Research Article

On Some Bounds and Exact Formulae for Connective Eccentric Indices of Graphs under Some Graph Operations

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The connective eccentric index of a graph is a topological index involving degrees and eccentricities of vertices of the graph. In this paper, we have studied the connective eccentric index for double graph and double cover. Also we give the connective eccentric index for some graph operations such as joins, symmetric difference, disjunction, and splice of graphs.

1. Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. Let n and m be the number of vertices and edges of G , respectively. We denote the degree of a vertex v of G by $d_G(v)$. For $u, v \in V(G)$, the distance between u and v in G is defined as the length of the shortest path between u and v in G and is denoted by $d_G(u, v)$. For a given vertex v of G , the eccentricity $\varepsilon_G(v)$ is the largest distance from v to any other vertices of G . The sum of eccentricities of all the vertices of G is denoted by $\theta(G)$ [1]. If any vertex $v \in V(G)$ is adjacent to all the other vertices of G then v is called a well-connected vertex. Thus, if $v \in V(G)$ is a well-connected vertex, then $\varepsilon_G(v) = 1$. For example, all the vertices of a complete graph are well connected.

Recently, a number of topological indices involving vertex degree and eccentricity were subject to a lot of mathematical as well as chemical studies. A topological index of this type, introduced by Gupta et al. [2], was named as the connective eccentric index and was defined as

$$C^\xi(G) = \sum_{v \in V(G)} \frac{d_G(v)}{\varepsilon_G(v)}. \quad (1)$$

Ghorbani [3] gave some bounds of connective eccentricity index and also computed this index for two infinite classes of

dendrimers. De [4] reported some bounds for this index in terms of some graph invariants such as maximum and minimum degree, radius, diameter, first Zagreb index, and first Zagreb eccentricity index. In [5], Ghorbani and Malekjani computed the eccentric connectivity index and the connective eccentric index of an infinite family of fullerenes. In [6], Yu and Feng also derived some upper or lower bounds for the connective eccentric index and investigated the maximal and the minimal values of connective eccentricity index among all n -vertex graphs with fixed number of pendent vertices.

In [3], Ghorbani showed that, for a vertex transitive graph G , the connective eccentric index is given by

$$C^\xi(G) = \sum_{j=1}^t |A_j| \frac{d_G(x_i)}{\varepsilon_G(x_i)}, \quad (2)$$

where $A_1, A_2, \dots, A_t$ are the orbits of $\text{Aut}(G)$ under its natural action on $V(G)$ and $x_i \in A_i$, $1 \leq i \leq t$. In particular, if G is a regular graph, then $C^\xi(G) = n\delta/r(G)$, where n is the number of vertices of G , which is a δ -regular graph, and $r(G)$ is the radius of G . Let K_n , C_n , Q_m , Π_m , A_m denote the complete graph with n vertices, the cycle on n vertices, m -dimensional hypercube, m -sided prism, and the m -sided antiprism, respectively. It can be easily verified that

the explicit formulae for the connective eccentric index of K_n , C_n , Q_m , Π_m , A_m are as follows.

Proposition 1. Consider the following

$$C^\xi(K_n) = n(n-1). \quad (3)$$

Proposition 2. Consider the following

$$C^\xi(C_n) = \frac{2n}{\lfloor n/2 \rfloor}. \quad (4)$$

Proposition 3. Consider the following

$$C^\xi(Q_m) = 2^m. \quad (5)$$

Proposition 4. Consider the following

$$C^\xi(\Pi_m) = \begin{cases} \frac{12m}{m+2}, & \text{when } m \text{ is even,} \\ \frac{12m}{m+1}, & \text{when } m \text{ is odd.} \end{cases} \quad (6)$$

Proposition 5. Consider the following

$$C^\xi(A_m) = \begin{cases} 16, & \text{when } m \text{ is even,} \\ \frac{16m}{m+1}, & \text{when } m \text{ is odd.} \end{cases} \quad (7)$$

Several studies on different topological indices related to graph operations of different kinds are available in the literature [7–11].

In this paper, first we calculate connective eccentric index of double graph and double cover and hence the explicit formulae for the connective eccentric indices of join, symmetric difference, disjunction, and splice of graphs are obtained. For the definitions and different results on graph operations, such as join, symmetric difference, and disjunction, readers are referred to the book of Imrich and Klavžar [12].

2. Main Results

In this section, first we define and then compute eccentric connectivity index of double graph and double cover graph.

