Research Article

New Approach on Robust and Reliable Decentralized $H_\infty$ Tracking Control for Fuzzy Interconnected Systems with Time-Varying Delay

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This paper investigates the robust and reliable decentralized $H_\infty$ tracking control issue for the fuzzy large-scale interconnected systems with time-varying delay, which are composed of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then, according to the Lyapunov direct method and the decentralized control theory of large-scale interconnected systems, the new linear matrix inequalities (LMIs)-based conditions with some free variables are derived to guarantee the $H_\infty$ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Moreover, there is no need for the precise failure parameters of the actuators, rather than the lower and upper bound. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Large-scale interconnected systems, such as electrical power systems, computer communication systems, economic systems, and process control systems, have attracted great interests from many researchers in recent years. Takagi-Sugeno (T-S) fuzzy model has become a popular and effective approach to control complex systems, and a lot of significant results on stabilization and $H_\infty$ control via linear matrix inequality (LMI) approach have been reported; see [1–4]. Compared with the centralized control, the decentralized scheme is preferred in the control design issue of the large-scale interconnected systems [5]. Recently, there are some works about stability and stabilization of fuzzy large-scale systems [6–9]. It is well known that delays appear in many dynamic systems, which are potential causes of system instability [10, 11]. The tasks of stabilization and tracking are two typical control problems. In general, tracking problems are more difficult than stabilization problems especially for nonlinear systems [12]. Reference [13] has given decentralized $H_\infty$ fuzzy model reference tracking control design, and the stable conditions in the sense of Lyapunov are given. The technology of descriptor model transformation is used in [14, 15]. A T-S fuzzy descriptor tracking control design for nonlinear systems with a guaranteed $H_\infty$ model reference tracking performance is discussed [16]. However, in practical situations, failure of actuators often occurs. Thus, an important requirement is to design a reliable controller such that the stability and performance of the closed-loop system can tolerate actuator failures [17–20].

In this paper, we investigate the robust and reliable decentralized $H_\infty$ tracking control issue for the fuzzy interconnected systems with time delay, which are composed of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then, according to the Lyapunov direct method and the decentralized control theory of large-scale interconnected systems, the new LMIs-based conditions with some free variables are derived to guarantee the $H_\infty$ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Moreover, there is no need for the precise failure parameters of the actuators, rather than the lower and upper bound. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

The innovation of this paper can be summarized as follows: (1) the more practical $H_\infty$ performance index is used,
which considers not only the effect of tracking error $e(t)$ but also the effect of control $u(t)$; (2) utilizing the descriptor model transformation, the new LMIs-based reliable $H_{\infty}$ performance conditions with some free variables are derived; (3) there is no need for the precise failure parameters of the actuators, rather than the lower and upper bound of failure parameters.

In the following sections, the identity matrices and zero matrices are denoted by $I$ and $0$, respectively. $X^T$ denotes the transpose of matrix $X$. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space. The standard notation $> (<)$ is used to denote the positive (negative-) definite matrices. Inequality $X > Y$ shows that the matrix $X - Y$ is positively definite. The symbol of $*$ denotes the transposed element in the symmetric position.

2. Systems Description

Suppose there are the interconnected systems consisting of $J$ interconnected subsystems $S_i$, $i = 1, \ldots, J$. Each rule of the subsystem $S_i$ is represented by a T-S fuzzy model as follows:

$$S_i^j: \begin{align} \text{If } \xi_{ij} &= M_{ij}^l \text{ and } \cdots \text{ and } \xi_{ij} = M_{ij}^u, \quad \text{then} \\ \dot{x}_i(t) &= A_i^j x_i(t) + B_i^j u_i(t) + B_i^u u_i(t) + \sum_{j=1, j \neq i}^{J} C_{ij} x_j(t - \tau_{ij}(t)) \\ x_i(t) &= \phi_i(t), \quad t \in [-\tau, 0] \end{align}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t)$, and $w_i(t)$ denote the state, input, and disturbance vector, respectively. $A_i^j, B_i^j, B_i^u$ denote system matrices and $C_{ij}$ denote the interconnection matrices between the $i$th and $j$th subsystem of the $j$th rule. $\tau_{ij}(t)$ are the time-varying delays, $\tau_{ij}(t) \leq \tau_{ij}, \tau_{ij}(t) \leq h_{ij}, \tau = \max_{i,j} \tau_{ij}$, and $r_i$ is the number of IF-THEN rules of the subsystem $S_i$. The initial conditions $\phi_i(t)$ are the differential functions for $t \in [-\tau, 0]$. Here, we denote $X_i^T = [x_1^T(t), \ldots, x_J^T(t)], \Phi_i(t) = [\phi_1^T(t), \ldots, \phi_J^T(t)].$

