Research Article

The Effects of Piezoelectricity on the Interaction of Waves in Fluid-Loaded Poroelastic Half-Space

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The effects of piezoelectricity on the interaction of waves at fluid-poroelastic interface are studied. The constitutive equations and governing equations are formulated and their solution is obtained. The boundary conditions are described at fluid-solid interface. The effects of various parameters on the angle of refraction, amplitude ratios, displacements, electric potentials, and vertical component of slowness are studied numerically for a particular model. The results obtained are in agreement with the general laws of physics.

1. Introduction

It is well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. Piezoelectric materials are acting as very important functional components in sonar projectors, fluid monitors, pulse generators, and surface acoustic wave devices. The field of piezoelectric materials has advanced rapidly due to an increasing awareness about capabilities of such materials, the development of new materials and transducer designs, and increasingly stringent design and control specifications in aerospace, aeronautics, industrial, automotive, biomedical, and nanosystems. Engineering applications in surface acoustic waves (SAW) devices, materials characterizations, and smart structures require the analysis of elastic wave interaction with piezoelectric materials. The problem of reflection-transmission in piezoelectric materials has been attracting unceasing attention in view of its theoretical interest and from application points of view also. The characteristics of the reflected and refracted waves at such boundaries give information about the resolution characteristics of acoustic transducers [1]. The reflection and refraction of plane waves in piezoelectric anisotropic materials have been mentioned in the texts of Auld [2], Dieulesaint and Royer [3], and Parton and Kudryavtsev [4].

Noorbehesht and Wade [5] obtained the analytical expressions for reflection and transmission coefficients of the waves at a boundary between piezoelectric materials and water and studied the effects of angle of incidence and material properties on these coefficients. Auld [6] studied the wave propagation in piezoelectric materials. The reflection of a transverse wave from the surface of a piezoelectric crystal of class 6 was studied by Alshits et al. [7]. Nayfeh and Chien [8] made an analytical study on ultrasonic wave interaction with fluid-loaded anisotropic piezoelectric substrates. The analytical expressions for the reflection and transmission coefficients were derived and propagation characteristics of the leaky wave and free wave are also identified. Every and Neiman [9] analyzed the reflection of electroacoustic waves at the boundary of a piezoelectric half-space. Zinchuk and Podlipenets [10, 11] obtained dispersion equations for acoustoelectric Rayleigh wave in a periodic layer piezoelectric half-space in a study for the 6 mm crystal class. Zaitsev and Kuznetsova [12] performed a detailed analysis of energy characteristics of bulk, surface, and plane acoustic waves in piezoelectric materials and structures. Le Clezio and Shuvalov [13] dealt theoretically and experimentally with the transmission of acoustic waves through a piezoelectric plate. Burkov et al. [14, 15] derived the basic equations describing the wave at the interface between acentric crystals

and studied the effects of a uniform external electric field on the reflection and transmission of waves. The role of electromagnetic effects in the reflection and transmission of acoustic waves in piezoelectric materials was discussed by Darinskii et al. [16]. Abd-Alla and Alsheikh [17] studied the reflection-transmission of quasi-longitudinal waves under initial stresses at an interface of two anisotropic piezoelectric media with different properties.

Porous piezoelectric materials are widely used for applications such as low frequency hydrophones, miniature, accelerometers, vibratory sensors, and contact microphones. Different theoretical models [18–21] were developed to study the effects of porosity and connectivity on the elastic, dielectric, and piezoelectric coefficients of piezoelectric composites. Different experimental studies [22–25] were made on manufacturing of porous microstructures and the properties of such porous piezoelectric materials. Vashishth and Gupta [26, 27] derived the constitutive equations for porous piezoelectric materials and studied the wave propagation in these materials. The general theorems of elasticity were generalized for the linear theory of porous piezoelectric materials by Vashishth and Gupta [28]. Recently, Vashishth and Gupta [29] studied the effects of piezoelectricity on the poroelastic plate sandwiched between fluid half-space and porous piezoelectric half-space. The effects of angle of incidence and frequency on the amplitude ratios are studied therein.