2.1. Connective Eccentric Index of Double Graph and Double Cover. Let us denote the double graph of a graph G by G^* , which is constructed from two copies of G in the following manner [13, 14]. Let the vertex set of G be $V(G) = \{v_1, v_2, \dots, v_n\}$, and the vertices of G^* are given by the two sets $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$. Thus, for each vertex $v_i \in V(G)$, there are two vertices x_i and y_i in $V(G^*)$. The double graph G^* includes the initial edge set of each copy of G , and, for any edge $v_i v_j \in E(G)$, two more edges $x_i y_j$ and $x_j y_i$ are added. The graph P_3 and its double graph P_3^* is shown in Figure 1.

Theorem 6. The connective eccentric index of the double graph G^* is given by $C^\xi(G^*) = 4C^\xi(G) - 2(n-1)\|n-1\|_G$, where $\|n-1\|_G$ is the number of vertices with degree $n-1$, that is, of eccentricity one.

FIGURE 1: The graph P_3 and its double graph P_3^* .

Proof. From the construction of double graph, it is clear that $d_{G^*}(x_i) = d_{G^*}(y_i) = 2d_G(v_i)$, where $v_i \in V(G)$ and $x_i, y_i \in V(G^*)$ are the corresponding clone vertices of v_i . Also we can write $\varepsilon_{G^*}(x_i) = \varepsilon_{G^*}(y_i) = \varepsilon_G(v_i)$ when $\varepsilon_G(v_i) \geq 2$ and $\varepsilon_{G^*}(x_i) = \varepsilon_{G^*}(y_i) = \varepsilon_G(v_i) + 1$ when $\varepsilon_G(v_i) = 1$.

Thus the connective eccentric index of double graph G^* is

$$\begin{aligned} C^\xi(G^*) &= \sum_{i=1}^n \frac{d_{G^*}(x_i)}{\varepsilon_{G^*}(x_i)} + \sum_{i=1}^n \frac{d_{G^*}(y_i)}{\varepsilon_{G^*}(y_i)} \\ &= 2 \left[\sum_{\varepsilon_G(v_i) \geq 2} \frac{2d_G(v_i)}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i)=1} \frac{2d_G(v_i)}{\varepsilon_G(v_i)+1} \right] \\ &= 4 \left[\sum_{\varepsilon_G(v_i) \geq 2} \frac{d_G(v_i)}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i)=1} \frac{d_G(v_i)}{2} \right] \\ &= 4 \left[\sum_{\varepsilon_G(v_i) \geq 2} \frac{d_G(v_i)}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i)=1} \frac{d_G(v_i)}{1} \right] \\ &\quad - 2 \sum_{\varepsilon_G(v_i)=1} d_G(v_i) \\ &= 4C^\xi(G) - 2(n-1)\|n-1\|_G. \end{aligned} \quad (8)$$

□

Let $G = (V, E)$ be a simple connected graph with $V = \{v_1, v_2, \dots, v_n\}$. The extended double cover of G , denoted by G^{**} , is the bipartite graph with bipartition (X, Y) where $X = \{x_1, x_2, \dots, x_n\}$ and $Y = \{y_1, y_2, \dots, y_n\}$ in which x_i and y_j are adjacent if and only if either v_i and v_j are adjacent in G or $i = j$. For example, the extended double cover of the complete graph is the complete bipartite graph. This construction of the extended double cover was introduced by Alon [13] in 1986. Extended double cover of P_3 is illustrated in Figure 2.

Theorem 7. The connective eccentric index of the extended double cover G^{**} satisfies the inequality

$$C^\xi(G^{**}) < 2C^\xi(G) - n(\|n-1\|_G - 2). \quad (9)$$

Proof. If G is a graph with n vertices and m edges, then, from definition, the extended double cover graph G^{**} consists of $2n$ vertices and $(n+2m)$ edges and $d_{G^{**}}(x_i) = d_{G^{**}}(y_i) = d_G(v_i) + 1$ and $\varepsilon_{G^{**}}(x_i) = \varepsilon_{G^{**}}(y_i) = \varepsilon_G(v_i) + 1$ for $i = 1, 2, \dots, n$.

FIGURE 2: The graph P_3 and its extended double cover P_3^{**} .

