Research Article

Combination of Two Nonlinear Techniques Applied to a 3-DOF Helicopter

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A combination of two nonlinear control techniques, fractional order sliding mode and feedback linearization control methods, is applied to 3-DOF helicopter model. Increasing of the convergence rate is obtained by using proposed controller without increasing control effort. Because the proposed control law is robust against disturbance, so we only use the upper bound information of disturbance and estimation or measurement of the disturbance is not required. The performance of the proposed control scheme is compared with integer order sliding mode controller and results are justified by the simulation.

1. Introduction

Helicopters are versatile flight vehicles that can perform aggressive maneuvers because of their unique thrust generation and operation principle. They can perform many missions that are dangerous or impossible for human to perform them. Helicopter is a multi-input multioutput (MIMO) highly nonlinear dynamical system, so most of the existing results to date have been based on the linearization model or through several linearization techniques [1, 2]. The linearization method provides local stability. In presence of disturbance or uncertainties the linearization can lead state variables of system to instability.

In recent years, many papers have been published about control design of helicopter. The sliding mode approach has been employed for helicopter's altitude regulation at hovering [3, 4]. H-infinity approach has been also used to design a robust control scheme for helicopter [5, 6]. In [5] a robust H-infinity controller has been presented using augmented plant and the performance and robustness of the proposed controller have been investigated in both time and frequency domain. The proposed controller in [6] is based on H-infinity loop shaping approach and it has been shown that the proposed controller is more efficient than classical controller such as PI and PID controller. A robust linear time-invariant

controller based on signal compensation has been presented in [7]. By suitably combining feedforward control actions and high-gain and nested saturation feedback laws, a new control scheme has been presented in [8]. Intelligent methods such as fuzzy [9] and neural network theory [10] have been used to design controller. Furthermore, in [11] a new intelligent control approach based on emotional model of human brain has been presented.

The history of fractional calculus goes back 300 years ago. For many years, it has remained with no applications. Recently, this branch of science has become an attractive discussion among control scholars [12]. Good references on fractional calculus have been presented in [13, 14]. The sliding mode control (SMC) has been also extended in [12, 15–19]. In [15] a PID controller based on sliding mode strategy is designed for linear fractional order systems. In [17] a single input fractional order model, described by a chain of integrators, is considered for nonlinear systems. In [16–18] sliding mode method has been applied to synchronize fractional order nonlinear chaotic systems. In [12, 19] sliding surface has been defined as an expressed manifold with fractional order integral.

In this paper, we propose a combination of two nonlinear control techniques, fractional order (FO) sliding mode and

feedback linearization control methods. The performance of the proposed control scheme is compared with integer order (IO) sliding mode controller and results are justified by the simulation.

This paper is organized as follows: in Section 2, the integer order SMC and feedback linearization techniques are briefly reviewed, and, in Section 3, some preliminaries and definitions of fractional order calculus are introduced. In Section 4, the 3-DOF helicopter model description is presented. In Section 5, the proposed controller scheme is given, and, in Section 6, the proposed approach is applied to 3-DOF helicopter model and experimental results are provided. Finally, conclusion is addressed in Section 7.

2. Review of IO SMC and Feedback Linearization

In this section, definitions of sliding mode and feedback linearization control methods are briefly reviewed for essential preparation of the combination of two nonlinear control techniques, FO SMC and feedback linearization.

2.1. Integer Order Sliding Mode Control.

Consider a second-order nonlinear dynamical system as

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = f(x) + g(x)u, \tag{1}$$

where $x = [x_1 \ x_2]^T$ is the system state vector, $f(x)$ and $g(x)$ are nonlinear functions of x, and $u \neq 0$ is the scalar input. Sliding mode control approach consists of two parts, correctivecontrol law (u_c) that compensates the deviations from the sliding surface to reach the sliding surfaceand equivalent control law (u_{eq}) that makes the derivative of the sliding surface equal zero to stay on the sliding surface. This control law is represented as

$$u = u_{eq} + u_c. \tag{2}$$

In general, the sliding surface is $S = x_2 + \beta x_1$, where $\beta > 0$.