If we utilize the singleton fuzzifier, product fuzzy inference, and central-average defuzzifier, (1) can be inferred as

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \mu_i^f(\xi_{ij}(t)) \left( A_i^j x_i(t) + B_i^j u_i(t) + B_i^u u_i(t) + \sum_{j=1, j \neq i}^{J} C_{ij} x_j(t - \tau_{ij}(t)) \right),$$

where

$$\lambda_i^f(\xi_{ij}(t)) = \prod_{q=1}^{q_i} M_{iq}^l(\xi_{iq}(t)),$$

$$\mu_i^f(\xi_{ij}(t)) = \frac{\lambda_i^f(\xi_{ij}(t))}{\sum_{l=1}^{r_i} \lambda_i^f(\xi_{ij}(t))},$$

where $\xi_{i1}, \ldots, \xi_{ij}$ are the premise variables and $M_{ij}^l(\xi_{ij}(t))$ is the grade of membership of $\xi_{ij}(t)$ in $M_{ij}^l$. It can be seen that $\lambda_i^f(\xi_{ij}(t)) \geq 0, \mu_i^f(\xi_{ij}(t)) \geq 0$ and $\sum_{l=1}^{r_i} \mu_i^f(\xi_{ij}(t)) = 1, \quad i = 1, \ldots, J, l = 1, \ldots, r_i.$

Consider the reference model for the $l$th subsystem as follows:

$$\dot{x}_r(l) = A_{rl} x_r(l) + B_{rl} v_l(t),$$

where $x_r(l)$ denote the reference states, $A_{rl}$ denote the specific asymptotically stable matrices, $B_{rl}$ denote the system matrices with appropriate dimensions, and $v_l(t)$ denotes the bounded reference input.

Lemma 1 (see [10]). For any constant symmetric matrix $M \in \mathbb{R}^{n \times n}$, $M = M^T > 0$, scalar $\alpha > 0$, vector function $\xi : [0, \alpha] \to \mathbb{R}^{n}$, such that the integrations in the following are well defined; then

$$\alpha \int_0^\alpha \xi^T(\beta) M \xi(\beta) d\beta \geq \left( \int_0^\alpha \xi(\beta) d\beta \right)^T M \left( \int_0^\alpha \xi(\beta) d\beta \right).$$

Lemma 2 (see [10]). For any matrix $Q \in \mathbb{R}^{n \times n}$, any constant $\varepsilon > 0$, and any positive definite matrix $T, \forall \xi \in \mathbb{R}^n, \eta \in \mathbb{R}^n$, the following result holds:

$$2\xi^T Q \eta \leq \varepsilon \xi^T Q^T Q \xi + \frac{1}{\varepsilon} \eta^T T \eta.$$

Lemma 3 (see [2]). For any real matrix $X_i, Y_i$, $1 \leq l \leq n$, and $S > 0$ with appropriate dimensions, one has

$$2 \sum_{l=1}^{n} \sum_{m=1}^{n} \mu_i M_i X_i^T S Y_i \leq \sum_{l=1}^{n} \mu_i (X_i^T S X_i + Y_i^T Y_i),$$

where $\mu_i$ ($1 \leq l \leq n$) are defined as $\mu_i (M(k)) \geq 0, \quad \sum_{i=1}^{n} \mu_i (M(k)) = 1$.

3. $H_{\infty}$ Tracking Control Design

According to the conventional parallel distributed compensation (PDC) concept, the fuzzy controllers corresponding to $S_i$ are used as follows:

$$F_i^j: \begin{align} \text{If } \xi_i &= M_{ij}^{l_1} \text{ and } \cdots \text{ and } \xi_i = M_{ij}^{l_u}, \quad \text{then} \\ \dot{u}_i(t) &= K_i^j (x_i(t) - x_{ri}(t)), \end{align}$$

where $K_i^j$ are the controllers gains of the $j$th rule for subsystem $S_i$.