Thus the connective eccentric index of extended double cover graph G^{**} is given by

$$\begin{aligned}
 C^\xi(G^{**}) &= \sum_{i=1}^n \frac{d_{G^{**}}(x_i)}{\varepsilon_{G^{**}}(x_i)} + \sum_{i=1}^n \frac{d_{G^{**}}(y_i)}{\varepsilon_{G^{**}}(y_i)} \\
 &= 2 \left[\sum_{\varepsilon_G(v_i) > 1} \frac{d_G(v_i) + 1}{\varepsilon_G(v_i) + 1} + \sum_{\varepsilon_G(v_i) = 1} \frac{d_G(v_i) + 1}{2} \right] \\
 &< 2 \sum_{\varepsilon_G(v_i) > 1} \frac{d_G(v_i) + 1}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i) = 1} \{d_G(v_i) + 1\} \\
 &= 2 \left[\sum_{\varepsilon_G(v_i) > 1} \frac{d_G(v_i)}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i) > 1} \frac{1}{\varepsilon_G(v_i)} \right] \\
 &\quad + \sum_{\varepsilon_G(v_i) = 1} d_G(v_i) + \sum_{\varepsilon_G(v_i) = 1} 1 \\
 &= 2 \left[\sum_{\varepsilon_G(v_i) > 1} \frac{d_G(v_i)}{\varepsilon_G(v_i)} + \sum_{\varepsilon_G(v_i) = 1} d_G(v_i) \right] \\
 &\quad + 2 \sum_{\varepsilon_G(v_i) > 1} \frac{1}{\varepsilon_G(v_i)} - \sum_{\varepsilon_G(v_i) = 1} d_G(v_i) + \sum_{\varepsilon_G(v_i) = 1} 1 \\
 &= 2C^\xi(G) + 2\{n - \|n - 1\|_G\} - (n - 1)\|n - 1\|_G \\
 &\quad + \|n - 1\|_G \\
 &= 2C^\xi(G) - n\{\|n - 1\|_G - 2\}.
 \end{aligned} \tag{10}$$

□

Now some exact formulae for the eccentric connectivity index of joins, symmetric difference, disjunction, and splice graphs are presented.

2.2. Join. The join $G_1 + G_2$ of two graphs G_1 and G_2 , with disjoint vertex sets V_1, V_2 and edge sets E_1, E_2 , is the graph union $G_1 \cup G_2$ together with all the edges joining V_1 and V_2 ; that is, $G_1 + G_2$ consists of the vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2 \cup \{xy : x \in V_1, y \in V_2\}$.

Theorem 8. Let G_1 and G_2 be two graphs without well-connected vertices. Then, $C^\xi(G_1 + G_2) = |E(G_1)| + |E(G_2)| + |V(G_1)||V(G_2)|$.

Proof. Let n_1 and n_2 be the numbers of vertices in G_1 and G_2 , respectively. Thus, $|V(G_1 + G_2)| = |V_1| + |V_2| = n_1 + n_2$. For vertices $u \in V_1$ and $v \in V_2$, it holds that $d_{G_1+G_2}(u) = d_{G_1}(u) + n_2$ and $d_{G_1+G_2}(v) = d_{G_2}(v) + n_1$. Since none of G_1 and G_2 contains well-connected vertices, then, for every $u \in V(G_1 + G_2)$, $\varepsilon_{G_1+G_2}(u) = 2$. So, from definition of connective eccentric index, we have

$$\begin{aligned}
 C^\xi(G_1 + G_2) &= \sum_{v \in V(G_1)} \frac{d_{G_1+G_2}(v)}{\varepsilon_{G_1+G_2}(v)} + \sum_{v \in V(G_2)} \frac{d_{G_1+G_2}(v)}{\varepsilon_{G_1+G_2}(v)} \\
 &= \frac{1}{2} \left[\sum_{v \in V(G_1)} \{d_{G_1}(v) + n_2\} + \sum_{v \in V(G_2)} \{d_{G_2}(v) + n_1\} \right]
 \end{aligned} \tag{11}$$

from where the desired result follows. □

Now we generalize the above result for n disjoint graphs $G_1, G_2, \dots, G_n$.

Theorem 9. Let $G_1, G_2, \dots, G_n$ be n graphs with disjoint vertex sets $V_i = V(G_i)$ and edge sets $E_i = E(G_i)$, $1 \leq i \leq n$, without well-connected vertices. Then,

$$\begin{aligned}
 C^\xi(G_1 + G_2 + \dots + G_n) &= \sum_{i=1}^n |E(G_i)| + \frac{1}{2} \left\{ \sum_{i=1}^n |V(G_i)| \right\}^2 \\
 &\quad - \frac{1}{2} \sum_{i=1}^n |V(G_i)|^2.
 \end{aligned} \tag{12}$$

Proof. From definition of join, we have $d_{G_1+G_2+\dots+G_n}(v_j) = d_{G_i}(v_j) + |V(G_1 + G_2 + \dots + G_n)| - |V(G_i)|$ and since none of $G_1, G_2, \dots, G_n$ contains any well-connected vertex, we have $\varepsilon_{G_1+G_2+\dots+G_n}(v_j) = 2$ for all $v_j \in V(G_i)$.