Survey of the literature related to porous piezoelectric materials reveals that a lot of experimental work has been done and is being done. Very few authors have established theoretical models for porous piezoelectric materials. A lot of work has also been done on reflection and transmission of waves in piezoelectric materials but theoretical work in porous piezoelectric materials is comparatively less reported and thus finds scopes for further investigations. In the present paper, the effects of piezoelectricity on the interaction of waves in fluid-loaded poroelastic half-space are studied. In the present model the porous piezoelectric plate is sandwiched between two fluid half-spaces. The formal expressions for the mechanical displacements, electric potentials, stresses, and electric displacements are obtained for the porous piezoelectric half-space. The effects of various parameters on the amplitude ratios, displacements, electric potentials, angle of refraction, and vertical component of slowness are also studied numerically for a particular model.

2. Geometry of the Problem

Let us consider a poro-piezoelectric half-space loaded with fluid half-space. Let a plane wave from the fluid half-space strike the interface, which results in one reflected wave and five transmitted waves in poro-piezoelectric half-space.

The constitutive equations for porous piezoelectric materials, having 6 mm symmetry, are [26]

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \\ \sigma^* \\ D_1 \\ D_2 \\ D_3 \\ D_1^* \\ D_2^* \\ D_3^* \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(c_{11} - c_{12})}{2} \\ m_{11} & m_{11} & m_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \zeta_{15} & 0 \\ 0 & 0 & 0 & \zeta_{15} & 0 & 0 \\ \zeta_{31} & \zeta_{31} & \zeta_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m_{11} & 0 & 0 & -e_{31} & 0 & 0 & -\zeta_{31} \\ m_{11} & 0 & 0 & -e_{31} & 0 & 0 & -\zeta_{31} \\ m_{33} & 0 & 0 & -e_{33} & 0 & 0 & -\zeta_{33} \\ 0 & 0 & -e_{15} & 0 & 0 & -\zeta_{15} & 0 \\ 0 & -e_{15} & 0 & 0 & -\zeta_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R & 0 & 0 & -\tilde{\zeta}_3 & 0 & 0 & -e_3^* \\ 0 & \xi_{11} & 0 & 0 & A_{11} & 0 & 0 \\ 0 & 0 & \xi_{11} & 0 & 0 & A_{11} & 0 \\ \tilde{\zeta}_3 & 0 & 0 & \xi_{33} & 0 & 0 & A_{33} \\ 0 & A_{11} & 0 & 0 & \xi_{11}^* & 0 & 0 \\ 0 & 0 & A_{11} & 0 & 0 & \xi_{11}^* & 0 \\ e_3^* & 0 & 0 & A_{33} & 0 & 0 & \xi_{33}^* \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{23} \\ 2\epsilon_{13} \\ 2\epsilon_{12} \\ \epsilon^* \\ E_1 \\ E_2 \\ E_3 \\ E_1^* \\ E_2^* \\ E_3^* \end{bmatrix}. \quad (1)$$

Here, c_{ij} are the elastic stiffness tensor components of porous bulk material. R is elastic constant corresponding to fluid phase. $e_{31}, e_{33}, e_{15}, \zeta_{31}, \zeta_{33}, \zeta_{15}, e_3^*, \tilde{\zeta}_3$ are piezoelectric coefficients. $\xi_{11}, \xi_{33}, A_{11}, A_{33}, \xi_{11}^*, \xi_{33}^*$ are the dielectric coefficients. m_{11}, m_{33} are elastic coupling coefficients. $\sigma_{ij}, \epsilon_{ij}, D_i$, and E_i are components of the stress tensor, strain tensor, electric displacement vector, and electric field vector for the solid phase of the porous aggregate, respectively. $\sigma^*, \epsilon^*, D_i^*$, and E_i^* are the stress, strain, electric displacement components, and electric field components corresponding to the fluid phase of the aggregate.

