_{d} - E_{d0},\) and \(x = [x_1 x_2 x_3 x_4 x_5]^T.\) We further consider that the model perturbations \(d_i, 1 \leq i \leq 5,\) may be applied in the power system (1) because the perturbation may come from the modeling errors, uncertainties, and disturbance in the control system. Then, (1) can be written as

\[
\dot{x}_1 = x_2 + d_1, \\
\dot{x}_2 = f_2(x_1, x_2) + z_2 x_3 - z_3 x_4 \sin (x_1 + \delta_0) - z_4 x_5 \sin (x_1 + \delta_0) + d_2, \\
\dot{x}_3 = f_3(x_3) + \left(\frac{C}{T_H} + \eta_1\right) \mu + d_3(x), \\
\dot{x}_4 = f_4(x_4, x_4) + \left(\frac{1}{T_{d0}} + \eta_2\right) E_{fd} + d_4(x), \\
\dot{x}_5 = f_5(x_1, x_5) + \left(\frac{1}{T_{d0}} + \eta_3\right) E_{fd} + d_5(x),
\]

(3a)

(3b)

(3c)

(3d)

(3e)

where \(P_{m0} = P_{c0}(0)\) [11],

\[
f_2(x_1, x_2) = -\frac{D}{H} x_2 + P_{m0} - P_{c}(x_1), \\
P_{c}(x_1) = \int (x_1 + \delta_0) + z_4 E_{d0} \cos (x_1 + \delta_0) + z_5 \sin 2(x_1 + \delta_0) \]

(4)

Remark 1. The assumptions of the mismatched perturbations \(d_i(x_i)\) and \(d_i(x_1, x_2, x_3, x_4, x_5)\) not in the range of any control effort \((\mu, E_{fd}, E_{fd}'),\) can be seen in some literatures [18, 19]. However, the stability analysis is not proposed in these works [18, 19]. \(d_i, 3 \leq i \leq 5,\) are the matched perturbations.

Assumption 2. The upper bounds of the following vanished perturbations [20] are assumed as

\[
|d_1(x_1)| \leq a_1 |x_1|, \\
|d_2(x_1, x_2, x_3, x_4, x_5)| \leq b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2| + a_3 |x_4| + a_4 |x_5|, \\
\]

(5)

where \(a_i, 1 \leq i \leq 4,\) are known positive constants. When \(x_4 = x_5 = 0,\) the upper bound of \(|d_2|\) can be computed as

\[
|d_2| \leq b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2|,
\]

(6)

where the system has been in the sliding mode. On the other hand, if the information of this upper bound is unknown,
the adaptive mechanism can be used to estimate these parameters.

3. Design of the Sliding Surface

For tackling the perturbation in (3a)–(3e), the sliding surface \( \sigma = [\sigma_1 \ \sigma_2 \ \sigma_3]^T \in \mathbb{R}^3 \) can be designed as

\[
\begin{align*}
\sigma_1 &= x_3 + \frac{1}{z_2} \left( (\epsilon_2 + \epsilon_1) \phi + x_1 - \epsilon_a x_2 + f_2 \right), \\
\sigma_2 &= x_4, \\
\sigma_3 &= x_5,
\end{align*}
\]

(7)

where \( \phi = x_4 - \alpha, \alpha = -\epsilon_1 x_1, \epsilon_0 = 3 \epsilon_1 + \alpha_1, \) and \( \epsilon_1 \) is a designed positive constant. \( \epsilon_2 = \alpha_3 + \alpha_4 \epsilon_1^2 + \beta_2 \epsilon_0^2/4 + \beta_3 \epsilon_1^2/4 \), where \( \alpha_0 \) is the known parameter defined in Appendix A.

**Theorem 3.** Consider the perturbed power system (3a)–(3e). If the sliding surface function is designed as (7), the trajectory of state \( x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T \) will reach zero asymptotically when the system is in the sliding mode.

