Research Article

Gravity Modulation Effects of Hydromagnetic Elastico-Viscous Fluid Flow past a Porous Plate in Slip Flow Regime

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The two-dimensional hydromagnetic free convective flow of elastico-viscous fluid (Walters liquid Model B') with simultaneous heat and mass transfer past an infinite vertical porous plate under the influence of gravity modulation effects has been analysed. Generalized Navier’s boundary condition has been used to study the characteristics of slip flow regime. Fluctuating characteristics of temperature and concentration are considered in the neighbourhood of the surface having periodic suction. The governing equations of fluid motion are solved analytically by using perturbation technique. Various fluid flow characteristics (velocity profile, viscous drag, etc.) are analyzed graphically for various values of flow parameters involved in the solution. A special emphasis is given on the gravity modulation effects on both Newtonian and non-Newtonian fluids.

1. Introduction

The analysis of viscoelastic fluid flow is one of the important fields of fluid dynamics. The complex stress-strain relationships of viscoelastic fluid flow mechanisms are used in geophysics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil-physics, biophysics, and paper and pulp technology. The viscosity of the viscoelastic fluid signifies the physics of the energy dissipated during the flow and its elasticity represents the energy stored during the flow. As Walters liquid (Model B') contains both viscosity and elasticity it is different from Newtonian fluid because the Newtonian fluid does not have the concept of elasticity as it discusses only the concept of energy dissipation.

The phenomenon of transient free convection flow from a vertical plate has been analysed by Siegel [1]. Gebhart [2] has studied the fluid motion in the presence of natural convection from vertical elements. Chung and Anderson [3] have investigated the nature of unsteady fluid flow including the effects of natural convection. Schetz and Eichhorn [4] have investigated the above unsteady problem in the vicinity of a doubly infinite vertical plate. Goldstein and Briggs [5] have studied the problem of free convection about vertical plates and circular cylinders. Two- and three-dimensional oscillatory convection in gravitationally modulated fluid layers have been investigated by Clever et al. [6, 7]. A study of thermal convection in an enclosure induced simultaneously by gravity and vibration has been done by Fu and Shieh [8]. Convection phenomenon of material processing in space has been described by Ostrach [9]. Li [10] has investigated the effect of magnetic fields on low frequency oscillating natural convection. Deka and Soundalgekar [11] have analysed the problem of free convection flow influenced by gravity modulation by using Laplace transform technique. Rajvanshi and Saini [12] have studied the free convection MHD flow past a moving vertical porous surface with gravity modulation at constant heat flux. The influence of combined heat and mass transfer and gravity modulation on unsteady flow past a porous vertical plate in slip flow regime has been examined by Jain and Rajvanshi [13].

In this study, an analysis is carried out to study the effects of gravity modulation on free convection unsteady flow of a viscoelastic fluid past a vertical permeable plate with slip flow regime under the action of transverse magnetic field. The velocity field and magnitude of shearing stress at the plate are obtained and illustrated graphically to observe the viscoelastic effects in combination with other flow parameters.
The constitutive equation for Walters liquid (Model B') is

\[ \sigma_{ik} = -pg_{ik} + \sigma'_{ik}, \quad \sigma'_{ik} = 2\eta_0e^{ik} - 2k_0\epsilon^{ik}, \]  

(1)

where \( \sigma^{ik} \) is the stress tensor, \( p \) is isotropic pressure, \( g_{ik} \) is the metric tensor of a fixed coordinate system \( x', v_i \) is the velocity vector, and the contravariant form of \( e^{ik} \) is given by

\[ e^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{im} - (\epsilon^{im} e^{ik}). \]  

(2)

It is the convected derivative of the deformation rate tensor \( e^{ik} \) defined by

\[ 2e^{ik} = v_{ik} + v_{ki}. \]  

(3)

Here \( \eta_0 \) is the limiting viscosity at the small rate of shear which is given by

\[ \eta_0 = \int_0^\infty N(r) \, dr, \quad k_0 = \int_0^\infty \tau N(r) \, dr, \]  

(4)

\( N(r) \) being the relaxation spectrum. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

\[ \int_0^\infty t^n N(r) \, dr, \quad n \geq 2 \]  

(5)

have been neglected.

