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Research Article

Some Results on Generalized Quasi-Einstein Manifolds

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This paper deals with generalized quasi-Einstein manifold satisfying certain conditions on conharmonic curvature tensor. Here we study some geometric properties of generalized quasi-Einstein manifold and obtain results which reveal the nature of its associated 1-forms.

1. Introduction

It is well known that a Riemannian or a semi-Riemannian manifold (M^n, g) $(n \geq 3)$ is an Einstein manifold if its Ricci tensor S of type $(0,2)$ is of the form $S = (r/n)g$, r being the (constant) scalar curvature of the manifold. Let $U_{S_1} = \{x \in M : S \neq (r/n)g \text{ at } x\}$. Then the manifold (M^n, g) is said to be a quasi-Einstein manifold [1–7] if, on $U_{S_1} \subset M$, we have

$$S = \alpha g + \beta A \otimes A, \tag{1}$$

where A is 1-form on U_{S_1} and α, β are some smooth functions on U_{S_1}. It is clear that the function β and the 1-form A are nonzero at every point on U_{S_1}. The scalars α, β are the associated scalars of the manifold. Also the 1-form A is called associated 1-form of the manifold defined by $g(X, \rho) = A(X)$ for any vector field X, ρ being a unit vector field called generator of the manifold. Such an n-dimensional quasi-Einstein manifold is denoted by $(QE)_n$. The quasi-Einstein manifolds have also been studied in [8–11].

Generalizing the notion of quasi-Einstein manifold, in [12], De and Ghosh introduced the notion of generalized quasi-Einstein manifolds and studied its geometrical properties with the existence of such notion by several nontrivial examples. A Riemannian manifold (M^n, g) $(n > 3)$ is said to be generalized quasi-Einstein manifold if its Ricci tensor S of type $(0,2)$ is not identically zero and satisfies the condition:

$$S(X,Y) = \alpha g(X,Y) + \beta A(X) A(Y) + \gamma B(X) B(Y), \tag{2}$$

where α, β, and γ are scalars of which $\beta \neq 0$, $\gamma \neq 0$ and A, B are nowhere vanishing 1-forms such that $g(X, \rho) = A(X)$, $g(X, \mu) = B(X)$ for any vector field X. The unit vectors ρ and μ corresponding to 1-forms A and B are orthogonal to each other. Also ρ and μ are the generators of the manifold. Such an n-dimensional manifold is denoted by $G(QE)_n$. The generalized quasi-Einstein manifolds have also been studied in [13–16].

In 2008, De and Gazi [17] introduced the notion of nearly quasi-Einstein manifolds. A nonflat Riemannian manifold (M^n, g) $(n > 2)$ is called a nearly quasi-Einstein manifold if its Ricci tensor of type $(0,2)$ is not identically zero and satisfies the condition:

$$S(X,Y) = \alpha g(X,Y) + \beta E(X,Y), \tag{3}$$

where α and β are nonzero scalars and E is a nonzero symmetric tensor of type $(0,2)$. An n-dimensional nearly quasi-Einstein manifold was denoted by $N(QE)_n$. The nearly quasi-Einstein manifolds have also been studied by Prakasha and Bagewadi [18].

The present paper is organized as follows. Section 2 deals with the preliminaries. Section 3 is concerned with conharmonic curvature tensor on $G(QE)_n$. In this section, it is proved that a conharmonically flat $G(QE)_n$ is one of the manifold of generalized quasiconstant curvature. Also, it is proved that if in a $G(QE)_n$ $(n \geq 3)$ the associated scalars are constants and the generators ρ and μ are vector

fields with the associated 1-forms A and B not being the 1-forms of recurrences, then the manifold is conharmonically conservative. In Section 4, we consider $G(QE)_n$ $(n \geq 3)$ satisfying the condition $\mathscr{L} \cdot S = 0$. In the last section we study some geometrical properties of a $G(QE)_n$.