**Proof.** Please see Appendix A. \( \square \)

4. Design of Controllers

According to (3a)–(3e), the robust controller can be designed as

\[
\begin{align*}
u &= \begin{bmatrix} \mu \\ E_{fd} \\ E_{fs} \end{bmatrix} = \begin{bmatrix} \mu_n + \mu_y \\ E_{fdn} + E_{fds} \\ E_{fsn} + E_{fsf} \end{bmatrix},
\end{align*}
\]

(8)

where

\[
\mu_n = -\frac{T_H \Sigma}{C},
\]

\[
\begin{align*}
&\frac{1}{z_2} \left( \epsilon_2 \phi + (\epsilon_2 + \epsilon_1) \\
&\times \left[ f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\
&\quad -z_4 x_5 \sin(x_1 + \delta_0) + \epsilon_a x_2 \right] + x_2 - \epsilon_a \\
&\times \left[ f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\
&\quad -z_4 x_5 \sin(x_1 + \delta_0) \right] + df_{2n} \right),
\end{align*}
\]

\[
\begin{align*}
E_{fdn} &= -T_{d0}^f f_4, \\
E_{fsn} &= -T_{d0}^f f_5,
\end{align*}
\]

\[
u_n = \begin{bmatrix} \mu_n \\ E_{fdn} \\ E_{fsn} \end{bmatrix},
\]

\[
u_s = \begin{bmatrix} \mu_s \\ E_{fds} \\ E_{fsf} \end{bmatrix} = \begin{bmatrix} \begin{cases} 0, & \text{if } ||\sigma|| = 0, \\ \frac{B^{-1} \sigma}{||\sigma||}, & \text{if } ||\sigma|| \neq 0, \end{cases} \end{bmatrix},
\]

where \( \epsilon_3 \) and \( \epsilon_4 \) are known positive constants. \( df_{2n}/dt \) and \( \dot{\epsilon}_3 \) can be divided into \( df_{2n}/dt = df_{2n} + df_{2d} \) and \( \dot{\epsilon}_3 = \dot{\epsilon}_{2n} + \dot{\epsilon}_{2d} \), respectively, where \( df_{2n} \) and \( \dot{\epsilon}_{2n} \) are the nominal parts of \( df_{2n}/dt \) and \( \dot{\epsilon}_3 \), respectively. \( df_{2d} \) and \( \dot{\epsilon}_{2d} \) are the parts of \( df_{2d}/dt \) and \( \dot{\epsilon}_3 \) which contain perturbations.

**Theorem 4.** Consider the system (3a)–(3e). If the controllers are designed as (8), the sliding variable \( \sigma \) will approach zero in a finite time.

**Proof.** Please see Appendix B. \( \square \)

5. Simulation

The system parameters are given in [11]. In the simulation, we assume that the desired reference signals are \( (\delta_0, \alpha_0, P_{m0}, E_{q0}, E_{d0}) = (0.5, 315^0, 0.6, 1.1, 1.1) \) and the initial conditions of the tracking errors are \( x = [15.3^0, 15, -0.42, -0.8, -0.8]^T \). We also assume that the disturbances \( d_1 = 0.2 x_1, d_2 = -0.1 x_1 + 0.2 x_2^2 + 0.15 x_2 - 0.3 x_3 + 0.45 x_4, d_3 = 0.4 - 0.8 x_1, d_4 = 0.5 + 0.3 x_1 - 0.2 x_3, x_4, d_5 = 0.15 - 0.2 \cos 3 x_4 \) are suddenly applied from 10 sec onwards. Figures 1, 2, 3, 4, and 5 show that the responses of \( \delta, \alpha, P_m, E_q, E_d \) of the proposed control scheme (SMC) can achieve the robust performance with a short transient time even if perturbations \( d_i, 1 \leq i \leq 5 \), exist, whereas the coordinated passivation...
control (CPC) may lose the asymptotical stability. Figure 6 demonstrates that the sliding variable $\sigma$ will approach zero in a finite time.

