Research Article

Sliding Mode Control for the Synchronous Generator

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Based on the Lyapunov stability theorem and sliding mode control technique, a design of the nonlinear controller is proposed for the dual-excited and steam-valving control of the synchronous generators with matched and mismatched perturbations in this paper. By using some constant gains designed in the sliding surface function, the perturbations in the power system can be suppressed, and the property of asymptotical stability of the rotor angle and the voltage can be achieved at the same time.

1. Introduction

To achieve a high degree of reliability in the power systems, many works [1–11] have studied the stability of generators. In general, there are two ways to stabilize the generators: the excited control [1–3] and the steam-valving control [4–7]. Using the excited control, Xie et al. [1] designed a linear matrix inequality (LMI) controller for a class of multimachine power systems with uncertain parameters to achieve the property of asymptotical stability. Galaz et al. [2] proposed a passivity-based controller and discussed the domain of attraction of the equilibria in power systems. Huang et al. [3] utilized a physical exact linearization method to design a controller for a dual-excited synchronous generators. For the steam-valving control, Zhang and Sun [4], Fu [5], Li et al. [6], and Li et al. [7] designed the adaptive backstepping controller for single machine infinite bus system in the presence of internal and external disturbances to achieve the property of asymptotical stability.

As for the systems with both steam turbine dynamics and the excited generator, Xi et al. [8] and Ma et al. [9] presented a novel nonlinear controller based on Hamiltonian energy theory steam for the turbine dynamics and single excited generator to achieve the property of asymptotical stability. The dual-excitation means the system has d -axis and q -axis field winding simultaneously. Each field voltage

can be adjusted separately and hence the control objectives can be achieved more flexibly. Based on the passive lemma, Wang and Lin [10] designed the bounded passivity controller for the synchronous generators to achieve the property of asymptotical stability. Using the coordinated passivation technique, Chen et al. [11] designed backstepping controller for steam-valving and dual-excited synchronous generators to achieve the property of asymptotical stability. However, the perturbations were not considered in the works [10, 11].

Sliding mode control (SMC) is well known to possess several advantages, for example, fast response, good transient performance, robustness of stability, and insensitivity to matched parameter variations and external disturbances [12, 13]. However, the property of asymptotical stability is in general hard to achieve by using the traditional SMC technique if the mismatched perturbations are presented in the systems [14]. A lot of researchers applied SMC techniques to solve the tracking problems with mismatched perturbations [15–17]. For example, Shieh and Shyu [15], Chen and Dunnigan [16], and Kwan [17] employed SMC techniques for an induction machine with an uncertain load torque; however, the mismatched perturbations considered in these works [15–17] belong to the unknown constants.

In this paper, we have proposed the nonlinear sliding mode controller for the dual-excited and steam-valving control of the synchronous generators with matched and

mismatched perturbations to achieve the property of asymptotical stability. Our proposed control scheme can be thought of as the extension work of [10, 11], where no perturbations are considered in the works [10, 11]. Furthermore, the mismatched perturbations considered in this paper can be time varying.

2. System Model

Consider a machine power system with dynamic equations [11] and model uncertainties given by

$$\dot{\delta} = \omega - \omega_0,$$

$$\dot{\omega} = -\frac{D}{H}(\omega - \omega_0) + \frac{\omega_0}{H}(P_m - P_e),$$

$$\dot{P}_m = \frac{P_{m0} - P_m}{T_{H\Sigma}} + \left(\frac{C}{T_{H\Sigma}} + \eta_1\right)\mu,$$

$$\dot{E}'_q = -\frac{X_{d\Sigma}}{T'_{d0}X'_{d\Sigma}}E'_q + \frac{X_{d\Sigma} - X'_{d\Sigma}}{T'_{d0}X'_{d\Sigma}}V_s \cos \delta + \left(\frac{1}{T'_{d0}} + \eta_2\right)E_{fd},$$

$$\dot{E}'_d = -\frac{X_{q\Sigma}}{T'_{q0}X'_{q\Sigma}}E'_d + \frac{X_{q\Sigma} - X'_{q\Sigma}}{T'_{q0}X'_{q\Sigma}}V_s \sin \delta + \left(\frac{1}{T'_{q0}} + \eta_3\right)E_{fq}, \quad (1)$$

where

$$P_e \triangleq \frac{E'_q}{X'_{d\Sigma}}V_s \sin \delta - \frac{E'_d}{X'_{q\Sigma}}V_s \cos \delta - \frac{X'_{d\Sigma} - X'_{q\Sigma}}{2X'_{q\Sigma}X'_{d\Sigma}}V_s^2 \cos 2\delta, \quad (2)$$