Walter [14] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 gm of polymer per litre and having density 0.98 gm/mL fits very nearly to this model.

2. Mathematical Formulation

An unsteady two-dimensional free convective flow of an electrically conducting elastico-viscous fluid past a vertical porous plate has been analysed in the presence of gravity modulation and slip flow regime A magnetic field of uniform strength \( B_0 \) is applied in the direction normal to the plate. Induced magnetic field is neglected by assuming very small values of magnetic Reynolds number (Crammer and Pai [15]).

The electrical conductivity of the fluid is also assumed to be of smaller order of magnitude. Let \( x' - \) axis be taken along the vertical plate and \( y' - \) axis is taken normal to the plate. Let \( T_w \) and \( C_w \) be, respectively, the temperature and the molar species concentration of the fluid at the plate and let \( T_{\infty} \) and \( C_{\infty} \) be, respectively, the equilibrium temperature and equilibrium molar species concentration of the fluid. The geometry of the problem is shown by Figure 1.

The governing equations of the fluid motion are as follows:

\[ \frac{\partial v'}{\partial y'} = 0 \implies v' = -V_0(1 + e^{i\omega t'}), \]

\[ \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta(T' - T_{\infty}) + g\beta(C' - C_{\infty}) + \frac{\partial^2 u'}{\partial y'^2}, \]

\[ -\frac{k_0}{\rho} \left( \frac{\partial^3 u'}{\partial y'^2 \partial t'} + v' \frac{\partial^3 u'}{\partial y'^3} \right) = \frac{\sigma_{B_0} u'}{\rho}, \]

\[ \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2}, \]

\[ \frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2}. \]  

(6)

In fact, nearly two hundred years ago Navier [16] proposed a more general boundary condition, which includes the possibility of fluid slip. Navier's proposed boundary condition assumes that the velocity at a solid surface is proportional to the shear rate at the surface.

The boundary conditions of the problem are

\[ u' \to h' \sigma'_{xy}, \quad T' = T_w + \epsilon(T_w - T_{\infty}) e^{i\omega t'}, \]

\[ C' = C_w + \epsilon(C_w - C_{\infty}) e^{i\omega t'}, \]

\[ u' \to 0, \quad T' \to T_{\infty}, \quad C' \to C_{\infty}, \]

\[ as \ y' \to 0, \quad as \ y' \to \infty, \]  

where

\[ \sigma'_{xy} = \eta_0(\frac{\partial u'}{\partial y'}) - k_0(\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v'(\frac{\partial^3 u'}{\partial y'^3})). \]

3. Method of Solution

The gravitational acceleration is considered as

\[ g = g_0 - ig_1 e^{i\omega t'}, \quad g_1 = e ag_0. \]

(8)
Let us introduce the following nondimensional quantities:

\[ y = \frac{y'}{V_0}, \quad t = \frac{t'}{V_0^2}, \quad \omega = \frac{4\omega'}{V_0^2}, \]

\[ u = \frac{u'}{V_0}, \quad \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C_\infty}{C_w - C_\infty}, \]

\[ Gr = \frac{\gamma_0\beta\nu (T_w - T_\infty)}{V_0^3}, \quad Gr = \frac{\gamma_0\beta\nu (C_w - C_\infty)}{V_0^3}, \]

\[ M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad k = \frac{k_0 V_0^2}{\rho \nu^2}, \]

\[ Pr = \frac{\eta_b C_p}{K}, \quad Sc = \frac{y}{D}, \quad h = h' \rho V_0^2, \]

where \( y \) is the displacement variable, \( t \) is the time, \( \omega \) is the frequency of oscillation, \( u \) is the dimension velocity, \( \theta \) is the dimensionless temperature, \( C \) is the dimensionless concentration, \( Gr \) is Grashof number for heat transfer, \( Gm \) is Grashof number for mass transfer, \( M \) is magnetic parameter, \( k \) is viscoelastic parameter, \( Pr \) is the Prandtl number, \( Sc \) is the Schmidt number, \( a \) is gravity modulation parameter, and \( h \) is the slip parameter.