2. Preliminaries

Consider a $G(QE)_n$ with associated scalars α, β, and γ and associated 1-forms A, B. Then from (2) we get

$$r = n\alpha + \beta + \gamma, \tag{4}$$

where r is the scalar curvature of the manifold. Since ρ and μ are orthogonal unit vector fields, $g(\rho, \rho) = 1$, $g(\mu, \mu) = 1$, and $g(\rho, \mu) = 0$. Again from (2), we have

$$\begin{aligned} S(X, \rho) &= (\alpha + \beta) A(X), \qquad & S(\rho, \rho) &= \alpha + \beta, \\ S(X, \mu) &= (\alpha + \gamma) B(X), \qquad & S(\mu, \mu) &= \alpha + \gamma. \end{aligned} \tag{5}$$

Let Q be the symmetric endomorphism of the tangent space at each point of the manifold corresponding to the Ricci tensor S. Then $g(QX, Y) = S(X, Y)$ for all X, Y.

The rank-four tensor $\mathscr{L}'$ that remains invariant under conharmonic transformation for an n-dimensional Riemannian manifold is given by [19]

$$\begin{aligned} &\mathscr{L}'(X, Y, Z, U) \\ &\quad = R'(X, Y, Z, U) - \frac{1}{n-2} \\ &\qquad \times [S(Y, Z) g(X, U) - S(X, Z) g(Y, U) \\ &\qquad\quad + g(Y, Z) S(X, U) - g(X, Z) S(Y, U)], \end{aligned} \tag{6}$$

where $\mathscr{L}'(X, Y, Z, U) = g(\mathscr{L}(X, Y)Z, U)$ and R' denotes the Riemannian curvature tensor of type (0, 4) defined by $R'(X, Y, Z, U) = g(R(X, Y)Z, U)$, where R is the Riemannian curvature tensor of type $(1, 3)$.

3. Conharmonic Curvature Tensor on $G(QE)_n$

A manifold of generalized quasiconstant curvature tensor is $G(QE)_n$. But the converse is not true, in general. In this section, we enquire under what conditions the converse will be true.

A manifold whose conharmonic curvature tensor vanishes at every point of the manifold is called conharmonically flat manifold. Thus this tensor represents the deviation of the manifold from conharmonic flatness. It satisfies all the symmetric properties of the Riemannian curvature tensor R'. There are many physical applications of the tensor $\mathscr{L}$. For example, in [20], Abdussattar showed that sufficient condition for a space-time to be conharmonic to a flat is either empty in which case it is flat or filled with a distribution represented by energy momentum tensor T possessing the algebraic structure of an electromagnetic field and conformal to a flat space-time [20].

Let us consider that the manifold under consideration is conharmonically flat. Then from (6) we have

$$\begin{aligned} &R'(X, Y, Z, U) \\ &\quad = \frac{1}{n-2} [S(Y, Z) g(X, U) - S(X, Z) g(Y, U) \\ &\qquad\quad + g(Y, Z) S(X, U) - g(X, Z) S(Y, U)]. \end{aligned} \tag{7}$$

Using (2) in (7), we obtain

$$\begin{aligned} &R'(X, Y, Z, U) \\ &\quad = \frac{2\alpha}{n-2} [g(Y, Z) g(X, U) - g(X, Z) g(Y, U)] \\ &\qquad + \frac{\beta}{n-2} [g(X, U) A(Y) A(Z) - g(Y, U) A(X) A(Z) \\ &\qquad\quad + g(Y, Z) A(X) A(U) - g(X, Z) A(Y) A(U)] \\ &\qquad + \frac{\gamma}{n-2} [g(X, U) B(Y) B(Z) - g(Y, U) B(X) B(Z) \\ &\qquad\quad + g(Y, Z) B(X) B(U) - g(X, Z) B(Y) B(U)]. \end{aligned} \tag{8}$$