$\delta, \omega, P_m, P_e, D, H, V_s, T_{H\Sigma}$, and C are the power angle, relative speed, mechanical input power, electromagnetic power, per-unit damping constant, inertia constant, infinite bus voltage, steam-valving control time constant, and power coefficient, respectively. μ, E_{fd} , and E_{fq} are steam-valving controller, d -axis field voltage, and q -axis field voltage, respectively. T'_{d0} and T'_{q0} are the d -axis and q -axis transient short-circuit time constants, respectively. $X'_{d\Sigma} = X'_d + X_T + X_L$, $X_{d\Sigma} = X_d + X_T + X_L$, $X'_{q\Sigma} = X'_q + X_T + X_L$, $X_{q\Sigma} = X_q + X_T + X_L$, where $X'_q, X_q, X'_d, X_d, X_L$, and X_T are the q -axis transient reactance, q -axis reactance, d -axis transient reactance, d -axis reactance, reactance of transmission line, and reactance of transformer, respectively. E'_q and E'_d are the q -axis internal transient voltage and q -axis internal transient voltage, respectively. Let $(\delta_0, \omega_0, P_{m0}, E'_{q0}, E'_{d0})$ be an operation point, and define the state variable by $x_1 = \delta - \delta_0, x_2 = \omega - \omega_0, x_3 = P_m - P_{m0}, x_4 = E'_q - E'_{q0}, x_5 = E'_d - E'_{d0}$, and $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$. We further consider that the model perturbations $d_i, 1 \leq i \leq 5$, may be applied in the power system (1) because the perturbation may come from the

modeling errors, uncertainties, and disturbance in the control system. Then, (1) can be written as

$$\dot{x}_1 = x_2 + d_1, \quad (3a)$$

$$\begin{aligned} \dot{x}_2 = & f_2(x_1, x_2) + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ & - z_4 x_5 \sin(x_1 + \delta_0) + d_2, \end{aligned} \quad (3b)$$

$$\dot{x}_3 = f_3(x_3) + \left(\frac{C}{T_{H\Sigma}} + \eta_1\right)\mu + d_3(\mathbf{x}), \quad (3c)$$

$$\dot{x}_4 = f_4(x_1, x_4) + \left(\frac{1}{T'_{d0}} + \eta_2\right)E_{fd} + d_4(\mathbf{x}), \quad (3d)$$

$$\dot{x}_5 = f_5(x_1, x_5) + \left(\frac{1}{T'_{q0}} + \eta_3\right)E_{fq} + d_5(\mathbf{x}), \quad (3e)$$

where $P_{m0} = P'_e(0)$ [11],

$$f_2(x_1, x_2) \triangleq -\frac{D}{H}x_2 + P_{m0} - P'_e(x_1),$$

$$\begin{aligned} P'_e(x_1) = & z_3 E'_{q0} \sin(x_1 + \delta_0) + z_4 E'_{d0} \cos(x_1 + \delta_0) \\ & + z_5 \sin 2(x_1 + \delta_0), \end{aligned} \quad (4)$$

$$f_3(x_3) \triangleq \frac{1}{T_{H\Sigma}}x_3,$$

$$f_4(x_1, x_4) \triangleq -z_6(x_4 + E'_{q0}) + z_7 \cos(x_1 + \delta_0),$$

$$f_5(x_1, x_5) \triangleq -z_8(x_5 + E'_{d0}) - z_9 \sin(x_1 + \delta_0),$$

$$\begin{aligned} z_2 = & \omega_0/H, z_3 = \omega_0 V_s / H X'_{q\Sigma}, z_4 = \omega_0 V_s / H X'_{d\Sigma}, z_5 = \\ & ((X'_{d\Sigma} - X'_{q\Sigma})/2 H X'_{q\Sigma} X'_{d\Sigma}) \omega_0 V_s^2, z_6 = X_{d\Sigma} / T'_{d0} X'_{d\Sigma}, z_7 = \\ & ((X_{d\Sigma} - X'_{d\Sigma})/T'_{d0} X'_{d\Sigma}) V_s, z_8 = X_{q\Sigma} / T'_{q0} X'_{q\Sigma}, \text{ and } z_9 = \\ & ((X_{q\Sigma} - X'_{q\Sigma})/T'_{q0} X'_{q\Sigma}) V_s. \end{aligned}$$

Remark 1. The assumptions of the mismatched perturbations $d_1(x_1)$ and $d_2(x_1, x_2, x_4, x_5)$, not in the range of any control effort (μ, E_{fd}, E_{fq}) , can be seen in some literatures [18, 19]. However, the stability analysis is not proposed in these works [18, 19]. $d_i, 3 \leq i \leq 5$, are the matched perturbations.

