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Research Article

On a Homotopy Perturbation Treatment of Steady Laminar Forced Convection Flow over a Nonlinearly Stretching Porous Sheet

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The steady two-dimensional laminar forced convection boundary layer flow of an incompressible viscous Newtonian fluid over a nonlinearly stretching porous (permeable) sheet with suction is considered. The sheet's permeability is also considered to be nonlinear. The boundary layer equations are transformed by similarity transformations to a nonlinear ordinary differential equation (ODE). Then the homotopy perturbation method (HPM) is used to solve the resultant nonlinear ODE. The dimensionless entrainment parameter and the dimensionless sheet surface shear stress are obtained for various values of the suction parameter and the nonlinearity factor of sheet stretching and permeability. The results indicate that the dimensionless sheet surface shear stress decreases with the increase of suction parameter. The results of present HPM solution are compared to the values obtained in a previous study by the homotopy analysis method (HAM). The HPM results show that they are in good agreement with the HAM results within 2% error.

1. Introduction

Boundary-layer flow of an incompressible fluid over a stretching sheet has many applications in engineering such as in liquid film condensation process, aerodynamic extrusion of plastic sheets, cooling process of metallic plate in a cooling bath, and glass and polymer industries.

In the last decade, many semianalytical methods have been used to solve the boundary layer flow problems. For example, He [1] proposed a new perturbation technique coupled with the homotopy technique, which requires no small parameters in the equations and can readily eliminate the limitations of the traditional perturbation techniques. He named this method as the homotopy perturbation method (HPM). Esmaeilpour and Ganji [2] presented the problem of forced convection over a horizontal flat plate and employed the HPM to compute an approximation to the solution of the system of nonlinear differential equations governing the problem. Xu [3] obtained an approximate solution of a boundary layer equation in unbounded domain by means of He's homotopy perturbation method (HPM). Fathizadeh and Rashidi [4] solved the convective heat transfer equations of boundary layer flow with pressure gradient over a flat plate using the HPM. They studied the effects of Prandtl number and pressure gradient on both temperature and velocity profiles in the boundary layer. Raftari and Yildirim [5] obtained by means of the HPM an approximate analytical solution of the magnetohydrodynamic (MHD) boundary layer flow of an upper-convected Maxwell (UCM) fluid over a permeable stretching sheet. Raftari et al. [6] obtained, by means of the HPM, an approximate solution of the magnetohydrodynamic (MHD) boundary layer flow. Liao [7] solved the boundary-layer flow over a stretched impermeable wall by means of another semianalytic technique, namely, the homotopy analysis method (HAM). He [8] compared the HAM with the HPM and stated that the difference is clear just as the Taylor series method is different from the perturbation methods. He also illustrated the effectiveness and convenience of HPM, which can powerfully use the modern perturbation methods. Liao [9] investigated the

steady-state boundary layer flows over a permeable stretching sheet by the HAM. He obtained two branches of solutions, in which one of them agrees well with the known numerical solutions, but the other is new and has not been reported in general cases. Mahgoub [10] investigated experimentally the non-Darcian forced convection heat transfer over a horizontal flat plate in a porous medium of spherical particles. He examined the effects of particle diameter and particles materials of different thermal conductivities with air as the working fluid. Dinarvand et al. [11] compared the HPM and HAM solutions for a nonlinear ordinary differential equation, arising from Berman's similarity problem for the steady two-dimensional flow of a viscous incompressible fluid through a channel with wall suction or injection.

In the present study, the steady boundary layer flow of an incompressible viscous fluid past a nonlinearly stretching and nonlinearly permeable flat horizontal sheet is considered. Using the similarity method, the boundary layer equations are transformed to a nonlinear ordinary differential equation (ODE). The HPM solution of the governing ODE is obtained and compared to the HAM results [9]. The results reveal that the HPM is very effective such that the analytical solution, obtained by using only two terms from HPM solution, coincides well with the HAM solution.