The equations of motion are

$$\begin{aligned} \sigma_{ij,j} &= \rho_{11}\ddot{u}_j + \rho_{12}\ddot{U}_j^*, \\ \sigma_{,i}^* &= \rho_{12}\ddot{u}_j + \rho_{22}\ddot{U}_j^*, \\ D_{i,i} &= 0, \\ D_{i,i}^* &= 0, \\ (i, j &= 1, 2, 3). \end{aligned} \quad (2)$$

Here, ρ_{11} , ρ_{12} , and ρ_{22} are the dynamical coefficients. u_i and U_i^* are the mechanical displacement components corresponding to solid and fluid phase.

Further,

$$\begin{aligned}\varepsilon_{ij} &= \frac{1}{2} (u_{i,j} + u_{j,i}), & \varepsilon^* &= U_{k,k}^*, \\ E_i &= -\psi_{,i}, & E_i^* &= -\psi_{,i}^*,\end{aligned}\quad (3)$$

where ψ and ψ^* are electric potential functions for solid and fluid phase of the porous aggregate.

Let us assume that

$$\begin{aligned}u_j &= A_j \exp(i\omega(p_x x_1 + p_z x_3 - t)), \\ U_j^* &= H_j \exp(i\omega(p_x x_1 + p_z x_3 - t)), \\ \psi &= \Theta \exp(i\omega(p_x x_1 + p_z x_3 - t)), \\ \psi^* &= \Omega \exp(i\omega(p_x x_1 + p_z x_3 - t)),\end{aligned}\quad (4)$$

where p_x and p_z are the horizontal and vertical components of the slowness vector. ω is the circular frequency of waves. A_j , H_j , Θ , and Ω are the associated amplitudes.

Equations (1)–(4) imply

$$\mathbf{WZ} = \mathbf{0}, \quad \mathbf{Z} = (A_1, A_3, H_1, H_3, \Theta, \Omega)^T. \quad (5)$$

The system (5) has a nontrivial solution if

$$\det(\mathbf{W}) = 0. \quad (6)$$

This leads to

$$c_1 p'^5 + c_2 p'^4 + c_3 p'^3 + c_4 p'^2 + c_5 p' + c_6 = 0, \quad (7)$$

where $p' = p_z^2$ and the coefficients c_i ($i = 1, 2, \dots, 6$) are given in the Appendix.

The five values of p' correspond to the slowness of five coupled waves termed as quasi P_1 , quasi S_1 , and quasi P_2 and another two waves due to piezoelectric interaction, propagating in a medium. The waves termed as quasi P_1 , quasi S_1 , and quasi P_2 will be denoted by qP_1 , qS_1 , qP_2 , respectively. The mechanical displacements and electric potentials in the poro-piezoelectric half-space can be written as

$$\begin{aligned}(u_1, u_3, U_1^*, U_3^*, \psi, \psi^*) \\ = \sum_{i=1}^5 (1, r_{1i}, r_{2i}, r_{3i}, r_{4i}, r_{5i}) Z_{1i} e^{i\omega(p_x x_1 + p_z x_3 - t)}.\end{aligned}\quad (8)$$

Using (1) and (8), we obtain

$$\begin{aligned}(\sigma_{31}, \sigma_{33}, \sigma^*, D_3, D_3^*) \\ = \sum_i i\omega (g_{1i}, g_{2i}, g_{3i}, g_{4i}, g_{5i}) Z_{1i} e^{i\omega(p_x x_1 + p_z x_3 - t)},\end{aligned}\quad (9)$$

where

$$r_{1i} = \frac{Z_{2i}}{Z_{1i}}, \quad r_{2i} = \frac{Z_{3i}}{Z_{1i}}, \quad r_{3i} = \frac{Z_{4i}}{Z_{1i}},$$

$$r_{4i} = \frac{Z_{5i}}{Z_{1i}}, \quad r_{5i} = \frac{Z_{6i}}{Z_{1i}},$$

$$g_{1i} = c_{55} p'_i + (c_{55} r_{1i} + e_{15} r_{4i} + \zeta_{15} r_{5i}) p_x,$$