Assumption 2. The upper bounds of the following vanished perturbations [20] are assumed as

$$|d_1(x_1)| \leq a_1 |x_1|,$$

$$\begin{aligned} |d_2(x_1, x_2, x_4, x_5)| \leq & b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2| \\ & + a_3 |x_4| + a_4 |x_5|, \end{aligned} \quad (5)$$

where $a_i, 1 \leq i \leq 4$, are known positive constants. When $x_4 = x_5 = 0$, the upper bound of $|d_2|$ can be computed as

$$|d_2| \leq b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2|, \quad (6)$$

where the system has been in the sliding mode. On the other hand, if the information of this upper bound is unknown,

the adaptive mechanism can be used to estimate these parameters.

3. Design of the Sliding Surface

For tackling the perturbation in (3a)–(3e), the sliding surface $\sigma \triangleq [\sigma_1 \ \sigma_2 \ \sigma_3]^T \in \mathbb{R}^3$ can be designed as

$$\begin{aligned}\sigma_1 &= x_3 + \frac{1}{z_2} [(\epsilon_2 + \epsilon_1)\phi + x_1 - \epsilon_a x_2 + f_2], \\ \sigma_2 &= x_4, \\ \sigma_3 &= x_5,\end{aligned}\quad (7)$$

where $\phi = x_2 - \alpha$, $\alpha = -\epsilon_a x_1$, $\epsilon_a = 3\epsilon_1 + a_1$, and ϵ_1 is a designed positive constant. $\epsilon_2 = a_2 + a_3^2/4\epsilon_1^2 + b_2^2 x_1^2/4\epsilon_1^2$, where a_6 is the known parameter defined in Appendix A.

Theorem 3. Consider the perturbed power system (3a)–(3e). If the sliding surface function is designed as (7), the trajectory of state $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ will reach zero asymptotically when the system is in the sliding mode.

Proof. Please see Appendix A. $\square$

4. Design of Controllers

According to (3a)–(3e), the robust controller can be designed as

$$\mathbf{u} = \begin{bmatrix} \mu \\ E_{fd} \\ E_{fq} \end{bmatrix} = \begin{bmatrix} \mu_n + \mu_s \\ E_{fdn} + E_{fds} \\ E_{fqn} + E_{fqs} \end{bmatrix}, \quad (8)$$

where

$$\begin{aligned}\mu_n &= -\frac{T_{H\Sigma}}{C} \\ &\times \left\{ f_3 + \frac{1}{z_2} \{ \dot{\epsilon}_{2n}\phi + (\epsilon_2 + \epsilon_1) \right. \\ &\quad \times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ &\quad \left. - z_4 x_5 \sin(x_1 + \delta_0) + \epsilon_a x_2] + x_2 - \epsilon_a \right. \\ &\quad \times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ &\quad \left. - z_4 x_5 \sin(x_1 + \delta_0)] \} + df_{2n} \right\}, \\ E_{fdn} &= -T'_{d0} f_4, \quad E_{fqn} = -T'_{q0} f_5, \\ \mathbf{u}_n &= [\mu_n \ E_{fdn} \ E_{fqn}]^T, \\ \mathbf{u}_s &= \begin{bmatrix} \mu_s \\ E_{fds} \\ E_{fqs} \end{bmatrix} = \begin{cases} -(\epsilon_3 + \epsilon_4) \frac{\mathbf{B}^{-1}\sigma}{\|\sigma\|}, & \text{if } \|\sigma\| \neq 0, \\ 0, & \text{if } \|\sigma\| = 0, \end{cases}\end{aligned}$$

FIGURE 1: Responses of the power angle δ .