According to Chen and Yano [21], a Riemannian manifold (M^n, g) $(n > 3)$ is said to be of quasiconstant curvature if it is conformally flat and its curvature tensor R' of type $(0, 4)$ has the form:

$$\begin{aligned} &R'(X, Y, Z, U) \\ &\quad = p [g(Y, Z) g(X, U) - g(X, Z) g(Y, U)] \\ &\qquad + q [g(X, U) T(Y) T(Z) - g(Y, U) T(X) T(Z) \\ &\qquad\quad + g(Y, Z) T(X) T(U) - g(X, Z) T(Y) T(U)], \end{aligned} \tag{9}$$

where T is a form defined by $g(X, \rho) = A(X)$ with ρ as a unit vector field and p, q are scalars of which $q \neq 0$.

In [12], De and Ghosh generalize the notion of quasiconstant curvature and prove the existence of such a manifold. A Riemannian manifold is said to be a manifold of generalized quasiconstant curvature, if the curvature tensor R' of type $(0, 4)$ satisfies the condition:

$$\begin{aligned} &R'(X, Y, Z, U) \\ &\quad = p [g(Y, Z) g(X, U) - g(X, Z) g(Y, U)] \\ &\qquad + q [g(X, U) T(Y) T(Z) - g(Y, U) T(X) T(Z) \\ &\qquad\quad + g(Y, Z) T(X) T(U) - g(X, Z) T(Y) T(U)] \\ &\qquad + s [g(X, U) D(Y) D(Z) - g(Y, U) D(X) D(Z) \\ &\qquad\quad + g(Y, Z) D(X) D(U) - g(X, Z) D(Y) D(U)], \end{aligned} \tag{10}$$

where p, q, and s are scalars and T and D are nonzero 1-forms.

We assume that the unit vector fields ρ and ρ_1 defined by $g(X,\rho) = T(X)$ and $g(X,\rho_1) = D(X)$ are orthogonal; that is, $g(\rho,\rho_1) = 0$. Now the relation (8) can be written as

$$\begin{aligned} R'(X,Y,Z,U) &= p_1 [g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \\ &\quad + q_1 [g(X,U)T(Y)T(Z) - g(Y,U)T(X)T(Z) \\ &\qquad + g(Y,Z)T(X)T(U) - g(X,Z)T(Y)T(U)] \\ &\quad + s_1 [g(X,U)D(Y)D(Z) - g(Y,U)D(X)D(Z) \\ &\qquad + g(Y,Z)D(X)D(U) - g(X,Z)D(Y)D(U)], \end{aligned} \tag{11}$$

where $p_1 = (2\alpha/(n-2))$, $q_1 = (\beta/(n-2))$, and $s_1 = (\gamma/(n-2))$. Comparing (10) and (11), it follows that the manifold is of generalized quasiconstant curvature. Thus we have the following theorem.

Theorem 1. *A conharmonically flat $G(QE)_n$ is one of generalized quasiconstant curvature.*

Next, differentiating (6) covariantly and then contracting we obtain

$$\begin{aligned} &(\operatorname{div}\cdot\mathscr{L})(X,Y)Z \\ &= (\operatorname{div}\cdot R)(X,Y)Z - \frac{1}{n-2}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)] \\ &\quad - \frac{1}{2(n-2)}[(\nabla_X r)g(Y,Z) - (\nabla_Y r)g(X,Z)], \end{aligned} \tag{12}$$

where div denotes the divergence.

Again, it is known that in a Riemannian manifold we have

$$(\operatorname{div}\cdot R)(X,Y)Z = (\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z). \tag{13}$$

Consequently by virtue of (13), the relation (12) takes the form:

$$\begin{aligned} &(\operatorname{div}\cdot\mathscr{L})(X,Y)Z \\ &= \frac{n-3}{n-2}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)] \\ &\quad - \frac{1}{2(n-2)}[(\nabla_X r)g(Y,Z) - (\nabla_Y r)g(X,Z)]. \end{aligned} \tag{14}$$