To guarantee the existence of sliding mode, the control law must satisfy the condition

$$\frac{1}{2} \frac{d}{dt} s^2 < 0. \tag{3}$$

To have a fast convergence, it is sufficient to modify the sliding surface.

2.2. Input-Output Linearization Control.

Consider the following nonlinear dynamical system:

$$\dot{x} = f(x) + g(x)u \qquad y = h(x). \tag{4}$$

The control method consists of the following steps.

(a) Differentiate y until u appears in one of the equations for the derivatives of y. Consider

$$y^{(r)} = \alpha(x) + \beta(x)u. \tag{5}$$

(b) Choose u to give $y(r) = v$, where v is the synthetic input. Consider

$$u = \frac{1}{\beta(x)} [-\alpha(x) + v]. \tag{6}$$

(c) Then the system has the following form:

$$\frac{Y(s)}{V(s)} = \frac{1}{s^r}. \tag{7}$$

Design a linear control law for this r-integrator linear system.

(d) Check internal dynamics.

3. Basic Description of Fractional Calculus

There exist many definitions of fractional derivative. Two of the most commonly used definitions are the Riemann-Liouville (RL) and Caputo (C) definitions [14].

Definition 1 (see [20]). The left RL fractional derivative is described by

$$^{RL}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{8}$$

where n is the first integer which is not less than α, that is, $n - 1 \leq \alpha < n$ and $\alpha > 0$, and $\Gamma(\cdot)$ is the gamma function.

Definition 2 (see [21]). The Laplace transform of the RL definition is described as

$$\int_0^\infty e^{-st} {}_a D_t^\alpha f(t) \, dt = s^\alpha F(s) - \sum_{k=0}^{n-1} s^k \left. {}_a D_t^{\alpha-k-1} f(t) \right|_{t=0}. \tag{9}$$

Lemma 3 (see [20]). *The following equality is satisfied for all $\alpha > 0$ and n is a natural number. Consider*

$$\frac{d^n}{dt^n} \left\{ {}^{RL}D^\alpha f(t) \right\} = {}^{RL}D^{n+\alpha} f(t),$$

$$^CD^\alpha \left\{ \frac{d^n}{dt^n} f(t) \right\} = {}^CD^{n+\alpha} f(t). \tag{10}$$

Lemma 4 (see [21]). *Fractional differentiation and fractional integration are linear operations. Consider*

$$_0D_t^\alpha \left(af(t) + bg(t) \right) = a \, {}_0D_t^\alpha f(t) + b \, {}_0D_t^\alpha g(t). \tag{11}$$

Lemma 5 (see [20]). *The following equality for left RL definition is satisfied for all $\alpha \in (0, 1)$:*

$$^{RL}D^\alpha \left\{ I^\alpha f(t) \right\} = f(t), \quad \alpha \in (0, 1). \tag{12}$$

However the opposite is not true, since

$$I^\alpha \left\{ {}^{RL}D^\alpha f(t) \right\} = f(t) - \frac{f_{1-\alpha}(0)}{\Gamma(\alpha)} t^{\alpha-1}, \tag{13}$$

where $f_{1-\alpha}(0) = \lim_{t \to 0} I^{1-\alpha} f(t)$.
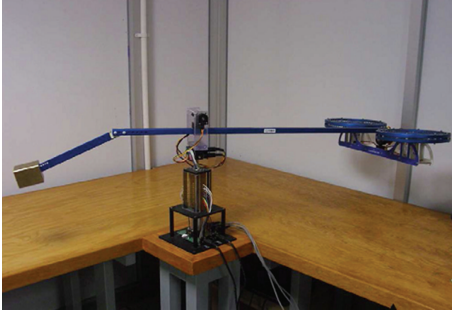
FIGURE 1: The 3-DOF helicopter system.