$$g_{2i} = (c_{33} r_{1i} + m_{33} r_{3i} + e_{33} r_{4i} + \zeta_{33} r_{5i}) p'_i + (c_{31} + m_{33} r_{2i}) p_x,$$

$$g_{3i} = (m_{33} r_{1i} + R r_{3i} + \tilde{\zeta}_3 r_{4i} + e_3^* r_{5i}) p'_i + (m_{11} + R r_{2i}) p_x,$$

$$g_{4i} = (e_{33} r_{1i} + \tilde{\zeta}_3 r_{3i} - \xi_{33} r_{4i} - a_{33} r_{5i}) p'_i + (e_{31} + \tilde{\zeta}_3 r_{2i}) p_x,$$

$$g_{5i} = (\zeta_{33} r_{1i} + e_3^* r_{3i} - a_{33} r_{4i} - \xi_{33}^* r_{5i}) p'_i + (\zeta_{31} + e_3^* r_{2i}) p_x. \quad (10)$$

$(Z_{1i}, Z_{2i}, Z_{3i}, Z_{4i}, Z_{5i}, Z_{6i})$ is the eigenvector of the system (5) corresponding to the i th value of p'_i .

The displacements (u_1^f, u_3^f) and normal stress (σ_{33}^f) in fluid half-space are

$$\begin{aligned}(u_1^f, u_3^f, \sigma_{33}^f) \\ = \sum_{p=1,2} \left(1, W_p^f, \frac{i\omega \rho_f^u}{p_x} \right) U_p^f e^{i\omega(p_x x_1 + (-1)^{p+1} q_f x_3 - t)},\end{aligned}\quad (11)$$

where $W_1^f = p_x c$ and $W_2^f = -p_x c$. ρ_f^u is the upper fluid density. q_f is the vertical slowness of wave propagating in the fluid half-space.

The boundary conditions on the interface $x_3 = 0$ are

$$\begin{aligned}D_3 = D_3^* = \sigma_{13} = 0, \quad \sigma_{33} = (1 - f) \sigma_{33}^f, \\ \sigma^* = f \sigma_{33}^f, \quad (1 - f) \dot{u}_3 + f \dot{U}_3^* = \dot{u}_3^f.\end{aligned}\quad (12)$$

Making use of (8)–(12), a nonhomogeneous system of order 6 is obtained. Solution of this system gives the reflected and transmitted amplitude ratios.

3. Numerical Observations

In this section, the vertical component of slowness, reflected and transmitted amplitude ratios, mechanical displacements, and electric potentials are calculated numerically for a particular model and their variation with different parameters is observed. Following Kar-Gupta and Venkatesh [30], the elastic, piezoelectric, and dielectric constants for PZT are given in Table 1.

Figure 1 shows the variation of angle of refraction with the angle of incidence. The refraction angles corresponding to qP_1 , qS_1 , and qP_2 modes are denoted by γ_1 , γ_2 , and γ_3 , respectively. The critical angles corresponding to qP_1 , qS_1 , and qP_2 modes are $\theta_1 = 13^\circ$, $\theta_2 = 31^\circ$, and $\theta_3 = 41^\circ$. Thus qP_1 wave becomes evanescent mode after critical angle $\theta_1 = 13^\circ$ and qS_1 wave does not propagate into the medium after $\theta_2 = 31^\circ$. qP_2 wave also becomes evanescent mode after

FIGURE 1: Variation of angle of refraction (γ_i) corresponding to propagating modes with the angle of incidence (θ); (a) qP_1 wave, (b) qS_1 wave, and (c) qP_2 wave.

TABLE 1: Elastic constants, piezoelectric constants, dielectric constants, and other parameters for PZT crystal.