Thus the connective eccentric index of $G_1 + G_2 + \dots + G_n$ is given by

$$\begin{aligned}
 C^\xi(G_1 + G_2 + \dots + G_n) &= \sum_{i=1}^n \sum_{j=1}^{|V(G_i)|} \frac{d_{G_1+G_2+\dots+G_n}(v_j)}{\varepsilon_{G_1+G_2+\dots+G_n}(v_j)} \\
 &= \sum_{i=1}^n \sum_{j=1}^{|V(G_i)|} \frac{d_{G_i}(v_j) + |V(G_1 + G_2 + \dots + G_n)| - |V(G_i)|}{2} \\
 &= \frac{1}{2} [|E(G_1)| + |E(G_2)| + \dots + |E(G_n)| \\
 &\quad + |V(G_1 + G_2 + \dots + G_n)| \\
 &\quad \times \{|V(G_1)| + |V(G_2)| + \dots + |V(G_n)|\} \\
 &\quad - \{|V(G_1)|^2 + |V(G_2)|^2 + \dots + |V(G_n)|^2\}]
 \end{aligned} \tag{13}$$

from where the desired result follows. □

FIGURE 3: The complete n -partite graph.

The following corollaries are direct consequences of the theorem.

Corollary 10. If nG denotes the join of n copies of G , then

$$C^{\xi}(nG) = n|E(G)| + \frac{n(n-1)}{2}|V(G)|^2. \quad (14)$$

Corollary 11. Let $G = K_{m_1, m_2, \dots, m_n}$ be the complete n -partite graph [see Figure 3] having $v = |V(G)|$ number of vertices. Here the vertex set can be partitioned into n subsets $V_1, V_2, \dots, V_n$ such that $V_1 \cup V_2 \cup \dots \cup V_n = V(G)$ and $V_i \cap V_j = \emptyset, i \neq j, 1 \leq i, j \leq n$. From definition of join it is clear that $G = K_{m_1, m_2, \dots, m_n}$ is the join of n empty graphs with $m_1, m_2, \dots, m_n$ number of vertices. Then the connective eccentric index of $G = K_{m_1, m_2, \dots, m_n}$ is given by

$$C^{\xi}(K_{m_1, m_2, \dots, m_n}) = \sum_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n \\ i \neq j}} m_i m_j. \quad (15)$$

2.3. Symmetric Difference. Let G_1 and G_2 be two graphs with vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$. Then the symmetric difference of G_1 and G_2 , denoted by $G_1 \oplus G_2$, is the graph with vertex set $V(G_1) \times V(G_2)$ in which any two vertices (u_1, u_2) are adjacent to (v_1, v_2) whenever u_1 is adjacent to v_1 in G_1 or u_2 is adjacent to v_2 in G_2 , but not both. From definition of symmetric difference, the degree of a vertex (u_1, u_2) of $G_1 \oplus G_2$ is given by [12]

$$d_{G_1 \oplus G_2}(u_1, u_2) = |V(G_2)|d_{G_1}(u_1) + |V(G_1)|d_{G_2}(u_2) - 2d_{G_1}(u_1)d_{G_2}(u_2). \quad (16)$$

Theorem 12. The connective eccentric index of the symmetric difference $G_1 \oplus G_2$ of two graphs G_1 and G_2 is given by

$$C^{\xi}(G_1 \oplus G_2) = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)|^2 - 4|E(G_1)||E(G_2)|, \quad (17)$$

where none of G_1 and G_2 contains well-connected vertices.

Proof. Since the distance between any two vertices of a symmetric difference cannot exceed two, if none of the components contains well-connected vertices, the eccentricity of all vertices is constant and is equal to two; that is, $\varepsilon_{G_1 \oplus G_2}(u_1, u_2) = 2$, for all vertices (u_1, u_2) [7, 8].

Thus the connective eccentric index of symmetric difference $G_1 \oplus G_2$ of two graphs G_1 and G_2 is given by

$$\begin{aligned} C^{\xi}(G_1 \oplus G_2) &= \sum_{(u_1, u_2) \in V(G_1 \oplus G_2)} \frac{d_{G_1 \oplus G_2}(u_1, u_2)}{\varepsilon_{G_1 \oplus G_2}(u_1, u_2)} \\ &= \frac{1}{2} \sum_{u_1 \in V(G_1)} \sum_{u_2 \in V(G_2)} [|V(G_2)|d_{G_1}(u_1) + |V(G_1)|d_{G_2}(u_2) \\ &\quad - 2d_{G_1}(u_1)d_{G_2}(u_2)] \\ &= \frac{1}{2} \left[|V(G_2)|^2 \sum_{u_1 \in V(G_1)} d_{G_1}(u_1) + |V(G_1)|^2 \sum_{u_2 \in V(G_2)} d_{G_2}(u_2) \right. \\ &\quad \left. - 2 \sum_{u_1 \in V(G_1)} d_{G_1}(u_1) \sum_{u_2 \in V(G_2)} d_{G_2}(u_2) \right] \end{aligned} \quad (18)$$