Now consider the associated scalars α, β, and γ as constants; then (4) yields that the scalar curvature r is constant, and hence $dr(X) = 0$ for all X. Consequently, (14) reduces to

$$(\operatorname{div}\cdot\mathscr{L})(X,Y)Z = \frac{n-3}{n-2}[(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)]. \tag{15}$$

Since α, β, and γ are constants, we have from (2) that

$$\begin{aligned} (\nabla_X S)(Y,Z) &= \beta[(\nabla_X A)(Y)A(Z) + A(Y)(\nabla_X A)(Z)] \\ &\quad + \gamma[(\nabla_X A)(Y)A(Z) + A(Y)(\nabla_X A)(Z)]. \end{aligned} \tag{16}$$

By virtue of (16), we get from (15) that

$$\begin{aligned} &(\operatorname{div}\cdot\mathscr{L})(X,Y)Z \\ &= \frac{n-3}{n-2}[\beta\{(\nabla_X A)(Y)A(Z) + A(Y)(\nabla_X A)(Z) \\ &\qquad - (\nabla_Y A)(X)A(Z) - A(X)(\nabla_Y A)(Z)\}] \\ &\quad + \gamma\{(\nabla_X B)(Y)B(Z) + B(Y)(\nabla_X B)(Z) \\ &\qquad - (\nabla_Y B)(X)B(Z) - B(X)(\nabla_Y B)(Z)\}. \end{aligned} \tag{17}$$

Next, if the generators ρ and μ of the manifold under consideration are recurrent vector fields [22], then we have $\nabla_X\rho = \pi_1(X)\rho$ and $\nabla_X\mu = \pi_2(X)\mu$, where π_1 and π_2 are the 1-forms of the recurrence such that π_1 and π_2 are different from A and B. Consequently, we get

$$\begin{aligned} (\nabla_X A)(Y) &= g(\nabla_X\rho, Y) = g(\pi_1(X)\rho, Y) = \pi_1(Y)A(Z), \\ (\nabla_X B)(Y) &= g(\nabla_X\mu, Y) = g(\pi_2(X)\mu, Y) = \pi_2(Y)B(Z). \end{aligned} \tag{18}$$

In view of (18), relation (17) reduces to

$$\begin{aligned} &(\operatorname{div}\cdot\mathscr{L})(X,Y)Z \\ &= \frac{2(n-3)}{(n-2)}[\beta\{\pi_1(X)A(Y) - \pi_1(Y)A(X)\}A(Z) \\ &\qquad \times\gamma\{\pi_2(X)B(Y) - \pi_2(Y)B(X)\}B(Z)]. \end{aligned} \tag{19}$$

Also, since $g(\rho,\rho) = g(\mu,\mu) = 1$, it follows that $(\nabla_X A)(\rho) = g(\nabla_X\rho,\rho) = 0$, and hence (18) reduces to $\pi_1(X) = 0$ for all X. Similarly, we have $\pi_2(X) = 0$. Hence, from (19), we have $(\operatorname{div}\cdot\mathscr{L})(X,Y)Z = 0$; that is, the manifold under consideration is conharmonically conservative. Hence, we can state the following theorem.

Theorem 2. *If, in a $G(QE)_n$ $(n > 3)$, the associated scalars are constants and the generators ρ and μ corresponding to the associated 1-forms A and B are not being the recurrence 1-forms, then the manifold is conharmonically conservative.*

4. $G(QE)_n$ $(n > 3)$ Satisfying the Condition $\mathscr{L}\cdot S = 0$

Let us consider a $G(QE)_n$ $(n > 3)$, satisfying the condition $\mathscr{L}\cdot S = 0$. Then we have

$$S(\mathscr{L}(X,Y)Z,U) + S(Z,\mathscr{L}(X,Y)U) = 0. \tag{20}$$

Setting $Z = \rho$ and $U = \mu$ in (20), we have

$$S(\mathscr{L}(X,Y)\rho,\mu) + S(\rho,\mathscr{L}(X,Y)\mu) = 0. \tag{21}$$

By using (5) in (21), we obtain

$$(\alpha+\gamma)B(\mathscr{L}(X,Y)\rho) + (\alpha+\beta)A(\mathscr{L}(X,Y)\mu) = 0. \tag{22}$$