6. Conclusions

In this paper, a sliding mode controller has been successfully designed for the dual-excited and steam-valving control of the synchronous generators with perturbations. Even though the dynamics of the controlled systems are affected by the nonlinear perturbations, some constant gains designed in the sliding surface can effectively overcome these perturbations and achieve asymptotical stability. The proposed control scheme also demonstrates the robustness against the perturbations in the simulation.
Appendices

A. The Dynamic of the System in the Sliding Mode

According to (3a) and (7), one can obtain the dynamics of $x_1$ as

$$\dot{x}_1 = x_2 + d_1 = \alpha + d_1 + x_2 - \alpha = -e_n x_1 + d_1 + \phi. \quad (A.1)$$

Choose the first Lyapunov function candidate as $V_1 = (1/2)x_1^2$. Using Assumption 2, the time derivative of $V_1$ along the trajectory of (A.1) can be given by

$$\dot{V}_1 = x_1 \dot{x}_1 = -e_n x_1^2 + x_1d_1 + x_1\phi$$

$$\leq -3e_1 x_1^2 - a_1 x_1^2 + |x_1| |d_1| + x_1\phi$$

$$= -3e_1 x_1^2 + x_1\phi. \quad (A.2)$$

When the system is in the sliding mode, $\sigma = 0$, from (8), it can be seen that $x_4 = x_2 = 0$ and

$$x_3 = \frac{-1}{z_2 \left[(e_2 + e_1)\phi + x_1 - e_n x_2 + f_2\right]} \quad (A.3)$$

Using (3b) and (A.3), the closed-loop reduced dynamics of $x_2$ can be rewritten as

$$\dot{x}_1 = f_2 + z_1 x_3 - z_3 x_4 \sin(x_1 + \delta_0)$$

$$- z_3 x_3 \sin(x_1 + \delta_0) + d_2$$

$$= -(e_2 + e_1)\phi - x_1 + e_n x_2 + d_2 \quad (A.4)$$

Let $d_{2d} = d_2 - e_n d_1$. It can be seen that

$$|d_{2d}| \leq |d_2| + |e_n d_1| \leq b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2| + e_n a_1 |x_1|$$

$$\leq a_3 |x_1| + b_2 |x_1|^2 + a_2 |x_2| \quad (A.5)$$

where $a_3 \geq b_1 + e_n (a_1 + a_2)$ in accordance with Assumption 2, (6), and (7). To show that the state trajectory of the state variable $x$ will approach zero asymptotically, one can select the 2nd Lyapunov function candidate as $V_2 = V_1 + (1/2)\dot{x}_1^2$. From Assumption 2, (A.2), and (A.5), we can obtain the time derivative of $V_2$ along the trajectory of (A.4) as

$$\dot{V}_2 = \dot{V}_1 + \phi (\dot{x}_2 - \dot{\alpha})$$

$$\leq -3e_1 x_1^2 - e_1 \phi^2 - e_2 \phi^2 + \phi d_{2d}$$

$$\leq -e_1 (x_1^2 + \phi^2) - e_2 \phi^2 + \left(a_2 + \frac{a_2^2}{4e_1^2} + \frac{b_2^2 x_1^2}{4e_1^2}\right)\phi^2$$

$$= -e_1 (x_1^2 + \phi^2) \quad (A.6)$$

where

$$- e_1 x_1^2 + |x_1| (a_3 |x_1| + a_2 |x_2|)$$

$$= -e_1 \left(|x_1| - \frac{a_3}{2e_1} |\phi|\right)^2 + \left(a_2 + \frac{a_2^2}{4e_1^2}\right)\phi^2$$

$$\leq \left(a_2 + \frac{a_2^2}{4e_1^2}\right)\phi^2,$$

$$- e_1 x_1^2 + b_2 |x_1|^2$$

$$= -e_1 x_1^2 \left(1 - \frac{b_2 |x_1|^2}{2e_1} |\phi|\right) + \frac{b_2^2 x_1^2}{4e_1^2}\phi^2$$

$$\leq \frac{b_2^2 x_1^2}{4e_1^2}\phi^2. \quad (A.7)$$

Equation (A.6) implies that $\phi$ and $x_1$ will approach zero as $t \to \infty$. From (7), $x_2 = (\phi + \alpha)$ also reaches zero as $t \to \infty$ because $\alpha = (-e_n x_1)$ approaches zero as $t \to \infty$. Using (3a)–(3e), it is also noted that $f_2(0,0) \equiv -(D/H)0 + P_{0m} - P_0(0) = 0$ as $t \to \infty$. Similarly, $x_3 = (-1/z_2 \left[(e_2 + e_1)\phi + x_1 - e_n x_2 + f_2\right])$ also reaches zero as $t \to \infty$ because $f_2$ and $e_n x_2$ approach zero as $t \to \infty$ in accordance with (A.3). Since $x_4 = x_2 = 0$ and $x_1, x_2, x_3 \to 0$ as $t \to \infty$, the state trajectory of the state variable $x = [x_1^1 x_2^3 x_3 x_4^4 x_5]^T$ will approach zero asymptotically when the system is in the sliding mode.