Then the nondimensional forms of the governing equations of motions are as follows:

\[
\frac{1}{4} \frac{\partial u}{\partial t} + \left( 1 + \epsilon A e^{i\omega t} \right) \frac{\partial u}{\partial y} = (Gr\theta + GmC) \left( 1 - i\epsilon A e^{i\omega t} \right) + \frac{\partial^2 u}{\partial y^2} - Mu
\]

\[
- k \left[ \frac{1}{4} \frac{\partial^2 u}{\partial y^2 \partial t} + \left( 1 + \epsilon A e^{i\omega t} \right) \frac{\partial^2 u}{\partial y^2} \right],
\]

\[
\frac{1}{4} \frac{\partial \theta}{\partial t} + \left( 1 + \epsilon A e^{i\omega t} \right) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2},
\]

\[
\frac{1}{4} \frac{\partial C}{\partial t} + \left( 1 + \epsilon A e^{i\omega t} \right) \frac{\partial C}{\partial y} = \frac{1}{Pr} \frac{\partial^2 C}{\partial y^2}.
\]

Boundary conditions in the dimensionless form are given as follows:

\[
u = h \left[ \frac{\partial u}{\partial y} - k \left( \frac{1}{4} \frac{\partial^2 u}{\partial y^2 \partial t} + \left( 1 + \epsilon A e^{i\omega t} \right) \frac{\partial^2 u}{\partial y^2} \right) \right],
\]

\[
\theta = 1 + \epsilon e^{i\omega t}, \quad C = 1 + \epsilon e^{i\omega t}
\]

at \( y = 0 \),

\[
u \to 0, \quad \theta \to 0, \quad C \to 0,
\]

as \( y \to \infty \).

Assuming small amplitude of oscillations (\( \epsilon \ll 1 \)) in the neighbourhood of the plate, the velocity, temperature, and concentration are considered as

\[
u = u_0 + \epsilon e^{i\omega t} u_1 + o(\epsilon^2),
\]

\[
\theta = \theta_0 + \epsilon e^{i\omega t} \theta_1 + o(\epsilon^2),
\]

\[
C = C_0 + \epsilon e^{i\omega t} C_1 + o(\epsilon^2).
\]

Using (12) in (10) and equating the like powers of \( \epsilon \) and neglecting the higher powers of \( \epsilon \) we get

\[
u'' u'' + u''_0 + u'_0 - Mu_0 = -(Gr\theta_0 + GmC_0),
\]

\[
u'' u'' + u''_0 + u'_0 - \left( M + \frac{i\omega}{4} \right) u'_0 = 0,
\]

\[
u'' = \left( M + \frac{i\omega}{4} \right) u'_0 + \theta_0 = \theta_0 = C_0 = C_1 = 0,
\]

 Relevant boundary conditions for solving the above equations are as follows:

\[
u_0 = h \left[ u_0' + ku''_0 \right],
\]

\[
u_1 = h \left[ u'_0 - k \left( \frac{i\omega}{4} u''_0 - u''_0 \right) \right],
\]

\[
\theta_0 = \theta_1 = C_0 = C_1 = 1,
\]

at \( y = 0 \),

\[
u_0 = u_1 = \theta_0 = \theta_1 = C_0 = C_1 = 0,
\]

as \( y \to \infty \).

Equations (14) are solved by using the boundary conditions (15) and their solutions are given by

\[
\theta_0 = e^{-\alpha y},
\]

\[
\theta_1 = C_1 e^{\alpha_y} + C_2 e^{-\alpha_y} + \frac{i4\Pr}{\omega} e^{\beta y},
\]

\[
C_0 = e^{-\alpha y},
\]

\[
C_1 = C_2 e^{\alpha_y} + C_3 e^{-\alpha_y} + \frac{i4\Sc}{\omega} e^{\beta y}.
\]