Next, putting $Z = \rho$ in (6) and then taking inner product with μ, we get

$$\begin{aligned}\mathscr{L}'(X,Y,\rho,\mu) &= R'(X,Y,\rho,\mu) - \frac{1}{n-2} \\ &\quad \times [S(Y,\mu)g(X,\rho) - S(X,\mu)g(Y,\rho) \\ &\quad + g(Y,\mu)S(X,\rho) - g(X,\mu)S(Y,\rho)],\end{aligned} \tag{23}$$

where $g(\mathscr{L}(X,Y)\rho,\mu) = \mathscr{L}'(X,Y,\rho,\mu) = B(\mathscr{L}(X,Y)\rho)$. Again from (5) we obtain from (23)

$$\begin{aligned}\mathscr{L}'(X,Y,\rho,\mu) &= R'(X,Y,\rho,\mu) \\ &\quad - \frac{2\alpha+\beta+\gamma}{(n-2)}[A(Y)B(X) - A(X)B(Y)].\end{aligned} \tag{24}$$

Again, plugging $Z = \mu$ in (6) and then taking inner product with ρ, we have

$$\begin{aligned}\mathscr{L}'(X,Y,\mu,\rho) &= R'(X,Y,\mu,\rho) \\ &\quad - \frac{2\alpha+\beta+\gamma}{(n-2)}[A(X)B(Y) - A(Y)B(X)],\end{aligned} \tag{25}$$

where $g(\mathscr{L}(X,Y)\mu,\rho) = \mathscr{L}'(X,Y,\mu,\rho) = A(\mathscr{L}(X,Y)\mu)$. From (24) and (25), one can get

$$A(\mathscr{L}(X,Y)\mu) = -B(\mathscr{L}(X,Y)\rho). \tag{26}$$

By taking account of (26) in (22), we obtain

$$(\gamma-\beta)B(\mathscr{L}(X,Y)\mu) = 0. \tag{27}$$

This implies either $\gamma = \beta$ or $B(\mathscr{L}(X,Y)\mu) = 0$.

Now if $\gamma = \beta$, then, from (2), we have

$$\begin{aligned}S(X,Y) &= \alpha g(X,Y) + \beta[A(X)A(Y) + B(X)B(Y)] \\ &= \alpha g(X,Y) + \beta E(X,Y),\end{aligned} \tag{28}$$

where $E(X,Y) = A(X)A(Y)+B(X)B(Y)$. That is, the manifold is a $N(QE)_n$.

On the other hand, if $B(L(X,Y)\rho) = 0$, then (24) gives

$$R'(X,Y,\rho,\mu) = \frac{(2\alpha+\beta+\gamma)}{(n-2)}[A(Y)B(X) - A(X)B(Y)]. \tag{29}$$

Thus we can state the following theorem.

Theorem 3. *If M is a $G(QE)_n$ $(n > 3)$ satisfying the condition $\mathscr{L}\cdot S = 0$, then either M is $N(QE)_n$ or the curvature tensor R of the manifold satisfies the property* (29).

5. Some Geometric Properties of $G(QE)_n$

In [23], Gray introduced two classes of Riemannian manifolds determined by the covariant differentiation of Ricci tensor. Class A consists of all Riemannian manifolds whose Ricci tensor S satisfies the condition:

$$(\nabla_X S)(Y,Z) = (\nabla_Y S)(X,Z); \tag{30}$$

that is, Ricci tensor S is a Codazzi type tensor.

Class B consists of all Riemannian manifolds whose Ricci tensor S satisfies the condition:

$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0; \tag{31}$$

that is, Ricci tensor S is cyclic parallel.