And the final output of the fuzzy controllers for each subsystem $S_i$ is

$$u_i(t) = \sum_{l=1}^{r_i} \mu_i^f(\xi_{ij}(t)) \left( x_i(t) - x_{ri}(t) \right).$$

Instead of actuator outage, a more general actuator failure model is adopted in this paper. Let $u_i^f(t)$ be the control input vector after failures have occurred. The following actuator failure model is adopted:

$$u_i^f(t) = \alpha_i u_i(t) = \alpha_i \sum_{j=1}^{r_i} \mu_i^f(\xi_{ij}(t)) (x_i(t) - x_{ri}(t)), \quad \text{with } 0 \leq \alpha_i \leq \alpha_i \leq \bar{\alpha}_i \leq 1, \quad i = 1, \ldots, J, l = 1, \ldots, r_i.$$

Note that the parameters $\alpha_i$ and $\bar{\alpha}_i$ characterize the admissible failures of the $j$th actuator in the $i$th subsystem.
Obviously, when $a_{ij} = a_{ij} = 0$, the failure model (10) corresponds to the case of the $j$th actuator outage. When $0 < a_{ij} < b$ and $a_{ij} ≤ 1$, it corresponds to the case of partial failure of the $j$th actuator. When $a_{ij} = a_{ij} = 1$, it implies that there is no failure in the $j$th actuator. Denote the matrix set $S_{p} = \{\lambda_{ij} : \lambda_{ij} = \text{diag}(\lambda_{i1}, \ldots, \lambda_{iL})\} \}$, where $\lambda_{ij} = a_{ij}$ or $\overline{a}_{ij}$, $i = 1, \ldots, I$, $j = 1, \ldots, m$, $L = 1, \ldots, 2^{|m|}$, $L \leq m_{i}$ is the number of no failure actuators for subsystem $S_{i}$.

**Lemma 4** (representation theorem [21]).\( S = \{\lambda | A\lambda = B, \lambda \geq 0\} \) is nonempty and bounded multiple-surface set, and there exist finite limit points $x^{(1)}, \ldots, x^{(k)}$; then, $x \in S$ if and only if $x = \sum_{k=1}^{k} \lambda_{i} x^{(k)}, \sum_{k=1}^{k} \lambda_{i} = 1$, $\lambda_{i} ≥ 0$, $i = 1, \ldots, k$.

Let $S_{1}, \ldots, S_{k}$ be the $k$ components of $S$, then, there exist finite limit points $x_{i}^{(1)} = (a_{i1}, \ldots, a_{i(m-1)}, \ldots, a_{i(m-2)}, \ldots, a_{i(m-|m|)})$. Then, there exist finite limit points $x^{(k)} = (a_{i1}, \ldots, a_{i(m-1)}, \ldots, a_{i(m-|m|)})$ for $S_{i}$.

Given any two points $x_{i}, y_{i} \in S_{i}, y_{i} = (y_{i1}, \ldots, y_{iL}), a_{ij} \leq x_{ij}, j = 1, \ldots, m_{i} - 1$, for any real number $\lambda \in [0, 1]$, then $\lambda a_{ij} \leq \lambda x_{ij} + (1 - \lambda)y_{ij} \leq \lambda a_{ij}$; that is $\lambda A_{ij} + (1 - \lambda)Y_{ij} \in S$. Therefore, $S_{i}$ is nonempty and bounded multiple-surface set.

Let the $l_{i}$ actuator outage of the $i$th subsystem, according to Lemma 4, one has

$$ a_{i} = \text{diag}(a_{i1}, \ldots, a_{i(m-1)}) \cdot 1 \cdot \ldots \cdot 1, $$

$$ = \xi_{i} \text{diag}(a_{i1}, \ldots, a_{i(m-1)}) \cdot 1 \cdot \ldots \cdot 1 \cdot \ldots \cdot 1, $$

$$ + \ldots + \xi_{i}^{(m-1)} \text{diag}(a_{i1}, \ldots, a_{i(m-1)}) \cdot 1 \cdot \ldots \cdot 1 \cdot \ldots \cdot 1, $$

where $\xi_{i} ≥ 0, \Sigma_{k=1}^{k} \xi_{i} = 1$.

Therefore, $a_{i} \in S_{u}$, where $S_{u}$ is the set of $a_{i} : a_{i} = \Sigma_{k=1}^{k} \xi_{i}^{(k)} \lambda_{i}$, $\xi_{i} ≥ 0, \Sigma_{k=1}^{k} \xi_{i} = 1$.