2. Mathematical Formulation

The boundary layer flow over a nonlinearly stretching sheet is considered. The governing equations of mass, momentum, and energy for the steady two-dimensional laminar forced convection boundary layer flow of an incompressible viscous Newtonian fluid over a stretching permeable sheet are as follows [9]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where u and v are the fluid velocity components in x and y directions, respectively, and υ is the kinematic viscosity of the fluid. The boundary conditions for fluid velocity are as follows:

$$u = U_w = a(x+b)^{\lambda}, \qquad v = V_w = A(x+b)^{(\lambda-1)/2} \quad \text{at } y = 0, \qquad u \longrightarrow 0 \quad \text{as } y \longrightarrow \infty, \tag{3}$$

where U_w represents the stretching velocity of the sheet and V_w denotes the velocity of fluid through the porous sheet. a, b, λ, and A are constants. If $A > 0$, then there is injection (blowing) through the sheet, and if $A < 0$, then there is suction of fluid through the sheet, respectively. If $a > 0$, then the sheet stretches in the positive x-direction, and if $a < 0$, then the sheet stretches in the negative x-direction, respectively.

The stream function ψ is defined such that it satisfies the continuity equation (1):

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{4}$$

To use similarity method, the following similarity transformations are applied to the problem:

$$\eta = \sqrt{\frac{a(1+\lambda)}{\upsilon}}(x+b)^{(\lambda-1)/2}y, \qquad f(\eta) = \frac{\psi}{a(x+b)^{(\lambda+1)/2}}\sqrt{\frac{a(1+\lambda)}{\upsilon}}, \tag{5}$$

where η is similarity variable and f is the dimensionless velocity variable. Using the similarity transformations (5), the boundary layer equations (1) and (2) transform to the following third order nonlinear ordinary differential equation (ODE):

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) - \beta f'^2(\eta) = 0 \tag{6}$$

and the boundary conditions transform to the following forms:

$$f(0) = \gamma, \qquad f'(0) = 1, \qquad f'(\infty) = 0, \tag{7}$$

where we have

$$\gamma = -\frac{2A}{\sqrt{a(\lambda+1)\upsilon}}, \qquad \beta = \frac{\lambda}{1+\lambda}. \tag{8}$$

Here, γ is the injection (blowing) or suction parameter. $\gamma < 0$ and $\gamma > 0$ correspond to the lateral injection ($V_w > 0$) and suction ($V_w < 0$) of fluid through the porous sheet, respectively, and $\gamma = 0$ indicates the impermeable sheet. β is the nonlinearity factor of the sheet permeability and stretching, and when $\beta = 0$, the sheet moves with a constant velocity. From (5), the fluid velocity components can be obtained as follows:

$$u(x,y) = a(x+b)^{\lambda}f'(\eta), \qquad v(x,y) = -\frac{1}{2}\sqrt{a(1+\lambda)\upsilon}(x+b)^{(\lambda-1)/2} \times \left[f(\eta) + (2\beta-1)\eta f'(\eta)\right]. \tag{9}$$

The fluid vertical velocity far away from the sheet is called the entrainment velocity of the fluid and is obtained as

$$v(x,+\infty) = -\frac{1}{2}\sqrt{a(1+\lambda)\upsilon}(x+b)^{(\lambda-1)/2}f(+\infty). \tag{10}$$

The shear stress on the surface of sheet can be written as

$$\tau_w = -\mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = -a\rho\sqrt{a(1+\lambda)\upsilon}(x+b)^{(3\lambda-1)/2}f''(0). \tag{11}$$

Thus $f(+\infty)$ and $f''(0)$ have physical meanings. $f(+\infty)$ is the dimensionless entrainment parameter and $f''(0)$ is the dimensionless sheet surface shear stress, respectively.

Table 1: Values of $f(+\infty)$ and $f''(0)$ for various γ when $\beta = 1$.

γ	α	$f(+\infty)$ (present HPM solution)	$f''(0)$ (present HPM solution)	$f(+\infty)$ (HAM [9])	$f''(0)$ (HAM [9])
0	0.70711	1.17851	−0.942814	1.28077378	−0.90637551
1	1.00000	1.91666	−1.16666	1.93111056	−1.17561409
2	1.36603	2.69936	−1.48804	2.69794357	−1.51344932
5	2.68614	5.36798	−2.74819	5.36649491	−2.77335832
10	5.09808	10.1955	−5.13077	10.19523827	−5.14619724