Elastic constants (GPa)	Piezoelectric constants (C/m ²)	Dielectric constants (nC/Vm)	Other parameters
$c_{11} = 148.0, c_{12} = 76.2,$ $c_{13} = 74.2, c_{33} = 131.0,$ $c_{44} = 25.3, m_{11} = 8.8,$ $m_{33} = 5.2, R = 20.$	$e_{15} = 9.3, e_{31} = -2.324,$ $e_{33} = 10.99, \zeta_{15} = 2.72,$ $\zeta_{31} = -0.48, \zeta_{33} = 5.32,$ $e_3^* = -3.6, \zeta_3 = -7.5.$	$\xi_{11} = 3.984, \xi_{33} = 2.081,$ $\xi_{11}^* = 11.8, \xi_{33}^* = 13.9,$ $A_{11} = 12.8, A_{33} = 15.1.$	$f = 0.2, c_f$ (m/s) = 1500, ρ (kg/m ³) = 7700, ρ_{11} (kg/m ³) = 5082, ρ_{12} (kg/m ³) = -1155, ρ_{22} (kg/m ³) = 4928.

critical angle $\theta_3 = 41^\circ$. All the incident energy gets reflected back after angle of incidence $\theta_3 = 41^\circ$.

Figure 2 shows the variation of absolute value of reflected and transmitted amplitude ratios with the horizontal component of slowness. A_R corresponds to reflected amplitude ratio and A_i ($i = 1, 2, \dots, 5$) denote the amplitude ratios corresponding to transmitted modes. Major portion of incident energy gets reflected back which signifies the fact

that the transmitting medium is much denser. The dips in the reflected amplitude ratio correspond to the excitation of pseudosurface mode. Thus we can find the values of incidence angles at which surface modes will be generated. This type of information is of practical importance for design of piezoelectric transducers. The variation of vertical component of slowness with the horizontal component of slowness is shown in Figure 3. The vertical component of

FIGURE 2: Variation of absolute value of amplitude ratios with the horizontal component of slowness.

FIGURE 3: Variation of vertical component of slowness with horizontal component of slowness; (a) qP_1 wave, (b) qS_1 wave, and (c) qP_2 wave.

the slowness associated with the qP_1 wave is minimum while that associated with the qP_2 wave is maximum, which verifies the fact that the velocity of qP_1 wave is maximum while that of the qP_2 wave is minimum. The value of the vertical component of the slowness helps in deciding the nomenclature of the waves propagating in the medium. It is also clear from this figure that the first of all, qP_1 , wave becomes nonpropagating.

Figure 4 displays the variation of real part of mechanical displacements and electric potentials with x_3 . It is

observed that mechanical displacements and electric potentials decrease with x_3 and become zero for large value of x_3 . Thus the amplitudes of the waves decrease as they travel far away from the interface and die out after a very long distance.

4. Conclusion

In the present paper the wave interaction of piezoelectric waves at fluid-porous piezoelectric solid half-space is studied. The analytical expression of the Christoffel equation is

FIGURE 4: Variation of real part of mechanical displacements and electric potentials with x_3 .

derived. The vertical slowness of the five coupled waves propagating in a medium is obtained from the solution of Christoffel equation. The analytical solution of the mechanical displacements, electric displacements, electric potentials, and stress in the porous piezoelectric half-space and fluid half-space is formulated. The behavior of angle of refraction, amplitude ratios, displacements, potentials, and vertical component of slowness relative to various parameters is studied numerically for particular model. It is found that major portion of incident energy gets reflected back in case of studied model which signifies the fact that the transmitting medium is much denser. The displacements and potentials decrease with the increase in x_3 and become zero for large value of x_3 .