Denote the tracking error by

$$ e_{i}(t) = x_{i}(t) - x_{i}(t). $$

Then, the whole closed-loop fuzzy interconnected systems become

$$ \ddot{x}_{i}(t) = \sum_{j=1, j \neq i}^{I} \sum_{l=1}^{L} \mu_{i}^{L_{j}} \left( \overline{A}_{dlm} \dot{x}_{i}(t) + \overline{C}_{jm} \overline{x}_{j}(t - \tau_{j}(t)) + \overline{E}_{ji} \overline{w}_{i}(t) \right), $$

where

$$ \overline{x}_{i}(t) = \left[ x_{i}(t) \right], \overline{w}_{i}(t) = \left[ u_{i}(t) \right], \overline{A}_{dlm} = \left[ \begin{array}{ll} A_{i}^j & B_{i}^j L_{i} K_{i}^m \\
A_{i}^j & A_{i} \end{array} \right], \overline{C}_{jm} = \left[ \begin{array}{ll} C_{jm} & 0 \\
C_{jm} & 0 \end{array} \right], \overline{E}_{ji} = \left[ \begin{array}{ll} 0 & 0 \end{array} \right], \overline{B}_{ji} = \left[ \begin{array}{ll} 0 & 0 \end{array} \right]. $$

Equation (13) can be transformed to the descriptor system form as follows:

$$ E \ddot{x}_{i} = \sum_{j=1, j \neq i}^{I} \sum_{l=1}^{L} \mu_{i}^{L_{j}} \left( \overline{A}_{dlm} \dot{x}_{i} + \overline{C}_{jm} \overline{x}_{j}(t - \tau_{j}(t)) + \overline{E}_{ji} \overline{w}_{i}(t) \right), $$

where

$$ \xi_{i} = \left[ \begin{array}{ll} x_{i} \end{array} \right], E = \left[ \begin{array}{ll} 1 & 0 \end{array} \right], \overline{A}_{dlm} = \left[ \begin{array}{ll} 0 & 1 \end{array} \right], \overline{C}_{jm} = \left[ \begin{array}{ll} 0 \end{array} \right], \overline{B}_{ji} = \left[ \begin{array}{ll} 0 \end{array} \right]. $$

**Definition 5.** The $H_{\infty}$ tracking control problem for the interconnected systems (13) is to design the controllers to minimize the prescribed level of disturbance attenuation $\Sigma_{i=1}^{I} y_{i} > 0, i = 1, \ldots, J$, if the following two conditions are satisfied.

(1) When $\overline{w}_{i} = 0$, the whole fuzzy interconnected systems (13) are asymptotically stable.

(2) For the zero initially condition $(\dot{x}_{i}^{(0)}(t) = 0, t \in [-\tau, 0])$, $\Sigma_{i=1}^{I} \int_{0}^{T} \overline{e}_{i}(t)Q_{i}e_{i}(t) + u_{i}(t)R_{i}u_{i}(t)dt - \int_{0}^{T} \overline{w}_{i}(t)P_{i}w_{i}(t)dt$.

**Remark 6.** From Definition 5, we can see that, compared with the condition $\Sigma_{i=1}^{I} \int_{0}^{T} \overline{e}_{i}(t)Q_{i}e_{i}(t)dt ≤ \Sigma_{i=1}^{I} \int_{0}^{T} \overline{w}_{i}(t)P_{i}w_{i}(t)dt$ commonly used for the $H_{\infty}$ tracking control problem, the practical $H_{\infty}$ performance index considers not only the effect of $e(t)$ but also the effect of $u(t)$. That is to say, the condition that the reducing of the tracking error $e(t)$ needs to cost the much bigger gain of the controller can be avoided effectively.