3. Application of HPM

The main feature of the HPM is that it deforms a difficult problem into a set of problems which are easier to solve. HPM produces analytical expressions for the solution of nonlinear differential equations [1]. The obtained analytical solution by HPM is in the form of an infinite power series. Using HPM [1], the original nonlinear ODE, which cannot be solved easily, is divided into some linear ODEs which are solved easily in a recursive manner by mathematical symbolic software MATHEMATICA. First, (6) and (7) are written in the following forms:

$$u''' + \frac{1}{2}uu'' - \beta u'^2 = 0, \tag{12}$$

$$u(0) = \gamma, \qquad u'(0) = 1, \qquad u'(\infty) = 0, \tag{13}$$

where the dependent variable is changed to u instead of f. Then a homotopy is constructed in the following form:

$$u''' - \alpha^2 u' + p\left(\frac{1}{2}uu'' - \beta u'^2 + \alpha^2 u'\right) = 0, \tag{14}$$

where p is a small parameter ($0 \le p \le 1$) and α is a constant which is further to be determined. It can be seen that when $p = 0$, (14) becomes $u''' - \alpha^2 u' = 0$ and when $p = 1$, (14) results in (12). According to HPM, the following polynomial in p is substituted in (14):

$$u = u_0 + pu_1 + p^2 u_2 + \cdots, \tag{15}$$

where u_0, u_1, and so forth are the analytical first, second, and so forth terms of the solution u. Substituting (15) in (14), performing some algebraic manipulation, and equating the identical powers of p to zero give

$$\begin{gathered} p^0 : u_0''' - \alpha^2 u_0' = 0, \qquad u_0(0) = \gamma, \\ u_0'(0) = 1, \qquad u_0'(\infty) = 0, \end{gathered} \tag{16}$$

$$\begin{gathered} p^1 : u_1''' - \alpha^2 u_1' + \frac{1}{2}u_0 u_0'' - \beta u_0'^2 + \alpha^2 u_0' = 0, \\ u_1(0) = 0, \qquad u_1'(0) = 0, \qquad u_1'(\infty) = 0. \end{gathered} \tag{17}$$

The equation for p^0, (16), has the following solution:

$$u_0(\eta) = \gamma + \frac{1}{\alpha}\left(1 - \exp(-\alpha\eta)\right). \tag{18}$$

Now, if the solution for u_0, (18), is substituted in the equation for p^1, (17) becomes

$$\begin{aligned} u_1''' - \alpha^2 u_1' = {} & \left(\frac{1}{2}\alpha\gamma + \frac{1}{2} - \alpha^2\right)\exp(-\alpha\eta) \\ & + \left(\beta - \frac{1}{2}\right)\exp(-2\alpha\eta). \end{aligned} \tag{19}$$

The equation for u_1, (19), can be solved in an unbounded domain under the boundary conditions $u_1(0) = 0$, $u_1'(0) = 0$, $u_1'(\infty) = 0$, as it is shown in the Appendix, which gives

$$\begin{aligned} u_1(\eta) = {} & \frac{-2 + 6\alpha^2 - 2\beta - 3\alpha\gamma}{12\alpha^3} + \frac{1 - 2\beta}{12\alpha^3}\exp(-2\alpha\eta) \\ & + \frac{-12\alpha^2 + 6\gamma\alpha + 8\beta + 2}{24\alpha^3}\exp(-\alpha\eta) \end{aligned} \tag{20}$$

in which $\alpha = \gamma/4 + \sqrt{(\gamma/4)^2 + (1/2)}$. It should be noted that, in fact, $\alpha = \gamma/4 \pm \sqrt{(\gamma/4)^2 + (1/2)}$, but here as α should be positive ($\alpha > 0$), thus $\alpha = \gamma/4 + \sqrt{(\gamma/4)^2 + (1/2)}$. Therefore the first-order approximate solution by HPM, that is, $f(\eta) = u(\eta) = u_0(\eta) + u_1(\eta)$, is as follows:

$$\begin{aligned} f(\eta) = {} & \left(\gamma + \frac{1}{\alpha} + \frac{-2 + 6\alpha^2 - 2\beta - 3\alpha\gamma}{12\alpha^3}\right) \\ & + \frac{1 - 2\beta}{12\alpha^3}\exp(-2\alpha\eta) \\ & + \left(\frac{-36\alpha^2 + 6\gamma\alpha + 8\beta + 2}{24\alpha^3}\right)\exp(-\alpha\eta). \end{aligned} \tag{21}$$

According to (21), the dimensionless entrainment parameter $f(+\infty)$ and the dimensionless sheet surface shear stress $f''(0)$ are obtained as follows:

$$\begin{aligned} f(+\infty) &= \frac{12\gamma\alpha^3 + 18\alpha^2 - 3\gamma\alpha - 2 - 2\beta}{12\alpha^3}, \\ f''(0) &= \frac{-36\alpha^2 + 6\gamma\alpha + 10 - 8\beta}{24\alpha}. \end{aligned} \tag{22}$$

4. Results and Discussion

The boundary layer flow over a stretching permeable sheet with suction is investigated using HPM. The dimensionless

Table 2: Values of $f(+\infty)$ and $f''(0)$ for various γ when $\beta = 10$.