Appendix

Consider

$$\begin{aligned}
 c_1 &= \beta_1 z_1 - \beta_5 z_{10} + \beta_9 z_{17} - \beta_{13} z_{26}, \\
 c_2 &= \beta_2 z_1 + \beta_1 z_2 - \beta_6 z_{10} - \beta_5 z_{11} + \beta_{10} z_{17} \\
 &\quad + \beta_9 z_{18} - \beta_{14} z_{26} - \beta_{13} z_{27}, \\
 c_3 &= \beta_3 z_1 + \beta_2 z_2 + \beta_1 z_3 - \beta_7 z_{10} - \beta_6 z_{11} + \beta_{11} z_{17} \\
 &\quad + \beta_{10} z_{18} + \beta_9 z_{19} - \beta_{15} z_{26} - \beta_{14} z_{27} - \beta_{13} z_{28}, \\
 c_4 &= \beta_4 z_1 + \beta_3 z_2 + \beta_2 z_3 - \beta_8 z_{10} - \beta_7 z_{11} + \beta_{12} z_{17} \\
 &\quad + \beta_{11} z_{18} + \beta_{10} z_{19} - \beta_{16} z_{26} - \beta_{15} z_{27} - \beta_{14} z_{28},
 \end{aligned}$$

$$\begin{aligned}
 c_5 &= \beta_4 z_2 + \beta_3 z_3 + \beta_{12} z_{18} + \beta_{11} z_{19} - \beta_8 z_{11} \\
 &\quad - \beta_{16} z_{27} - \beta_{15} z_{28},
 \end{aligned}$$