FIGURE 2: Schematic diagram of the 3-DOF helicopter.

4. System Description

In this work, a 3-DOF helicopter model is considered (see Figure 1).

The following notations are used to describe the 3-DOF helicopter system dynamics:

θ: is angular position of the pitch axis, rad;

φ: is angular position of the roll axis, rad;

ψ: is angular position of the yaw axis, rad;

J_θ: is moment of inertia of the system around the pitch axis, kg m^2;

J_φ: is moment of inertia of the helicopter body about the roll axis, kg m^2;

J_ψ: is moment of inertia of the helicopter body about the yaw axis, kg m^2;

V_l: is voltage applied to the left motor, V;

V_r: is voltage applied to the right motor, V;

K_f: is force constant of the motor combination, N;

l_a: is distance between the base and the helicopter body, m;

l_h: is distance from the pitch axis to either motor, m;

T_g: is effective gravitational torque, N m;

K_p: is constant of proportionality of the gravitational force, N.

The 3-DOF helicopter control system consists of two DC motors at the end of the arm as shown in Figure 1. Figure 2 shows a physical model of the 3-DOF helicopter. The following equations (see Figure 2) describe the 3-DOF helicopter dynamics:

$$\ddot{\psi} = -\frac{K_p l_a}{J_\psi} \sin(\varphi), \tag{14}$$

$$\ddot{\theta} = -\frac{T_g}{J_\theta} + \frac{K_f l_a}{J_\theta}(V_l + V_r + \xi_\varphi), \tag{15}$$

$$\ddot{\varphi} = \frac{K_f l_h}{J_\varphi}(V_l - V_r), \tag{16}$$
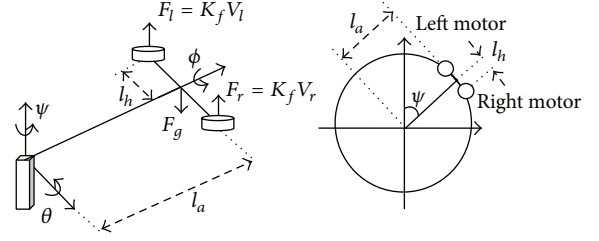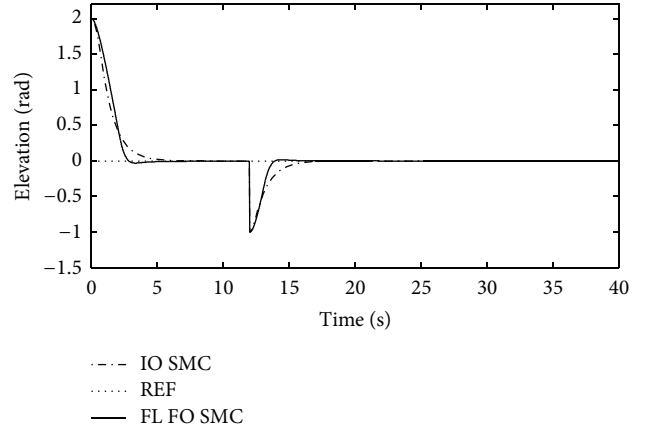
$$y = [\psi \ \theta \ \varphi]^T. \tag{17}$$
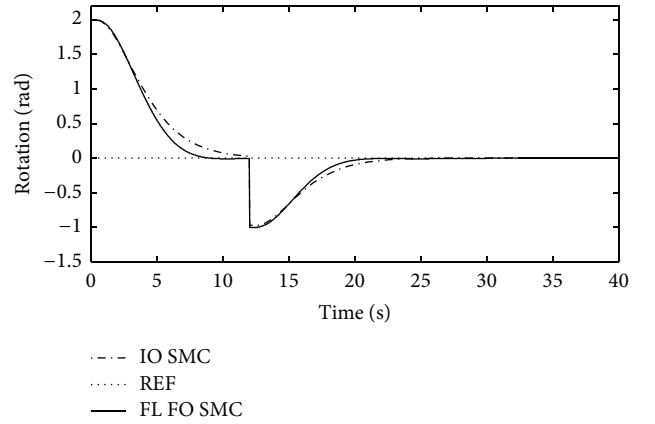

FIGURE 3: Stability curve of elevation.