First suppose that the associated scalars are constants and the Ricci tensor S is of Codazzi type. Then from (2) we obtain

$$\begin{aligned}(\nabla_X S)(Y,Z) &= \beta[(\nabla_X A)(Y)A(Z) + (\nabla_X A)(Z)A(Y)] \\ &\quad + \gamma[(\nabla_X B)(Y)B(Z) + (\nabla_X B)(Z)B(Y)].\end{aligned} \tag{32}$$

Interchanging X and Y in (32), we have

$$\begin{aligned}(\nabla_Y S)(X,Z) &= \beta[(\nabla_Y A)(X)A(Z) + (\nabla_Y A)(Z)A(X)] \\ &\quad + \gamma[(\nabla_Y B)(X)B(Z) + (\nabla_Y B)(Z)B(X)].\end{aligned} \tag{33}$$

Since S is of Codazzi type, we have from (32) and (33) that

$$\begin{aligned}&\beta[(\nabla_X A)(Y)A(Z) + (\nabla_X A)(Z)A(Y)] \\ &\quad + \gamma[(\nabla_X B)(Y)B(Z) + (\nabla_X B)(Z)B(Y)] \\ &= \beta[(\nabla_Y A)(X)A(Z) + (\nabla_Y A)(Z)A(X)] \\ &\quad + \gamma[(\nabla_Y B)(X)B(Z) + (\nabla_Y B)(Z)B(X)].\end{aligned} \tag{34}$$

Putting $Z = \rho$ in (34) and using $(\nabla_X A)(\rho) = 0$ and $(\nabla_X B)(\rho) = 0$, since ρ is a unit vector, we get

$$(\nabla_X B)(Y) - (\nabla_Y B)(X) = 0. \tag{35}$$

This shows that $dB(X,Y) = 0$.

Again, putting $Z = \mu$ in (34) and using $(\nabla_X A)(\mu) = 0$ and $(\nabla_X B)(\mu) = 0$, since μ is a unit vector, we get

$$(\nabla_X A)(Y) - (\nabla_Y A)(X) = 0. \tag{36}$$

This shows that $dA(X,Y) = 0$.

Thus we can state the following theorem.

Theorem 4. *In a $G(QE)_n$, if the associated scalars are constants and the Ricci tensor is of Codazzi type, then the associated 1-forms A and B are closed.*

Next, suppose that the generators ρ and μ are Killing vector fields in a $G(QE)_n$ and the associated scalars are constants. Then

$$(\mathbb{L}_U g)(X,Y) = 0, \qquad (\mathbb{L}_V g)(X,Y) = 0, \tag{37}$$

where $\mathbb{L}$ denotes the Lie derivative, which implies that

$$g(\nabla_X U, Y) + g(X, \nabla_Y U) = 0,$$
$$g(\nabla_X V, Y) + g(X, \nabla_Y V) = 0. \tag{38}$$

From (38) it follows that

$$(\nabla_X A)(Y) + (\nabla_Y A)(X) = 0,$$
$$(\nabla_X B)(Y) + (\nabla_Y B)(X) = 0, \tag{39}$$

since $g(\nabla_X U, Y) = (\nabla_X A)(Y)$ and $g(\nabla_X V, Y) = (\nabla_X B)(Y)$.

Similarly, we have

$$(\nabla_X A)(Z) + (\nabla_Z A)(X) = 0,$$
$$(\nabla_X B)(Z) + (\nabla_Z B)(X) = 0,$$
$$(\nabla_Y A)(Z) + (\nabla_Z A)(Y) = 0,$$
$$(\nabla_Y B)(Z) + (\nabla_Z B)(Y) = 0 \tag{40}$$

for all X, Y, and Z. Now from (2) and using the relations (39)-(40) we have

$$(\nabla_X S)(Y, Z) + (\nabla_Y S)(Z, X) + (\nabla_Z S)(X, Y) = 0. \tag{41}$$

Therefore we can state the following theorem.

Theorem 5. *In a $G(QE)_n$, if the generators are Killing vector fields and the associated scalars are constants, then the Ricci tensor of the manifold is cyclic parallel.*

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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