$$c_6 = \beta_4 z_3 + \beta_{12} z_{19} - \beta_{16} z_{28},$$

$$\beta_1 = q_1 z_{12} - q_4 z_{14} + q_7 z_{15};$$

$$\beta_2 = q_2 z_{12} + q_1 z_{13} - q_5 z_{14} + q_8 z_{15} + q_7 z_{16};$$

$$\beta_3 = q_3 z_{12} + q_2 z_{13} - q_6 z_{14} + q_9 z_{15} + q_8 z_{16};$$

$$\beta_4 = q_3 z_{13} + q_9 z_{16};$$

$$\beta_5 = q_1 z_4 - q_4 z_6 + q_7 z_8;$$

$$\beta_6 = q_2 z_4 + q_1 z_5 - q_5 z_6 - q_4 z_7 + q_8 z_8 + q_7 z_9;$$

$$\beta_7 = q_3 z_4 + q_2 z_5 - q_6 z_6 - q_5 z_7 + q_9 z_8 + q_8 z_9;$$

$$\beta_8 = q_3 z_5 - q_6 z_7 + q_9 z_9;$$

$$\beta_9 = r_1 z_4 - r_4 z_6 + r_7 z_8;$$

$$\beta_{10} = r_2 z_4 + r_1 z_5 - r_5 z_6 - r_4 z_7 + r_8 z_8 + r_7 z_9;$$

$$\beta_{11} = r_3 z_4 + r_2 z_5 - r_6 z_6 - r_5 z_7 + r_9 z_8 + r_8 z_9;$$

$$\beta_{12} = r_3 z_5 - r_6 z_7 + r_9 z_9;$$

$$\beta_{13} = s_1 z_4 - s_4 z_6 + s_7 z_8;$$

$$\beta_{14} = s_2 z_4 + s_1 z_5 - s_5 z_6 - s_4 z_7 + s_8 z_8 + s_7 z_9;$$

$$\beta_{15} = s_3 z_4 + s_2 z_5 - s_6 z_6 - s_5 z_7 + s_9 z_8 + s_8 z_9;$$

$$\beta_{16} = s_3 z_5 - s_6 z_7 + s_9 z_9;$$

$$\begin{aligned}
q_1 &= z_{22}z_{33} - z_{24}z_{31}; \\
q_2 &= z_{22}z_{34} + z_{23}z_{33} - z_{25}z_{31} - z_{24}z_{32}; \\
q_3 &= z_{23}z_{34} - z_{25}z_{32}; \\
q_4 &= z_{20}z_{33} - z_{24}z_{29}; \\
q_5 &= z_{20}z_{34} + z_{21}z_{33} - z_{24}z_{30} - z_{25}z_{29}; \\
q_6 &= z_{21}z_{34} - z_{25}z_{30}; \quad q_7 = z_{20}z_{31} - z_{22}z_{29}; \\
q_8 &= z_{20}z_{32} + z_{21}z_{31} - z_{23}z_{29} - z_{22}z_{30}; \\
q_9 &= z_{21}z_{32} - z_{23}z_{30}; \\
r_1 &= z_{14}z_{33} - z_{15}z_{31}; \\
r_2 &= z_{14}z_{34} - z_{16}z_{31} - z_{15}z_{32}; \\
r_3 &= -z_{16}z_{32}; \quad r_4 = z_{12}z_{33} - z_{15}z_{29}; \\
r_5 &= z_{12}z_{34} + z_{13}z_{33} - z_{16}z_{29} - z_{15}z_{30}; \\
r_6 &= z_{13}z_{34} - z_{16}z_{30}; \quad r_7 = z_{12}z_{31} - z_{14}z_{29}; \\
r_8 &= z_{12}z_{32} - z_{14}z_{30} + z_{13}z_{31}; \quad r_9 = z_{13}z_{32}; \\
s_1 &= z_{14}z_{24} - z_{15}z_{22}; \\
s_2 &= z_{14}z_{25} - z_{16}z_{22} - z_{15}z_{23}; \\
s_3 &= -z_{16}z_{23}; \quad s_4 = z_{12}z_{24} - z_{15}z_{20}; \\
s_5 &= z_{12}z_{25} + z_{13}z_{24} - z_{16}z_{20} - z_{15}z_{21}; \\
s_6 &= z_{13}z_{25} - z_{16}z_{21}; \quad s_7 = z_{12}z_{22} - z_{14}z_{20}; \\
s_8 &= z_{12}z_{23} - z_{14}z_{21} + z_{13}z_{22}; \quad s_9 = z_{13}z_{23}; \\
z_1 &= e_{33}y_1 + \zeta_{33}y_6; \\
z_2 &= (e_{15}y_1 + \zeta_{15}y_6) p_x^2 + (c_{44} + c_{13}) p_x \\
&\quad + e_{33}y_2 + \zeta_{33}y_7; \\
z_3 &= (e_{15}y_2 + \zeta_{15}y_7) p_x^2; \quad z_4 = e_{33}y_3 + \zeta_{33}y_8 + c_{33}; \\
z_5 &= (e_{15}y_3 + \zeta_{15}y_8 + c_{44}) p_x^2 - \rho_{11}; \\
z_6 &= e_{33}y_4 + \zeta_{33}y_9 + m_{33}p_x; \\
z_7 &= (e_{15}y_4 + \zeta_{15}y_9) p_x^2; \\
z_8 &= e_{33}y_5 + \zeta_{33}y_{10} + m_{33}; \\
z_9 &= (e_{15}y_5 + \zeta_{15}y_{10}) p_x^2 - \rho_{12}; \\
z_{10} &= \tilde{\zeta}_3 y_1 + e_3^* y_6; \quad z_{11} = m_{11}p_x + \tilde{\zeta}_3 y_2 + e_3^* y_7; \\
z_{12} &= m_{33} + \tilde{\zeta}_3 y_3 + e_3^* y_8; \\
z_{13} &= -\rho_{12}; \quad z_{14} = R p_x + \tilde{\zeta}_3 y_4 + e_3^* y_9; \\
z_{15} &= R + \tilde{\zeta}_3 y_5 + e_3^* y_{10}; \\
z_{16} &= -\rho_{22}; \quad z_{17} = -y_1 \xi_{33} - y_6 A_{33};
\end{aligned}$$