FIGURE 4: Stability curve of rotation.

Let define $u_1 = V_l + V_r$ and $u_2 = V_l - V_r$; for the sake of simplicity we define state vector as $x = [\psi \ \dot{\psi} \ \theta \ \dot{\theta} \ \varphi \ \dot{\varphi}]^T$ and the model (14)–(16) can be rewritten as

$$\dot{x}_1 = x_2 \qquad \dot{x}_2 = -\frac{T_g}{J_\theta} + \frac{K_f l_a}{J_\theta}(u_1 + \xi_\varphi),$$

$$\dot{x}_3 = x_4 \qquad \dot{x}_4 = -\frac{K_p l_a}{J_\psi} \sin(x_5), \tag{18}$$

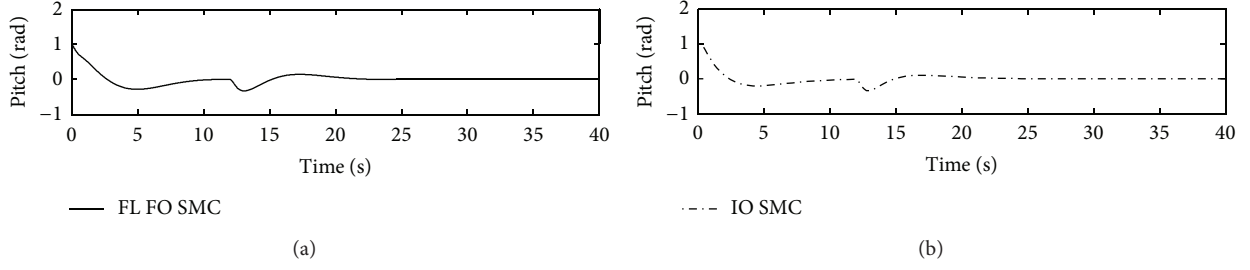$$\dot{x}_5 = x_6 \qquad \dot{x}_6 = \frac{K_f l_h}{J_\varphi} u_2.$$

(a)

(b)

FIGURE 5: Stability curve of pitch.

Furthermore, let vectors X_1 and X_2 represent the partitions of the state vector, respectively. Thus, $x = [X_1{}^T, X_2{}^T]^T$, where $X_1 \in R^2$ and $X_2 \in R^4$. Hence, $\dot{X}_1$-subsystem and $\dot{X}_2$-subsystem are presented as follows:

$$\dot{X}_1\text{-subsystem:} \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{T_g}{J_\theta} + \dfrac{K_f l_a}{J_\theta}\left(u_1 + \xi_\varphi\right), \end{cases}$$

$$\dot{X}_2\text{-subsystem:} \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -\dfrac{K_p l_a}{J_\psi}\sin(x_5) \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \dfrac{K_f l_h}{J_\varphi}u_2. \end{cases} \quad (19)$$

5. Design of Controller

It will be seen that $\dot{X}_1$-subsystem and $\dot{X}_2$-subsystem have no coupling. We know feedback linearization scheme is suitable approach for multivariable systems but is not robust inherently. Sliding mode control is well known as a robust nonlinear control scheme by disturbance rejection capability. Because feedback linearization is a simple method and the FO SMC method is robust against disturbance, so we combine these two approaches. Firstly, feedback linearization method is applied; secondly, FO SMC method is used. Our control approach is formed in two steps. In the first step, a fractional order sliding mode controller for $\dot{X}_1$-subsystem is designed; then, since the $\dot{X}_2$-subsystem is multivariable and nonlinear, feedback linearization control is applied. For $\dot{X}_2$-subsystem, controller must be robust against disturbance. After linearization, a fractional order sliding mode controller is proposed for linearized subsystem. This helicopter model is an underactuated system by three outputs and two inputs, so we can control two outputs and third output only is kept limited.