γ	α	$f(+\infty)$ (present HPM solution)	$f''(0)$ (present HPM solution)	$f(+\infty)$ (HAM [9])	$f''(0)$ (HAM [9])
0	0.70711	0.56407	−2.68544	0.658388	−2.608148
1	1.00000	1.41666	−2.86666	1.541900	−2.824345
2	1.36603	2.21091	−3.18419	2.457505	−3.067872
3	1.78078	3.28118	−3.55903	3.392704	−3.340911
5	2.68614	5.29059	−3.86503	5.300584	−3.973654
10	5.09808	10.1842	−5.71923	10.181382	−5.928155

Table 3: Values of $f(+\infty)$ and $f''(0)$ for various β when $\gamma = 1$ ($\alpha = 1$).

β	$f(+\infty)$ (present HPM solution)	$f''(0)$ (present HPM solution)	$f(+\infty)$ (HAM [9])	$f''(0)$ (HAM [9])
−0.656192	2.19270	−0.414603	2.3	−0.381592
−0.395769	2.14929	−0.601410	2.2	−0.569180
0	2.08333	−0.833333	2.093662	−0.786400
1	1.91666	−1.16666	1.931110	−1.175614
3	1.62333	−1.75333	1.764465	−1.699431
5	1.45000	−2.25000	1.670900	−2.088843

entrainment parameter and the dimensionless sheet surface shear stress are obtained and compared to the values obtained by the HAM solution [9]. Table 1 shows the values of the dimensionless entrainment parameter, $f(+\infty)$, and the values of the dimensionless sheet surface shear stress, $f''(0)$, using the present HPM method and the HAM solution [9], for various values of γ when $\beta = 1$. It can be seen that the present HPM solution agrees well with the HAM solution [9] for both $f(+\infty)$ and $f''(0)$ within ±2% error. It can also be seen that at $\beta = 1$, when γ increases, $f(+\infty)$ increases. This means that the increase of suction through the sheet causes, according to (10), the increase of the fluid vertical downward velocity far away from the sheet. It is also observed that at $\beta = 1$, $f''(0)$ decreases with the increase of γ; that is, the increase of suction through the sheet causes the increase of sheet surface shear stress.

Table 2 demonstrates the values of $f(+\infty)$ and the values of $f''(0)$ using the present HPM solution and the HAM solution [9], for various values of γ when $\beta = 10$. It can be seen that at $\beta = 10$, when suction parameter γ increases, $f(+\infty)$ increases while $f''(0)$ decreases, which means that the suction increase causes the boost of fluid far-away vertical velocity and sheet surface shear stress.

Table 3 shows the values of $f(+\infty)$ and $f''(0)$ using the present HPM solution and the HAM solution [9], for various values of β when $\gamma = 1$. It is observed that at $\gamma = 1$, the increase of β results in the reduction of $f(+\infty)$. This means that when there is suction through the sheet, the increase of nonlinearity factor of sheet permeability and stretching causes the fluid entrainment velocity to decrease. It may also be seen that at $\gamma = 1$, the increase of β decreases $f''(0)$; that is, the increase of nonlinearity factor β decreases fluid friction at the sheet surface.