The presence of elasticity in the governing fluid motion constitutes a third-order differential equation (13) but for
Newtonian fluid $k = 0$; then the differential equation reduces to of order two. And also it is seen that as there are insufficient boundary conditions for solving (13), we use multiparameter perturbation technique. Since $k$, which is a measure (dimensionless) of relaxation time, is very small for a viscoelastic fluid with small memory, thus following Ray Mahapatra and Gupta [17] and Reza and Gupta [18], we use the multiparameter perturbation technique, and we consider

$$u_0 = u_{00} + ku_{01} + o(k^2),$$

$$u_1 = u_{10} + ku_{11} + o(k^2).$$

Using the perturbation scheme (17) in (13) and equating the like powers of $k$ and neglecting the higher orders power of $k$, we get

$$u''_0 + u'_0 - Mu_0 = - (G_ru_0 + GmC_0),$$

$$u''_0 + u'_0 - Mu_0 = -u''_{00},$$

$$u''_{10} + u'_{10} - (M + i\omega) u_{10} = \alpha (Gr\theta_0 + GmC_0) - Au'_{10} - (Gr\theta_1 + GmC_1),$$

$$u''_{11} + u'_{11} - (M + i\omega) u_{11} = -Au'_{10} - Au''_{00} - u''_{10} - \frac{i\omega}{4} u''_{10}.$$

The modified boundary conditions for solving the above equations are

$$u_{00} = hu_0', \quad u_{01} = h [u_{01}' + u_{00}'], \quad u_{10} = hu_{10}',$$

$$u_{11} = h [u_{11}' - \frac{i\omega}{4} u_{10}' + Au_{00}'' + u_{10}'']$$

at $y = 0$,

$$u_{00} \to 0, \quad u_{01} \to 0, \quad u_{10} \to 0, \quad u_{11} \to 0$$

as $y \to \infty$.

The solutions of (18) relevant to the above boundary conditions are presented as follows:

$$u_{00} = C_6 e^{-\alpha_0 y} + A_6 e^{-A_1 y} + A_3 e^{-A_3 y},$$

$$u_{01} = C_7 e^{-\alpha_0 y} + A_7 e^{-A_2 y} + A_4 e^{-A_3 y} + A_5 e^{-A_4 y},$$

$$u_{10} = C_{12} e^{-\alpha_{10} y} + (A_6 + iA_7) e^{-Pr y}$$

$$+ (A_8 + iA_9) e^{-Sc y} + iA_{10} e^{-\alpha_{10} y},$$

$$u_{11} = C_{14} e^{-\alpha_{11} y} + (A_8 + iA_{13}) e^{-A_1 y} + (A_9 + iA_{14}) e^{-A_2 y}$$

$$+ (A_{15} + iA_{16}) e^{-A_3 y} + (A_{17} + iA_{18}) e^{-A_4 y},$$

$$u_{10} = C_{14} e^{-\alpha_{10} y} + (A_{36} + iA_{37}) e^{-\alpha_{10} y} + iA_{38} ye^{-\alpha_3 y}$$

$$+ (A_{39} + iA_{40}) e^{-A_1 y} + (A_{41} + iA_{42}) e^{-A_2 y}$$

$$+ (A_{43} + iA_{44}) ye^{-A_3 y} + (A_{45} + iA_{46}) e^{-Pr y}$$

$$+ (A_{47} + iA_{48}) e^{-Sc y} + (A_{49} + iA_{50}) e^{-\alpha_{10} y}$$

$$+ (A_{51} + iA_{52}) e^{-\alpha_3 y}.$$

The constants of the solutions are not presented here for the sake of brevity.

Knowing the velocity field, the shearing stress at the plate is defined as

$$Sh = \left[ \frac{\partial u}{\partial y} - k \left\{ \frac{1}{4} \frac{\partial^2 u}{\partial y^2} \right\} \right]_{y=0}.$$

The effects of gravity modulation on free convective flow of an elastico-viscous fluid with simultaneous heat and mass transfer past an infinite vertical porous plate under the influence of transverse magnetic field have been analyzed. Figures 2 to 5 represent the pattern of velocity profile against $y$ for various values of flow parameters involved in the solution. The effect of viscoelasticity is exhibited through the nondimensional parameter “$k$.” The nonzero values of $k$ characterize viscoelastic fluid, where $k = 0$ characterizes Newtonian fluid flow mechanisms. In this study, main emphasis is given on the effect of viscoelasticity on the governing fluid motion and also on the difference in flow pattern of both Newtonian and non-Newtonian fluids in the presence of various physical properties considered in this problem.