Consider $\dot{X}_1$-subsystem as

$$\ddot{x}_1 = f + gu_1. \quad (20)$$

Let define sliding surface as $S_1 = {}^{RL}D^\alpha \ddot{e}_1 + \lambda_1 e_1$ and the error signal is defined as $e_1 = x_{1d} - x_1$. Also, disturbance is imposed to roll and yaw channel.

The fractional order sliding mode control input signal can be defined as follows for subsystem (20), [12]:

$$u_1(t) = g^{-1}\left[\ddot{x}_d(t) - f + {}^{RL}D^{-1-\alpha}\left(\lambda_1 \dot{e}_1\right) \right. \\ \left. + {}^{RL}D^{-1-\alpha}\left(k_1 S_1 + k_1 \operatorname{sign}(S_1)\right)\right], \quad (21)$$

where k_1 and λ_1 are positive constants.

In [12], it has been shown that this control input signal guarantees the stability of the closed-loop $\dot{X}_1$-subsystem and the tracking error converges to zero in finite time. Then, in order to control $\dot{X}_2$-subsystem, at first we use input-output feedback linearization method. Feedback linearization control law is defined as

$$u_2(t) = \left(-\frac{K_p l_a}{J_\psi}\cos(x_5)\right)^{-1}\left[V - \frac{K_p l_a}{J_\psi}x_6 \sin(x_5)\right]. \quad (22)$$

V is new control law that will be designed for linearized system. So we want to obtain overall control law that it has robustness behavior and good performance. We use fractional order control instead of integer order to design control input V.

The sliding mode control law, V, is designed as follows:

$$V = x_{3d}^{(4)} - \lambda_2^3 {}^{RL}D^\beta \dot{e}_2 - 3\lambda_2^2 {}^{RL}D^\beta \ddot{e}_2 \\ - 3\lambda_2 {}^{RL}D^\beta e_2^{(3)} - k_2 \operatorname{Sgn}(S_2) . \quad (23)$$

Let the sliding surface equation be proposed as follows:

$$S_2 = e_2^{(3)} + \lambda_2^3 {}^{RL}D^\beta e_2 + 3\lambda_2^2 {}^{RL}D^\beta \dot{e}_2 + 3\lambda_2 {}^{RL}D^\beta \ddot{e}_2 . \quad (24)$$

The error signal is defined as $e_2 = x_3 - x_{3d}$ and the Lyapunov function is to be defined as

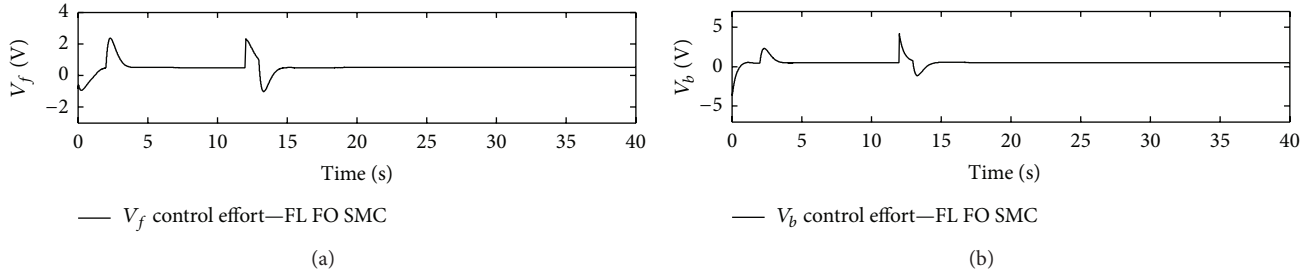$$\widetilde{V} = \frac{1}{2}S_2^2. \quad (25)$$