$$\begin{aligned}\epsilon_4 &= \frac{\epsilon_u (\|\mathbf{u}_n\| + \|\mathbf{B}^{-1}\| \epsilon_3) + \epsilon_1}{1 - \epsilon_u \|\mathbf{B}^{-1}\|}, \\ \mathbf{B} &= \begin{bmatrix} \frac{C}{T_{H\Sigma}} & 0 & 0 \\ 0 & \frac{1}{T'_{d0}} & 0 \\ 0 & 0 & \frac{1}{T'_{q0}} \end{bmatrix},\end{aligned}\quad (9)$$

where ϵ_3 and $\epsilon_u < 1$ are known positive constants. df_2/dt and $\dot{\epsilon}_2$ can be divided into $df_2/dt \triangleq df_{2n} + df_{2d}$ and $\dot{\epsilon}_2 = \dot{\epsilon}_{2n} + \dot{\epsilon}_{2d}$, respectively, where df_{2n} and $\dot{\epsilon}_{2n}$ are the nominal parts of df_2/dt and $\dot{\epsilon}_2$, respectively. df_{2d} and $\dot{\epsilon}_{2d}$ are the parts of df_2/dt and $\dot{\epsilon}_2$ which contain perturbations.

Theorem 4. Consider the system (3a)–(3e). If the controllers are designed as (8), the sliding variable σ will approach zero in a finite time.

Proof. Please see Appendix B. $\square$

5. Simulation

The system parameters are given in [11]. In the simulation, we assume that the desired reference signals are $(\delta_0, \omega_0, P_{m0}, E'_{q0}, E'_{d0}) = (0.5, 315^\circ, 0.6, 1.1, 1.1)$ and the initial conditions of the tracking errors are $\mathbf{x} = [15.3^\circ, 15, -0.42, -0.8, -0.8]^T$. We also assume that the disturbances $d_1 = 0.2x_1$, $d_2 = -0.1x_1 + 0.2x_1^2 + 0.15x_2 - 0.3x_4 + 0.45x_5$, $d_3 = 0.4 - 0.8x_1x_4$, $d_4 = 0.5 + 0.3x_1 - 0.2x_1x_4$, and $d_5 = 0.15 - 0.2 \cos 3x_4$ are suddenly applied from 10 sec onwards. Figures 1, 2, 3, 4, and 5 show that the responses of δ , ω , P_m , E'_q , E'_d of the proposed control scheme (SMC) can achieve the robust performance with a short transient time even if perturbations d_i , $1 \leq i \leq 5$, exist, whereas the coordinated passivation

FIGURE 2: Responses of the relative speed ω . FIGURE 5: Responses of the d -axis internal transient voltage E'_d . FIGURE 3: Responses of the mechanical power P_m . FIGURE 6: The sliding variable σ . FIGURE 4: Responses of the q -axis internal transient voltage E'_q .

control (CPC) may lose the asymptotical stability. Figure 6 demonstrates that the sliding variable σ will approach zero in a finite time.

6. Conclusions

In this paper, a sliding mode controller has been successfully designed for the dual-excited and steam-valving control of the synchronous generators with perturbations. Even though the dynamics of the controlled systems are affected by the nonlinear perturbations, some constant gains designed in the sliding surface can effectively overcome these perturbations and achieve asymptotical stability. The proposed control scheme also demonstrates the robustness against the perturbations in the simulation.

Appendices

A. The Dynamic of the System in the Sliding Mode

According to (3a) and (7), one can obtain the dynamics of x_1 as

$$\dot{x}_1 = x_2 + d_1 = \alpha + d_1 + x_2 - \alpha = -\epsilon_a x_1 + d_1 + \phi. \quad (\text{A.1})$$

Choose the first Lyapunov function candidate as $V_1 = (1/2)x_1^2$. Using Assumption 2, the time derivative of V_1 along the trajectory of (A.1) can be given by

$$\begin{aligned} \dot{V}_1 &= x_1 \dot{x}_1 = -\epsilon_a x_1^2 + x_1 d_1 + x_1 \phi \\ &\leq -3\epsilon_1 x_1^2 - a_1 x_1^2 + |x_1| |d_1| + x_1 \phi \\ &= -3\epsilon_1 x_1^2 + x_1 \phi. \end{aligned} \quad (\text{A.2})$$

When the system is in the sliding mode, $\sigma = 0$, from (8), it can be seen that $x_4 = x_5 = 0$ and

$$x_3 \triangleq \frac{-1}{z_2 [(\epsilon_2 + \epsilon_1)\phi + x_1 - \epsilon_a x_2 + f_2]}. \quad (\text{A.3})$$