**Theorem 7.** For the given constant $\delta, \eta_{j}$, if there exist matrices $Y_{j}^{1}, Y_{j}^{2}, \ldots, Y_{j}^{r}, \ldots, Y_{j}^{q}, Y_{i}^{1}, Y_{i}^{2}, \ldots, Y_{i}^{r}, \ldots, Y_{i}^{q}, T_{i}^{1}, T_{i}^{2}, \ldots, T_{i}^{r}, \ldots, T_{i}^{q}, P_{i}^{1}, P_{i}^{2}, \ldots, P_{i}^{r}, \ldots, P_{i}^{q}, P, q = 1, 2$, and positive definite matrices $W_{2m}^{1}, W_{1}^{1}, P_{1}^{1}, \ldots, W_{1}^{q}, P_{1}^{q}, E_{2}^{1}, E_{2}^{2}, \ldots, E_{2}^{r}, E_{2}^{r}, P_{1}^{1}, P_{1}^{q}, P_{q}^{1}$, $\delta, \eta_{j}, s_{j}^{\alpha}, s_{j}^{\beta}, s_{j}^{\gamma}, \ldots, s_{j}^{\delta}$, satisfying following eigenvalue problem (17), then the whole interconnected nonlinear systems are asymptotically stable with the $H_{\infty}$ tracking performance index $y_{i}, i = 1, \ldots, J, j \neq i, l, m = 1, \ldots, r_{j}, L = 1, \ldots, 2^{m-1}$.

$$ \min \sum_{i=1}^{I} y_{i} \quad \text{subject to} \quad \text{(18) and (19)} $$
\[
\begin{align*}
\bar{\Pi}^{11} & = \delta_i A_i^1 Y_{2i}^1 + \delta_i (A_i^1 Y_{2i}^1)^T + \delta_i B_{i1}^1 \lambda_i W_{2i}^m \\
& \quad + \delta_i (B_{i1}^1 \lambda_i W_{2i}^m)^T + S_{ji}^{11}, \\
\bar{\Pi}^{12} & = \delta_i A_i^1 Y_{2i}^2 + \delta_i \eta_i B_{i2}^1 \lambda_i W_{2i}^m + \delta_i (A_i^1 - A_{ri}) Y_{2i}^3 \\
& \quad + \delta_i \eta_i (A_i^1 - A_{ri}) (Y_{2i}^3)^T + S_{ji}^{12}, \\
\bar{\Pi}^{22} & = \delta_i (A_i^1 - A_{ri}) Y_{2i}^1 + \delta_i (A_i^1 - A_{ri}) Y_{2i}^2 + \delta_i (A_i^1 - A_{ri}) Y_{2i}^3 \\
& \quad + \delta_i \eta_i (A_i^1 - A_{ri}) (Y_{2i}^3)^T + S_{ji}^{22}, \\
\end{align*}
\]

where

\[
\begin{align*}
T_1^{11} & = \frac{T_1^{11}}{p_i}, \\
T_1^{12} & = \frac{T_1^{12}}{p_i}, \\
T_2^{12} & = \frac{T_2^{12}}{p_i}, \\
S_{ji}^{11} & = \frac{S_{ji}^{11}}{p_i}, \\
S_{ji}^{12} & = \frac{S_{ji}^{12}}{p_i}, \\
S_{ji}^{22} & = \frac{S_{ji}^{22}}{p_i},
\end{align*}
\]

\[
\begin{align*}
\bar{\Pi}^{13} & = (A_i^1 Y_{2i}^1)^T + (B_{i2}^1 \lambda_i W_{2i}^m)^T + P_i - \delta_i Y_{2i}^1, \\
\bar{\Pi}^{14} & = ((A_i^1 - A_{ri}) Y_{2i}^2)^T + (A_i^1 Y_{2i}^3 + B_{i2}^1 \lambda_i W_{2i}^m)^T \\
& \quad + P_i - \delta_i Y_{2i}^3, \\
\bar{\Pi}^{23} & = (A_i^1 Y_{2i}^1)^T + \eta_i (B_{i2}^1 \lambda_i W_{2i}^m)^T + (P_i^2)^T - \delta_i Y_{2i}^3, \\
\bar{\Pi}^{24} & = ((A_i^1 - A_{ri}) Y_{2i}^2)^T + \eta_i (A_i^1 Y_{2i}^3 + B_{i2}^1 \lambda_i W_{2i}^m)^T \\
& \quad + P_i^2 - \delta_i \eta_i Y_{2i}^3, \\
\bar{\Pi}^{33} & = -Y_{2i}^1)^T - Y_{2i}^1 + \tau_{ji} T_1^{11}, \\
\bar{\Pi}^{34} & = -Y_{2i}^3)^T - Y_{2i}^3 + \tau_{ji} T_1^{12}, \\
\bar{\Pi}^{44} & = -\eta_i (Y_{2i}^1)^T - \eta_i Y_{2i}^3 + \tau_{ji} T_1^{22}, \\
\bar{\Pi}^{15} & = E_{i1}, \\
\bar{\Pi}^{16} & = E_{i1}^1, \\
\bar{\Pi}^{17} & = E_{i1}^2, \\
\bar{\Pi}^{25} & = E_{i1}^3, \\
\bar{\Pi}^{26} & = E_{i1}^4, \\
\bar{\Pi}^{45} & = E_{i2}, \\
\bar{\Pi}^{46} & = E_{i2}^1, \\
\end{align*}
\]
\[\begin{align*}
\Pi^{55} &= (E_{11}^{ij})^T + E_{11}^{ij}, \\
\Pi^{56} &= (E_{21}^{ij})^T + E_{12}^{ij}, \\
\Pi^{66} &= (E_{12}^{ij})^T + E_{22}^{ij}, \\
\Pi^{77} &= -E_{11}^{ij} + G_{ij} Y_{12}, \\
\Pi^{88} &= -E_{21}^{ij} + G_{ij} Y_{22}, \\
\Pi^{99} &= -E_{12}^{ij} + (E_{21}^{ij})^T + (E_{12}^{ij})^T,
\end{align*}\]