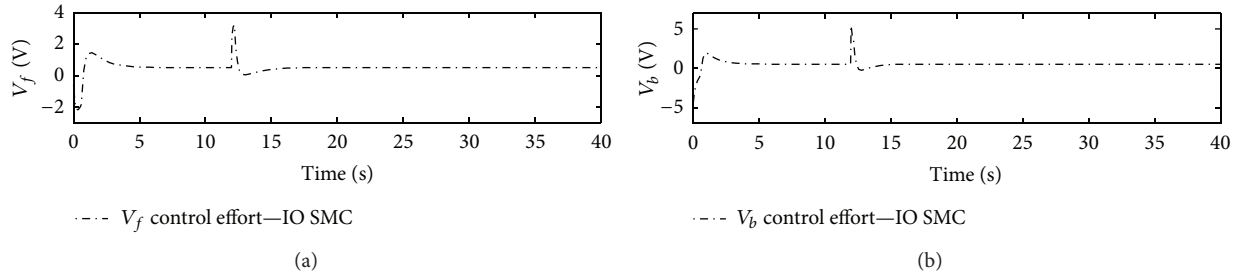FIGURE 6: Force control inputs of proposed controller (stability).

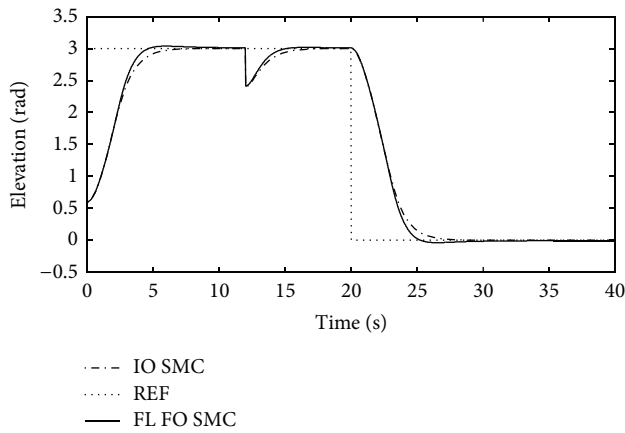FIGURE 7: Force control inputs of IO SMC (stability).
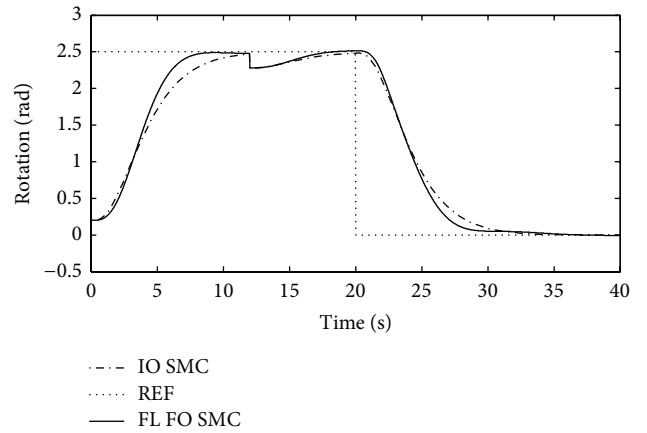

FIGURE 8: Tracking curve of elevation.

FIGURE 9: Tracking curve of rotation.

Consider the sufficient condition for the existence finite time convergence is

$$\frac{1}{2}\frac{d}{dt}S_2^2(t) \le -k_2 |S_2(t)|, \tag{26}$$

where k_2 is a positive constant. Taking time derivative of both sides of (25), according to Lemmas 3 and 5, then we have

$$\dot{\tilde{V}} = S_2 \dot{S}_2 = S_2 \left(e_2^{(4)} + \lambda_2^3 \,{}^{\text{RL}}D^{\beta+1} e_2 \right.$$

$$\left. + 3\lambda_2^2 \,{}^{\text{RL}}D^{\beta+1}\dot{e}_2 + 3\lambda_2 \,{}^{\text{RL}}D^{\beta+1}\ddot{e}_2 \right)$$

$$= S_2 \left(V - x_{3d}^{(4)} + \lambda_2^3 \,{}^{\text{RL}}D^{\beta+1} e_2 \right.$$

$$\left. + 3\lambda_2^2 \,{}^{\text{RL}}D^{\beta+1}\dot{e}_2 + 3\lambda_2 \,{}^{\text{RL}}D^{\beta+1}\ddot{e}_2 \right). \tag{27}$$