from where the desired result follows. $\square$

2.4. Disjunction. The disjunction $G_1 \vee G_2$ of two graphs G_1 and G_2 is the graph with vertex set $V(G_1) \times V(G_2)$ in which (u_1, u_2) is adjacent to (v_1, v_2) whenever u_1 is adjacent to v_1 in G_1 or u_2 is adjacent to v_2 in G_2 . Obviously, the degree of a vertex (u_1, u_2) of $G_1 \vee G_2$ is given by [7, 8]

$$d_{(G_1 \vee G_2)}(u_1, u_2) = [|V(G_2)|d_{G_1}(u_1) + |V(G_1)|d_{G_2}(u_2) - d_{G_1}(u_1)d_{G_2}(u_2)]. \quad (19)$$

Theorem 13. The connective eccentric index of the disjunction $G_1 \vee G_2$ of two graphs G_1 and G_2 is given by

$$C^{\xi}(G_1 \vee G_2) = |E(G_1)||V(G_2)|^2 + |E(G_2)||V(G_1)|^2 - 2|E(G_1)||E(G_2)|, \quad (20)$$

where none of G_1 and G_2 contains well-connected vertices.

Proof. Since the distance between any two vertices of a disjunction cannot exceed two, if none of the components contains well-connected vertices, the eccentricity of all vertices is constant and equal to two [7, 8]. Then the connective

eccentric index of the disjunction $G_1 \vee G_2$ of two graphs G_1 and G_2 is computed as

$$\begin{aligned} C^\xi(G_1 \vee G_2) &= \sum_{(u,v_j) \in V(G_1 \vee G_2)} \frac{d_{G_1 \vee G_2}(u_1, u_2)}{\varepsilon_{G_1 \vee G_2}(u_1, u_2)} \\ &= \frac{1}{2} \sum_{u_1 \in V(G_1)} \sum_{v_2 \in V(G_2)} [|V(G_2)| d_{G_1}(u_1) + |V(G_1)| d_{G_2}(u_2) \\ &\quad - d_{G_1}(u_1) d_{G_2}(u_2)] \end{aligned} \quad (21)$$

from where the desired result follows. $\square$

2.5. Splice. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs $V_1 \cap V_2 = \emptyset$. Let $v_1 \in V_1$ and $v_2 \in V_2$ be two given vertices of G_1 and G_2 , respectively. A splice of G_1 and G_2 at the vertices v_1 and v_2 is obtained by identifying the vertices v_1 and v_2 in the union of G_1 and G_2 and is denoted by $S(G_1, G_2, v_1, v_2)$ [15]. Different topological indices of the splice graphs have already been computed [16]. Let $u \in V(S)$; then $\varepsilon_1(u)$ denotes the eccentricity of u as a vertex of G_1 , $\varepsilon_2(u)$ denotes the eccentricity of u as a vertex of G_2 , and $\varepsilon(u)$ denotes the eccentricity of u as a vertex of S . Let $d_S(x)$ be the degree of the vertex $x \in V(S)$. Then the connective eccentric index of $S(G_1, G_2, v_1, v_2)$ is computed as follows.

Theorem 14. *The connective eccentric index of splice of G_1 and G_2 is given by*

$$\begin{aligned} C^\xi(S) &= \sum_{x \in V_1} \frac{d_S(x)}{\max\{d_{G_1}(x, v_1) + \varepsilon_1(x) \varepsilon_2(v_2)\}} \\ &\quad + \sum_{y \in V_2} \frac{d_S(y)}{\max\{d_{G_2}(y, v_2) + \varepsilon_1(v_1) \varepsilon_2(y)\}}. \end{aligned} \quad (22)$$

Proof. For any vertex $x \in V_1$, Sharafadini and Gutman [16] showed that

$$\varepsilon(x) = \max\{d_{G_1}(x, v_1) + \varepsilon_1(x) \varepsilon_2(v_2)\}. \quad (23)$$

Similarly, for any vertex $y \in V_2$,

$$\varepsilon(y) = \max\{d_{G_2}(y, v_2) + \varepsilon_1(v_1) \varepsilon_2(y)\}. \quad (24)$$

Thus, the desired result follows from the definition of connective eccentric index. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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