Using (3b) and (A.3), the closed-loop reduced dynamics of x_2 can be rewritten as

$$\begin{aligned} \dot{x}_2 &= f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ &\quad - z_4 x_5 \sin(x_1 + \delta_0) + d_2 \\ &\triangleq -(\epsilon_2 + \epsilon_1)\phi - x_1 + \epsilon_a x_2 + d_2. \end{aligned} \quad (\text{A.4})$$

Let $d_{2d} = d_2 - \epsilon_a d_1$. It can be seen that

$$\begin{aligned} |d_{2d}| &\leq |d_2| + |\epsilon_a d_1| \leq b_1 |x_1| + b_2 |x_1|^2 + a_2 |x_2| + \epsilon_a a_1 |x_1| \\ &\triangleq a_3 |x_1| + b_2 |x_1|^2 + a_2 |\phi|, \end{aligned} \quad (\text{A.5})$$

where $a_3 \triangleq b_1 + \epsilon_a(a_1 + a_2)$ in accordance with Assumption 2, (6), and (7). To show that the state trajectory of the state variable $\mathbf{x}$ will approach zero asymptotically, one can select the 2nd Lyapunov function candidate as $V_2 = V_1 + (1/2)\phi^2$. From Assumption 2, (A.2), and (A.5), we can obtain the time derivative of V_2 along the trajectory of (A.4) as

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \phi(\dot{x}_2 - \dot{\alpha}) \\ &\leq -3\epsilon_1 x_1^2 - \epsilon_1 \phi^2 - \epsilon_2 \phi^2 + \phi d_{2d} \\ &\leq -\epsilon_1 (x_1^2 + \phi^2) - \epsilon_2 \phi^2 + \left(a_2 + \frac{a_3^2}{4\epsilon_1^2} + \frac{b_2^2 x_1^2}{4\epsilon_1^2} \right) \phi^2 \\ &= -\epsilon_1 (x_1^2 + \phi^2), \end{aligned} \quad (\text{A.6})$$

where

$$\begin{aligned} &-\epsilon_1 x_1^2 + |\phi| (a_3 |x_1| + a_2 |\phi|) \\ &= -\epsilon_1 \left(|x_1| - \frac{a_3}{2\epsilon_1} |\phi| \right)^2 + \left(a_2 + \frac{a_3^2}{4\epsilon_1^2} \right) \phi^2 \\ &\leq \left(a_2 + \frac{a_3^2}{4\epsilon_1^2} \right) \phi^2, \\ &-\epsilon_1 x_1^2 + b_2 |\phi| |x_1|^2 \\ &= -\epsilon_1 x_1^2 \left(1 - \frac{b_2 |x_1|^2}{2\epsilon_1} |\phi| \right)^2 + \frac{b_2^2 x_1^2}{4\epsilon_1^2} \phi^2 \\ &\leq \frac{b_2^2 x_1^2}{4\epsilon_1^2} \phi^2. \end{aligned} \quad (\text{A.7})$$

Equation (A.6) implies that ϕ and x_1 will approach zero as $t \rightarrow \infty$. From (7), $x_2 = (\phi + \alpha)$ also reaches zero as $t \rightarrow \infty$ because $\alpha = (-\epsilon_a x_1)$ approaches zero as $t \rightarrow \infty$. Using (3a)–(3e), it is also noted that $f_2(0, 0) \triangleq -(D/H) \cdot 0 + P_{m0} - P'_e(0) = 0$ as $t \rightarrow \infty$. Similarly, $x_3 = (-1/z_2 [(\epsilon_2 + \epsilon_1)\phi + x_1 - \epsilon_a x_2 + f_2])$ also reaches zero as $t \rightarrow \infty$ because f_2 and $\epsilon_a x_2$ approach zero as $t \rightarrow \infty$ in accordance with (A.3). Since $x_4 = x_5 = 0$ and $x_1, x_2, x_3 \rightarrow 0$ as $t \rightarrow \infty$, the state trajectory of the state variable $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5]^T$ will approach zero asymptotically when the system is in the sliding mode.