B. The Proof of the Reaching Mode

From (7) and (8), one can obtain the time derivative of the sliding variable $\sigma$ as

$$\dot{\sigma} = \zeta + Bu, \quad (B.1)$$

where $\sigma \equiv [\dot{\sigma}_1 \dot{\sigma}_2 \dot{a}_3]^T$,

$$\dot{\sigma}_1 = x_3 + \frac{1}{z_2} \left[\dot{e}_2 \phi + (e_2 + e_1)\phi + \dot{x}_1 - e_n \dot{x}_2 + \dot{f}_2\right]$$

$$= f_3 + \frac{C}{T_{H\Sigma}} \left(\mu_n + \mu_s\right) + n_1 \mu + d_3$$

$$+ \frac{1}{z_2} \left[(e_{2n} + e_{2d})\phi + (e_2 + e_1)\phi x_1 + \dot{x}_1 - e_n \dot{x}_2 + \dot{f}_2\right] \times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) - z_3 x_3 \sin(x_1 + \delta_0) + a_2 (x_1 + d_1)] + x_2 + d_1 - e_n$$

$$\times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) - z_4 x_3 \sin(x_1 + \delta_0) + d_2 + df_{2n} + df_{2d}]$$

$$= \zeta + \frac{C}{T_{H\Sigma}} \mu,$$
\[ \dot{\sigma}_2 = \dot{x}_4 = f_4 + \frac{1}{d_0} \left( E_{f_d n} + E_{f_d s} + \eta_2 E_{f_d} + d_4 \right) = \xi_2 + \frac{1}{d_0} E_{f_d s}, \]
\[ \dot{\sigma}_3 = \dot{x}_5 = f_5 + \frac{1}{d_0} \left( E_{f_q n} + E_{f_q p} + \eta_3 E_{f_q} + d_5 \right) = \xi_3 + \frac{1}{d_0} E_{f_q s}, \]

and \( \xi \doteq [\xi_1 \xi_2 \xi_3]^T \).

\[ \begin{align*}
\xi_1 & \doteq \eta_1 \mu + d_3 + \frac{1}{\varepsilon_2} \left( \dot{e}_\Delta + (e_2 + e_1) (d_2 - e_d d_1) \right) + d_4 - e_a d_2 + df_\Delta, \\
\xi_2 & \doteq \eta_2 E_{f_d} + d_4, \\
\xi_3 & \doteq \eta_3 E_{f_q} + d_5.
\end{align*} \tag{B.2} \]

The lumped perturbations \( \xi \) can be assumed to satisfy the constraints
\[ \|\xi\| \leq \varepsilon_3 + e_\mu \|u\|. \tag{B.4} \]

See [12]. To prove that the sliding variable \( \sigma \) will approach zero in a finite time, we define a Lyapunov function candidate as \( V_\sigma = (1/2) \sigma^T \sigma \). By using (8) and (B.1), one can obtain the time derivative of \( V_\sigma \) as
\[ \dot{V}_\sigma = \sigma^T \dot{\sigma} = \sigma^T (\xi + B u_\xi) \leq \|\sigma\| (e_3 + e_\mu \|u\|) - (e_4 + e_\mu) \|\sigma\| \]
\[ = (-e_4 + e_\mu \|u\|) \|\sigma\| \]
\[ \leq \left[ -e_4 + e_\mu \left( \|u_\xi\| + \|B^{-1}\| (e_3 + e_\mu) \right) \right] \|\sigma\| \]
\[ = \left[ -e_4 (1 - e_\mu \|B^{-1}\|) + e_\mu \left( \|u_\xi\| + \|B^{-1}\| e_3 \right) \right] \|\sigma\| \]
\[ = -e_1 \|\sigma\| < 0. \tag{B.5} \]

The preceding equation indicates that the values of \( \sigma \) will approach zero in a finite time.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

**References**


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