Replacing (23) into (27) gives

$$\dot{\tilde{V}} = S_2 \dot{S}_2 = S_2 \left(-k_2 \operatorname{sgn}(S_2) \right) \le -\left(k_2 + F \right)|S_2|, \tag{28}$$

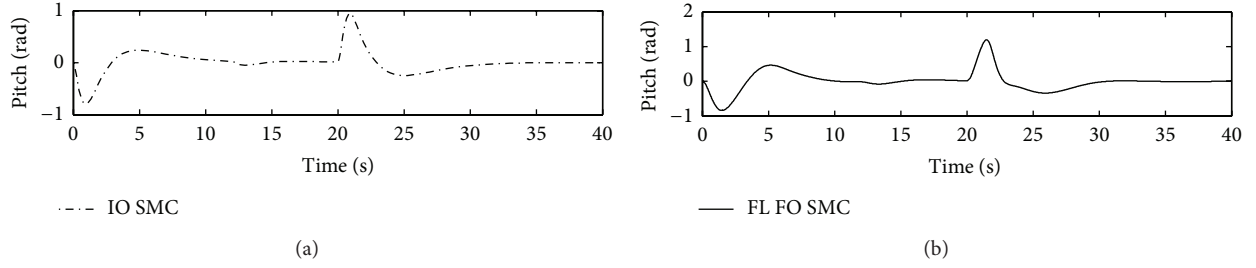where F is maximum of disturbance. So, total feedback linearization fractional order sliding mode control (FL FO

FIGURE 10: Tracking curve of pitch.

SMC) law is achieved by combination of (22) and (23) that is represented as follows:

$$u_2(t) = \left(-\frac{K_p l_a}{J_\psi} \cos(x_5) \right)^{-1}$$

$$\times \left[x_{3d}^{(4)} - \lambda_2^3 \, {}^{RL}D^\beta \dot{e}_2 - 3\lambda_2^2 \, {}^{RL}D^\beta \ddot{e}_2 \right.$$

$$\left. - 3\lambda_2 \, {}^{RL}D^\beta e_2^{(3)} - k_2 \mathrm{Sgn}(S_2) \right.$$

$$\left. - \frac{K_p l_a}{J_\psi} x_6 \sin(x_5) \right]. \tag{29}$$

These two control laws, u_1 and u_2, are applied to 3-DOF helicopter model and simulation results are shown in the next section.

6. Simulation Results

To see the performance of the proposed controller, we have simulated the controlled system. The results have been compared with integer order sliding mode controller. The helicopter nominal parameters are shown in Table 1. We choose $\alpha = 0.1$, $\beta = 0.15$ in this simulation. Furthermore, a step disturbance is applied in $t = 12$ sec.

To investigate the stability of the closed-loop system, the initial conditions are applied and the reference input is considered to be zero. The time response of the states of the system (18) in the presence of the control laws (21) and (29) is illustrated in Figures 3, 4, and 5. It can be seen that under the proposed controller the output converges to zero and when a disturbance applies the proposed controller is able to nullify the output again. It is clear that the convergence rate in the proposed controller is more than the integer order controller.

The force control inputs are given in Figures 6, and 7. Although the proposed controller is faster than integer order controller in states convergence rate, it requires less control signals energy compared with the integer order SMC controller.

The simulation results of the reference tracking under the disturbance are illustrated in Figures 8, 9, and 10. These figures show that the helicopter can follow the reference trajectory under the proposed control method, and the output response under the proposed control scheme is faster

TABLE 1: Parameters' values.