B. The Proof of the Reaching Mode

From (7) and (8), one can obtain the time derivative of the sliding variable σ as

$$\dot{\sigma} = \zeta + \mathbf{B}u_s, \quad (\text{B.1})$$

where $\dot{\sigma} \triangleq [\dot{\sigma}_1 \ \dot{\sigma}_2 \ \dot{\sigma}_3]^T$,

$$\begin{aligned} \dot{\sigma}_1 &= \dot{x}_3 + \frac{1}{z_2} [\dot{\epsilon}_2 \phi + (\epsilon_2 + \epsilon_1) \dot{\phi} + \dot{x}_1 - \epsilon_a \dot{x}_2 + \dot{f}_2] \\ &= f_3 + \frac{C}{T_{H\Sigma}} (\mu_n + \mu_s) + \eta_1 \mu + d_3 \\ &\quad + \frac{1}{z_2} \{ (\dot{\epsilon}_{2n} + \dot{\epsilon}_{2d}) \phi + (\epsilon_2 + \epsilon_1) \\ &\quad \times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ &\quad - z_4 x_5 \sin(x_1 + \delta_0) + d_2 - \epsilon_a (x_2 + d_1)] \\ &\quad + x_2 + d_1 - \epsilon_a \\ &\quad \times [f_2 + z_2 x_3 - z_3 x_4 \sin(x_1 + \delta_0) \\ &\quad - z_4 x_5 \sin(x_1 + \delta_0) + d_2] + df_{2n} + df_{2d} \} \\ &= \zeta_1 + \frac{C}{T_{H\Sigma}} \mu_s, \end{aligned}$$

$$\begin{aligned}
\dot{\sigma}_2 = \dot{x}_4 &= f_4 + \frac{1}{T'_{d0}} (E_{fdn} + E_{fds}) + \eta_2 E_{fd} + d_4 \\
&= \zeta_2 + \frac{1}{T'_{d0}} E_{fds}, \\
\dot{\sigma}_3 = \dot{x}_5 &= f_5 + \frac{1}{T'_{q0}} (E_{fqn} + E_{fqs}) + \eta_3 E_{fq} + d_5 \\
&= \zeta_3 + \frac{1}{T'_{q0}} E_{fqs}
\end{aligned} \tag{B.2}$$

$$\text{and } \zeta \triangleq [\zeta_1 \quad \zeta_2 \quad \zeta_3]^T,$$

$$\begin{aligned}
\zeta_1 &\triangleq \eta_1 \mu + d_3 + \frac{1}{z_2} [\dot{e}_{2d} \phi + (\epsilon_2 + \epsilon_1) (d_2 - \epsilon_a d_1) \\
&\quad + d_1 - \epsilon_a d_2 + d f_{2d}], \\
\zeta_2 &\triangleq \eta_2 E_{fd} + d_4, \\
\zeta_3 &\triangleq \eta_3 E_{fq} + d_5.
\end{aligned} \tag{B.3}$$

The lumped perturbations ζ can be assumed to satisfy the constraints

$$\|\zeta\| \leq \epsilon_3 + \epsilon_u \|\mathbf{u}\|. \tag{B.4}$$

See [12].

To prove that the sliding variable σ will approach zero in a finite time, we define a Lyapunov function candidate as $V_\sigma = (1/2) \sigma^T \sigma$. By using (8) and (B.1), one can obtain the time derivative of V_σ as

$$\begin{aligned}
\dot{V}_\sigma &= \sigma^T \dot{\sigma} = \sigma^T (\zeta + \mathbf{B} \mathbf{u}_s) \leq \|\sigma\| (\epsilon_3 + \epsilon_u \|\mathbf{u}\|) - (\epsilon_4 + \epsilon_3) \|\sigma\| \\
&= (-\epsilon_4 + \epsilon_u \|\mathbf{u}\|) \|\sigma\| \\
&\leq \{-\epsilon_4 + \epsilon_u [\|\mathbf{u}_n\| + \|\mathbf{B}^{-1}\| (\epsilon_3 + \epsilon_4)]\} \|\sigma\| \\
&= [-\epsilon_4 (1 - \epsilon_u \|\mathbf{B}^{-1}\|) + \epsilon_u (\|\mathbf{u}_n\| + \|\mathbf{B}^{-1}\| \epsilon_3)] \|\sigma\| \\
&= -\epsilon_1 \|\sigma\| < 0.
\end{aligned} \tag{B.5}$$

The preceding equation indicates that the values of σ will approach zero in a finite time.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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