$$\begin{aligned}
z_{18} &= (-y_1 \xi_{11} - y_6 A_{11}) p_x^2 + (e_{15} + e_{31}) p_x \\
&\quad - \xi_{33} y_2 - A_{33} y_7; \\
z_{19} &= -(y_2 \xi_{11} + y_7 A_{11}) p_x^2; \\
z_{20} &= e_{33} - y_3 \xi_{33} - y_8 A_{33}; \\
z_{21} &= (e_{15} - y_3 \xi_{11} - y_8 A_{11}) p_x^2; \\
z_{22} &= \tilde{\zeta}_3 p_x - A_{33} y_9 - \xi_{33} y_4; \\
z_{23} &= -(y_4 \xi_{11} + y_9 A_{11}) p_x^2; \\
z_{24} &= -y_{10} A_{33} - y_5 \xi_{33} + \tilde{\zeta}_3; \\
z_{25} &= -(y_5 \xi_{11} + y_{10} A_{11}) p_x^2; \\
z_{26} &= -y_1 A_{33} - y_6 \xi_{33}^*; \\
z_{27} &= (-y_1 A_{11} - y_6 \xi_{11}^*) p_x^2 + (\zeta_{15} + \zeta_{31}) p_x \\
&\quad - y_2 A_{33} - y_7 \xi_{33}^*; \\
z_{28} &= -(y_2 A_{11} + y_7 \xi_{11}^*) p_x^2; \\
z_{29} &= \zeta_{33} - y_3 A_{33} - y_8 \xi_{33}^*; \\
z_{30} &= (\zeta_{15} - y_3 A_{11} - y_8 \xi_{11}^*) p_x^2; \\
z_{31} &= e_3^* p_x - \xi_{33}^* y_9 - A_{33} y_4; \\
z_{32} &= -(y_4 A_{11} + y_9 \xi_{11}^*) p_x^2; \\
z_{33} &= -y_{10} \xi_{33}^* - y_5 A_{33} + e_3^*; \\
z_{34} &= -(y_5 A_{11} + y_{10} \xi_{11}^*) p_x^2; \\
y_1 &= \frac{-c_{44}d}{p_x}; \\
y_2 &= -\frac{(c_{11}p_x^2 - \rho_{11})d}{p_x} - \frac{(m_{11}p_x^2 - \rho_{12})}{p_x \tilde{\zeta}_3} \\
&\quad + \frac{(m_{11}p_x^2 - \rho_{12})(e_{15} + e_{31})d}{p_x \tilde{\zeta}_3}; \\
y_3 &= \frac{(m_{33}(e_{15} + e_{31})d - m_{33})}{\tilde{\zeta}_3} - (c_{13} + c_{44})d; \\
y_4 &= -\frac{(m_{11}p_x^2 - \rho_{12})d}{p_x} - \frac{(R p_x^2 - \rho_{22})}{p_x \tilde{\zeta}_3} \\
&\quad + \frac{(R p_x^2 - \rho_{22})(e_{15} + e_{31})d}{p_x \tilde{\zeta}_3}; \\
y_5 &= \frac{(R(e_{15} + e_{31})d - R)}{\tilde{\zeta}_3} - m_{11}d;
\end{aligned}$$

$$\begin{aligned}
y_6 &= \frac{c_{44}\tilde{\zeta}_3 d}{p_x e_3^*}; \\
y_7 &= \frac{(c_{11}p_x^2 - \rho_{11})d\tilde{\zeta}_3}{p_x e_3^*} - \frac{(m_{11}p_x^2 - \rho_{12})(e_{15} + e_{31})d}{p_x e_3^*}; \\
y_8 &= \frac{(c_{13} + c_{44})d\tilde{\zeta}_3}{e_3^*} - \frac{m_{33}(e_{15} + e_{31})d}{e_3^*}; \\
y_9 &= \frac{(m_{11}p_x^2 - \rho_{12})d\tilde{\zeta}_3}{p_x e_3^*} - \frac{(Rp_x^2 - \rho_{22})(e_{15} + e_{31})d}{p_x e_3^*}; \\
y_{10} &= \frac{m_{11}d\tilde{\zeta}_3}{e_3^*} - \frac{R(e_{15} + e_{31})d}{e_3^*}; \\
d &= \frac{e_3^*}{(e_3^*(e_{15} + e_{31}) - \tilde{\zeta}_3(\zeta_{15} + \zeta_{31}))}.
\end{aligned}
\tag{A.1}$$

Conflict of Interests

There is no conflict of interests regarding the publication of this paper.

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