Parameter	Value	Unit
J_θ	0.91	(kgm²)
J_φ	0.0364	(kgm²)
J_ψ	0.91	(kgm²)
K_f	0.5	(N)
K_p	0.5	(N)
l_a	0.66	(m)
l_h	0.177	(m)
T_g	0.33	(Nm)

than another one in spite of disturbance. Also the pitch angle remained limited.

The control efforts have been shown in Figures 11 and 12. According to these figures it can be seen that although the proposed controller is faster than integer order controller, it needs less control effort. To implement the controller it is essential that the control effort has an acceptable value.

According to these simulations, it can be seen that the tracking performance of proposed controller is more efficient, with less control effort compared with the integer order sliding mode controller in spite of disturbances.

7. Conclusion

In this paper, we presented a combination of two nonlinear control techniques applied to 3-DOF helicopter. Also we considered disturbance in roll and yaw channel. In fact we have used feedback linearization fractional order sliding mode theory to control roll and yaw channel. It is desirable that the outputs track the reference input in less time. In practical application, a high-gain controller may be undesirable. So in this paper a fractional order controller was developed to increase the convergence rate in less control signal energy. To verify the performance of the proposed controller, it was compared with integer order sliding mode theory. The results show that the proposed method also simplifies the design process due to the use of feedback linearization control mechanism for multivariable systems, the sliding mode method advantages, such as robustness is retained. The
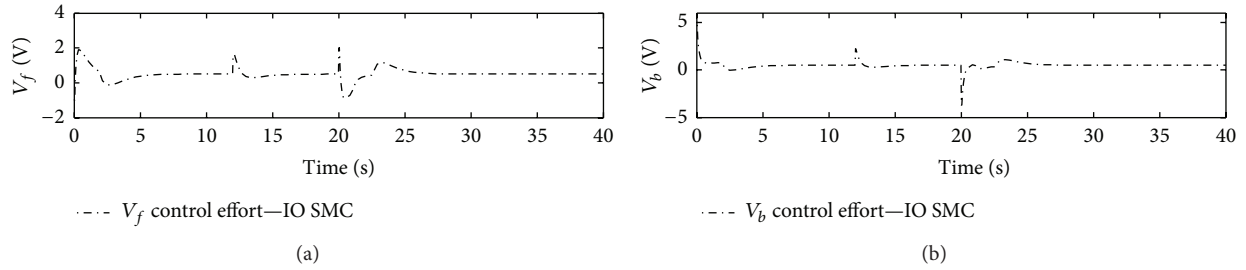
(a)

(b)

FIGURE 11: Force control inputs of proposed controller (tracking).
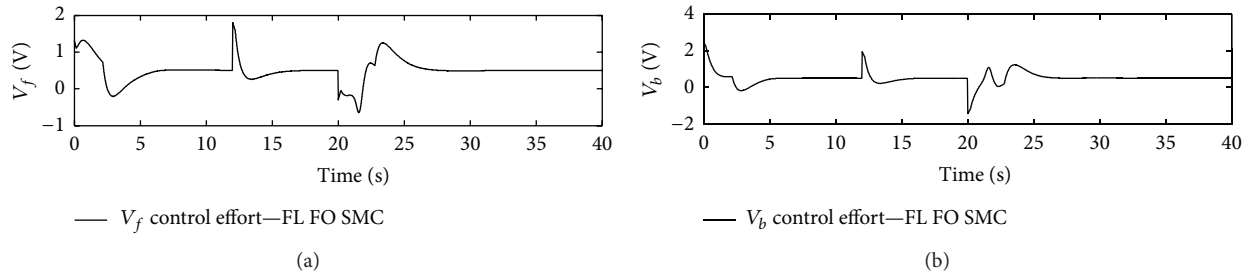

(a)

(b)

FIGURE 12: Force control inputs of pitch (tracking).

reason is that after linearization, a fractional order sliding mode controller is designed for linearized subsystem.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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