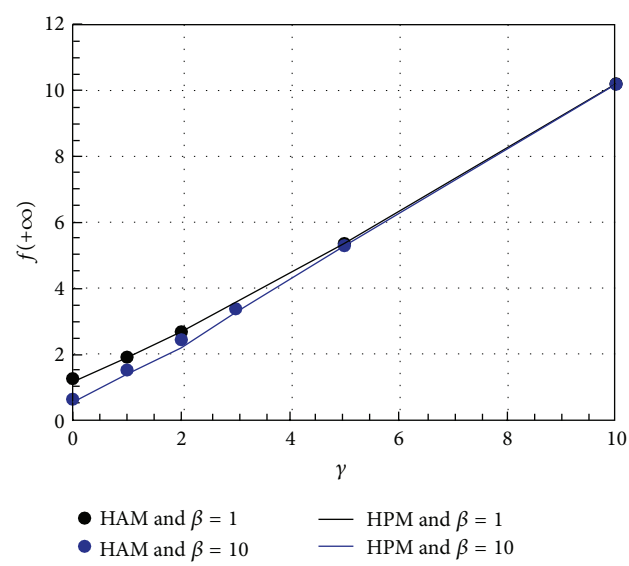

Figure 1: Variation of $f(+\infty)$ with γ for various β ($\beta = 1$ & 10) using present HPM solution and HAM [9].

Table 4 depicts $f(+\infty)$ and $f''(0)$ values using the HPM solution of the present paper and the HAM solution [9], for various values of β when $\gamma = 10$. It is observable that $f(+\infty)$ values reduce and $f''(0)$ decreases with the increase of β at a constant γ ($\gamma = 10$). Thus it can be concluded that in situations where lower sheet surface friction is desired, lower values of suction parameter γ and nonlinearity factor β can be helpful.

Figure 1 indicates the variation of $f(+\infty)$ with γ for various β using present HPM solution and HAM [9]. The

Table 4: Values of $f(+\infty)$ and $f''(0)$ for various β when $\gamma = 10$ ($\alpha = 5.09808$).

β	$f(+\infty)$ (present HPM solution)	$f''(0)$ (present HPM solution)	$f(+\infty)$ (HAM [9])	$f''(0)$ (HAM [9])
−5.949297	10.2043	−4.67640	10.21	−4.413988
−1.495596	10.1987	−4.96760	10.2	−4.899776
0	10.1968	−5.06539	10.197086	−5.049351
1	10.1955	−5.13077	10.195238	−5.146197
3	10.1930	−5.26154	10.191759	−5.333035
5	10.1905	−5.39231	10.188534	−5.511670
10	10.1842	−5.71923	10.181382	−5.928155
20	10.1716	−6.37307	10.169894	−6.663634
30	10.1590	−7.02691	10.160896	−7.307361
40	10.1465	−7.68075	10.153542	−7.886163

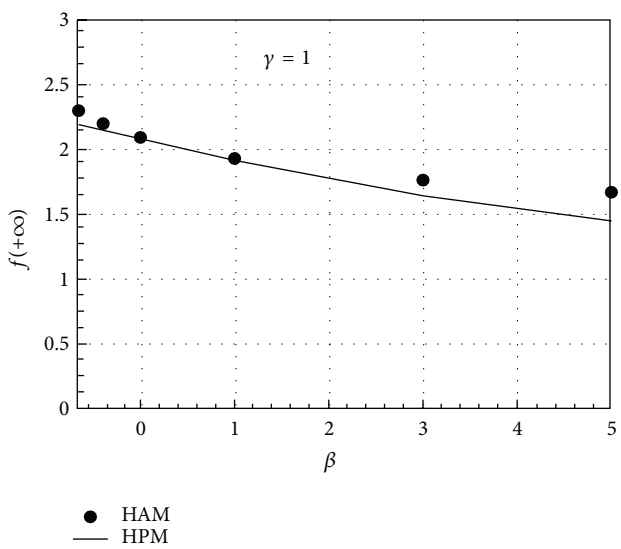

Figure 2: Variation of $f(+\infty)$ with β by present HPM solution and HAM [9] for $\gamma = 1$.

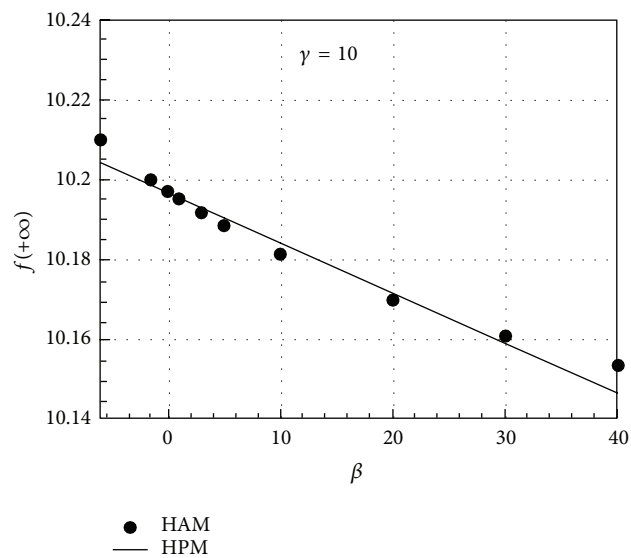

Figure 3: Variation of $f(+\infty)$ with β by present HPM solution and HAM [9] for $\gamma = 10$.