**Proof.** We choose the following Lyapunov function for the whole interconnected system (13):

\[V (X_i) = V_1 (X_i) + V_2 (X_i) + V_3 (X_i),\]

\[V_1 (X_i) = \sum_{i=1}^{l} \xi_i (t) E_i^T \tilde{P}_i \xi_i (t),\]

\[V_2 (X_i) = \sum_{i=1}^{l} \int_{t-	au_{ij}(t)}^{t} \bar{X}_i (s) S_i \bar{X}_i (s) ds,
\]

\[V_3 (X_i) = \sum_{i=1}^{l} \int_{t-	au_{ij}(t)}^{t} \bar{X}_i (s) T_i \bar{X}_i (s) ds d\theta,
\]

where \(S_i, T_i\) are positive definite matrices, \(E_i^T \tilde{P}_i = \tilde{P}_i^T E_i \geq 0\), and denote

\[\tilde{P}_i = \begin{bmatrix} P_{0i} & 0 \\ P_{1i} & P_{2i} \end{bmatrix}, \quad P_{0i} = P_{0i}^T,\]

Computing the time derivative of \(V(X_i)\), we have

\[\begin{align*}
\dot{V}_1 (X_i) &= \sum_{i=1}^{l} \sum_{j=1}^{l} \sum_{m=1}^{r_i} \mu_{ijkl}^m \left( \xi_i^T \left( \bar{A}_{ilm}^T P_i + \bar{A}_{ilm} P_i^T \right) \xi_i + \xi_i^T \tilde{P}_i \bar{X}_j (t-	au_{ij}(t)) \right) \\
&+ \left( \sum_{j=1, j \neq i}^{l} \bar{X}_j (t-	au_{ij}(t))^T C_{jil} \bar{P}_j \xi_i + \xi_i^T \tilde{P}_i \bar{P}_j \bar{X}_j \right) \\
&= \sum_{i=1}^{l} \sum_{j=1, j \neq i}^{l} \sum_{m=1}^{r_i} \mu_{ijkl}^m \left[ \bar{X}_j (t-	au_{ij}(t))^T \bar{X}_j (t-	au_{ij}(t)) \right]^T \left[ \begin{bmatrix} \xi_i \\ \bar{X}_j (t-	au_{ij}(t)) \end{bmatrix} \right]
\end{align*}\]

\[\dot{V}_2 (X_i) \leq \sum_{i=1}^{l} \sum_{j=1, j \neq i}^{l} \left( \bar{X}_i (t-	au_{ij}(t))^T S_i \bar{X}_i (t-	au_{ij}(t)) \right),\]

\[\dot{V}_3 (X_i) = \sum_{i=1}^{l} \sum_{j=1, j \neq i}^{l} \left( \tau_{jil} \bar{X}_j (t-	au_{ij}(t))^T T_i \bar{X}_i - \int_{t-	au_{ij}(t)}^{t} \bar{X}_i (s) T_i \bar{X}_i (s) ds \right) \]

\[\leq \sum_{i=1}^{l} \sum_{j=1, j \neq i}^{l} \left( \tau_{jil} \bar{X}_j (t-	au_{ij}(t))^T \bar{X}_i - \int_{t-	au_{ij}(t)}^{t} \bar{X}_i (s) T_i \bar{X}_i (s) ds \right).
\]