It is seen from Figure 2 that both Newtonian and non-Newtonian fluid flows accelerate asymptotically in the neighborhood of the plate and then they experience a decline trend as we move away from the plate. The nonzero value of velocity profile at $y = 0$ represents the strength of slip at the plate. Also, it can be concluded that the growth in viscoelasticity slows down the speed of fluid flow. Effects of gravity modulation parameter $a$ on the governing fluid motions are shown in Figure 3 and it is observed that as $a$ increases, the speed of fluid flow increases along with the increasing values of $k = (0, 0.2$ and $0.4)$. Application of transverse magnetic field leads to the generalization of Lorentz force and its effect is displayed by the nondimensional parameter $M$. Figure 4 shows the impact of magnetic parameter on the fluid flow against the displacement variable $y$. As magnetic parameter increases, the strength of Lorentz force rises and, as a result, the flow is retarded. It is also noticed that, as $M$ decreases, the effect of viscoelasticity is seen prominent.
Grashof number is defined as the ratio of buoyancy force to viscous force and its positive value identifies the flow past an externally cooled plate and the negative value characterizes the flow past an externally heated plate. Figure 5 shows that as Gr increases, the resistance of fluid flow diminishes, and as a consequence the speed enhances along with the increasing values of visco-elastic parameter. Again, when the flow passes an externally heated plate (Gr = −10), a back flow is noticed in case of both Newtonian and non-Newtonian fluids.

4. Results and Discussions

Knowing the velocity field, it is important from a practical point of view to know the effect of viscoelastic parameter on shearing stress or viscous drag. Figures 6 to 8 depict the magnitude of shearing stress at the plate of viscoelastic fluid in comparison with the Newtonian fluid for the various values of flow parameters involved in the solution. The figures notify that the growth in viscoelasticity subdues the magnitude of viscous drag at the surface. Prandtl number plays a significant role in heat transfer flow problems as it helps to study the simultaneous effect of momentum and thermal diffusion in fluid flow. Effect of Prandtl number on the magnitude of shearing stress is seen in Figure 6 and it can be concluded that the magnitude of shearing stress increases rapidly for Pr < 8 of both Newtonian and non-Newtonian fluids, but for higher values of Pr (>8) it increases steadily. In mass transfer problems, the importance of Schmidt number cannot be neglected as it studies the combined effect of momentum and mass diffusion. The same phenomenon as that in case of growth in Prandtl number is experienced against Schmidt number for various values of viscoelastic parameter (Figure 7). Effect of gravity modulation parameter a on the viscous drag is shown in Figure 8 and it is revealed that as a increases, the magnitude of shearing stress increases.

The rate of heat transfer and rate of mass transfer are not significantly affected by the presence of viscoelasticity of the governing fluid flow.

5. Conclusions

From the present study, we make the following conclusions.

(i) Both Newtonian and non-Newtonian fluid flows accelerate asymptotically in the neighbourhood of the plate.

(ii) Growth in viscoelasticity slows down the speed of fluid flow.
(iii) Effect of viscoelasticity is seen prominent during the lower order magnitude of magnetic parameter.

(iv) A back flow is experienced in the motion governing fluid flow past an externally heated plate.

(v) As gravity modulation parameter \( a \) increases, the magnitude of shearing stress experienced by Newtonian as well as non-Newtonian fluid flows increase.

Conflict of Interests

The author does not have any conflict of interests regarding the publication of paper.

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References


\begin{align*}
\text{Figure 6: Sc} &= 7, \text{Gm} = 10, \text{Gr} = 7, M = 2, \gamma = 0.01, a = 1, h = 0.3, \\
\omega &= 5, A = 6, \omega t = \pi.
\end{align*}