present HPM solution is observed to agree quite well with the HAM results [9]. It is seen that $f(+\infty)$ augments somewhat linearly with the increase of γ. The reason is that when γ increases, the suction through the porous sheet increases. Hence the fluid vertical velocity on the sheet boosts which results in increases of the fluid vertical velocity far away from the sheet.

Figure 2 demonstrates the variation of $f(+\infty)$ with β using present HPM solution and HAM [9] for $\gamma = 1$. The HPM results are in quite good agreement with the HAM ones. It is observed that $f(+\infty)$ reduces linearly with the increase of β. This result is due to the fact that when β increases, the velocity of the fluid in direction perpendicular to the sheet increases.

Figure 3 illustrates the variation of $f(+\infty)$ with β using present HPM solution and HAM [9] for $\gamma = 10$. The difference with Figure 2 is in the values of $f(+\infty)$, where $\gamma = 10$ results in higher values of $f(+\infty)$, while the slope for reduction of $f(+\infty)$ is lower in $\gamma = 10$ (Figure 3).

Figure 4 depicts the variation of $f''(0)$ with γ for various values of β ($\beta = 1$ & 10) using present HPM solution and HAM [9]. For both $\beta = 1$ and $\beta = 10$, there is a good match between the results of HPM with the ones of HAM. It is also apparent that $f''(0)$ diminishes with a growth in γ. When γ is increased, the resultant higher suction leads to a reduction in the stickiness of fluid particles to the sheet. Hence the fluid friction at sheet surface experiences moderation.

Figure 5 shows variation of $f''(0)$ with β using present HPM solution and HAM [9] for $\gamma = 1$. The results of the two methods, HPM and HAM [9], indicate a quite good match. With an increase in β, a decrease in $f''(0)$ is observed, which can be attributed to lower fluid-surface interaction by the higher nonlinearity of sheet stretching and permeability.

The variation of $f''(0)$ with β using present HPM solution and HAM [9] for $\gamma = 10$ is shown in Figure 6. In this case, the same conclusions made in Figure 5 are also quite acceptable. However, for $\gamma = 10$ (Figure 6), lower values of $f''(0)$ with a lower slope of the curve can be observed.

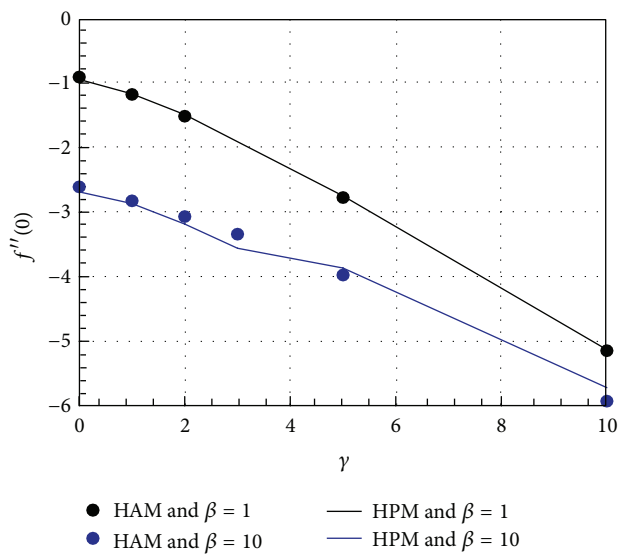

Figure 4: Variation of $f''(0)$ with γ for various β ($\beta = 1$ & 10) using present HPM solution and HAM [9].

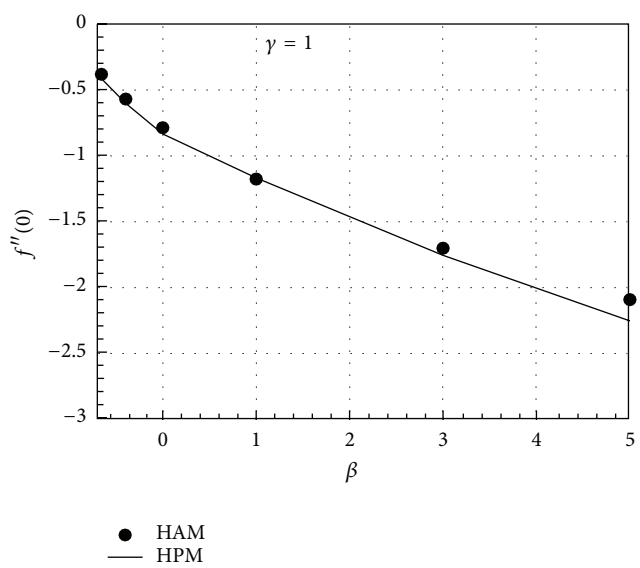

Figure 5: Variation of $f''(0)$ with β by present HPM solution and HAM [9] for $\gamma = 1$.

5. Conclusions

The steady 2-D boundary layer flow of an incompressible fluid on a nonlinearly stretching permeable sheet is considered. The HPM solution of the boundary layer flow is obtained. The dimensionless entrainment parameter and the dimensionless sheet surface shear stress are obtained and compared to the values of a previous HAM solution which show that the present HPM solution with only two terms agrees well with the results of HAM. The results obtained are as follows.

(1) The increase of suction through the sheet causes the increase of fluid vertical downward velocity far away from the sheet.

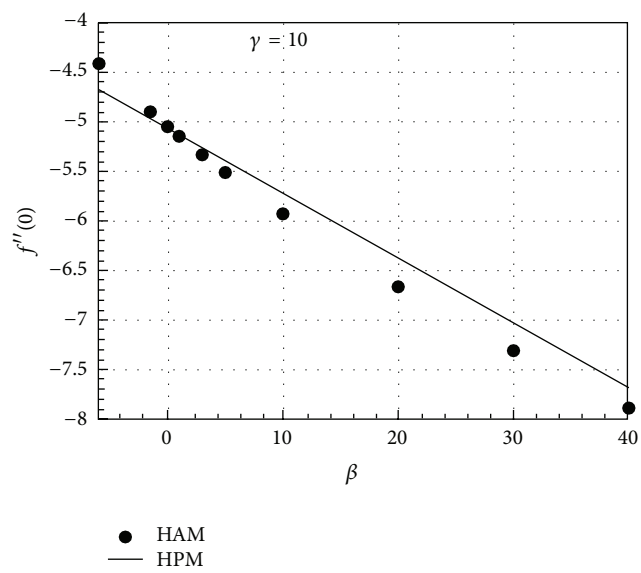

Figure 6: Variation of $f''(0)$ with β by present HPM solution and HAM [9] for $\gamma = 10$.

(2) When there is suction through the sheet, the increase of nonlinearity factor of sheet permeability and stretching causes the fluid entrainment velocity to decrease.

(3) The fluid friction at the sheet surface decreases with the increase of nonlinearity factor of sheet stretching and permeability.

Appendix

The equation for $u_1(\eta)$, (19) is solved using the symbolic software MATHEMATICA under the boundary conditions $u_1(0) = 0$, $u_1'(0) = 0$ as

$$\begin{aligned} u_1''' - \alpha^2 u_1' &= \left(\frac{1}{2}\alpha\gamma + \frac{1}{2} - \alpha^2\right)\exp(-\alpha\eta) \\ &\quad + \left(\beta - \frac{1}{2}\right)\exp(-2\alpha\eta), \\ u_1(0) &= 0, \qquad u_1'(0) = 0 \end{aligned} \tag{A.1}$$

which gives the following solution:

$$\begin{aligned} u_1(\eta) = {} & \frac{-3 + 4\alpha^2 + 2\beta - 2\alpha\gamma + 8\alpha^2 C(2)}{4\alpha^3} \\ & + \frac{1 - 2\beta}{12\alpha^3}\exp(-2\alpha\eta) \\ & + \frac{3 - 6\alpha^2 + 3\alpha\gamma - 8\alpha^2 C(2)}{8\alpha^3}\exp(-\alpha\eta) \\ & + \frac{7 - 6\alpha^2 - 8\beta + 3\gamma\alpha - 24\alpha^2 C(2)}{24\alpha^3}\exp(\alpha\eta) \\ & + \frac{-2\alpha^2 + 1 + \alpha\gamma}{4\alpha^2}\eta\exp(-\alpha\eta), \end{aligned} \tag{A.2}$$

where $C(2)$ is the integration constant. If the boundary condition $u_1'(\infty) = 0$ is applied to the solution (A.2), it gives $C(2)$ and α as

$$\frac{7 - 6\alpha^2 - 8\beta + 3\gamma\alpha - 24\alpha^2 C(2)}{24\alpha^2} = 0$$
$$\longrightarrow C(2) = \frac{7}{24\alpha^2} - \frac{1}{4} - \frac{\beta}{3\alpha^2} + \frac{\gamma}{8\alpha}, \tag{A.3}$$
$$\frac{-2\alpha^2 + 1 + \alpha\gamma}{-4\alpha} = 0 \longrightarrow \alpha = \frac{\gamma \pm \sqrt{\gamma^2 + 8}}{4}.$$

Now, if $C(2)$ from (A.3) is substituted in (A.2), $u_1(\eta)$ becomes

$$\begin{aligned} u_1(\eta) = {} & \frac{-2 + 6\alpha^2 - 2\beta - 3\alpha\gamma}{12\alpha^3} + \frac{1 - 2\beta}{12\alpha^3}\exp(-2\alpha\eta) \\ & + \frac{2 - 12\alpha^2 + 6\alpha\gamma + 8\beta}{24\alpha^3}\exp(-\alpha\eta) \\ & + \frac{-2\alpha^2 + 1 + \alpha\gamma}{4\alpha^2}\eta\exp(-\alpha\eta). \end{aligned} \tag{A.4}$$

Here it can be checked and seen that for (A.4), $u_1(0) = 0$, $u_1'(0) = 0$. If the third boundary condition $u_1'(\infty) = 0$ is applied to (A.4), it gives the value of $\alpha = \gamma/4 \pm \sqrt{(\gamma/4)^2 + (1/2)}$. This value of α removes the secular term from the ordinary differential equation (ODE) for $u_1(\eta)$. If α is substituted in the last term of (A.4), the last term vanishes, and $u_1(\eta)$ takes its final form as

$$\begin{aligned} u_1(\eta) = {} & \frac{-2 + 6\alpha^2 - 2\beta - 3\alpha\gamma}{12\alpha^3} + \frac{1 - 2\beta}{12\alpha^3}\exp(-2\alpha\eta) \\ & + \frac{2 - 12\alpha^2 + 6\alpha\gamma + 8\beta}{24\alpha^3}\exp(-\alpha\eta). \end{aligned} \tag{A.5}$$

Nomenclature

a: Stretching rate (—)
A: Permeability rate (—)
b: Displacement value of the sheet stretching and permeability (m)
f: Dimensionless velocity variable ($= (\psi/a(x+b)^{(\lambda+1)/2})\sqrt{a(1+\lambda)/\upsilon}$) (—)
p: Small parameter in (14) (—)
u: Velocity in x-direction (m s^{-1}), dependent variable in (12) instead of f (—)
U_w: Velocity of the sheet (m s^{-1})
v: Velocity in y-direction (m s^{-1})
V_w: Velocity of fluid through the porous sheet (m s^{-1})
x: Horizontal coordinate (m)
y: Vertical coordinate (m).

Greek Symbols

α: Constant in (14) (—)
β: Nonlinearity factor of the sheet stretching and permeability ($= \lambda/(1+\lambda)$) (—)
γ: Suction or injection (blowing) parameter ($= -2A/\sqrt{a(\lambda+1)\upsilon}$) (—)
η: Similarity variable ($= (x+b)^{(\lambda-1)/2} y\sqrt{a(1+\lambda)/\upsilon}$) (—)
λ: Exponent of sheet stretching and permeability velocity (—)
μ: Dynamic viscosity (N s m^{-2})
υ: Kinematic viscosity (m^2 s^{-1})
ρ: Density (kg m^{-3})
τ: Shear stress (N m^{-2})
τ_w: Shear stress on sheet surface (N m^{-2})
ψ: Stream function (m^2 s^{-1}).

Subscripts

∞: Infinity
f: Fluid
w: Sheet surface.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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