From the Leibniz-Newton formula, the following equation is considered:

\[2 \left( \bar{X}_i^T E_{11} + \bar{X}_i^T E_{12} + \bar{X}_i^T E_{ij3} + \bar{X}_i^T (t-	au_{ij}(t)) E_{ij4} + \bar{X}_i^T E_i \bar{X}_j \right) \times \left( \bar{X}_i - \bar{X}_j (t-	au_{ij}(t)) - \int_{t-	au_{ij}(t)}^{t} \bar{X}_j (s) ds \right) = 0.
\]

From (24), we have

\[2 \left( \bar{X}_i^T E_{11} + \bar{X}_i^T E_{12} + \bar{X}_i^T E_{ij3} + \bar{X}_i^T (t-	au_{ij}(t)) E_{ij4} \right. \]

\[\left. + \bar{X}_i^T E_i \bar{X}_j \right) \left( \bar{X}_i - \bar{X}_j (t-	au_{ij}(t)) \right) = \bar{X}_i^T \bar{X}_j \left( t-	au_{ij}(t) \right) \bar{X}_j,
\]

(24)
where
\[
\Gamma_{ij} = \begin{bmatrix}
\xi_i \\
X_j \\
X_j(t - \tau_{ij})
\end{bmatrix}, \quad \Gamma_{ij} = \begin{bmatrix}
\xi_i \\
X_j \\
X_j(t - \tau_{ij})
\end{bmatrix},
\]

\[
\Phi_{ij} = \begin{bmatrix}
E_{ij} \\
E_{ij3} + E_{ij}^T \\
E_{ij4} - E_{ij4}^T \\
E_{ij5}
\end{bmatrix},
\]

\[
\sum_{\lambda=1}^{\lambda} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds \leq \tau_{ij(t)} E_{ij}^T E_{ij} \mathcal{J}_j + \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds,
\]

where
\[
E_i = [E_{i1} E_{i2} E_{i3} E_{i4} E_{i5}]^T.
\]

According to Lemma 3, we have
\[
\epsilon_i^T(t) Q \epsilon_i(t) + u_i^T(t) R u_i(t)
\]

\[
= \sum_{l=1}^{L} \sum_{m=1}^{M} \mu_l \mu_m e_i^T(t) \left( \begin{bmatrix} 0 & K_l \end{bmatrix}^T R K_l^T \right) e_i(t)
\]

\[
\leq \sum_{l=1}^{L} \sum_{m=1}^{M} \mu_l \mu_m e_i^T(t) \left( \begin{bmatrix} 0 & K_l \end{bmatrix}^T R K_l^T \right) e_i(t).
\]

Letting \( P_i = \delta_i P_2 \) and using (15), we have
\[
V + \sum_{i=1}^{I} \epsilon_i^T(t) Q \epsilon_i(t) + \sum_{i=1}^{I} u_i^T(t) R u_i(t)
\]

\[
= \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds + \epsilon_i^T(t) Q \epsilon_i(t) + \epsilon_i^T(t) R \epsilon_i(t) - \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds
\]

\[
\leq - \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds
\]

where
\[
\Pi_{ijm} = \begin{bmatrix}
\Pi_{ijm}^{11} & \Pi_{ijm}^{12} & \Pi_{ijm}^{13} & \Pi_{ijm}^{14} & \Pi_{ijm}^{15} \\
* & \Pi_{ijm}^{22} & \Pi_{ijm}^{23} & \Pi_{ijm}^{24} & \Pi_{ijm}^{25} \\
* & * & \Pi_{ijm}^{33} & \Pi_{ijm}^{34} & \Pi_{ijm}^{35} \\
* & * & * & \Pi_{ijm}^{44} & \Pi_{ijm}^{45} \\
\end{bmatrix} + \tau_{ij0} \mathcal{J}_j(x_j(s)) ds.
\]

From (29), we can see that, if \( \Pi_{ijm} < 0 \), then \( \Pi'_{ijm} < 0 \), where
\[
\Pi_{ijm}^{11} = \delta_i P_2^T A_{ijm} + \delta_i P_2^T R_i + \overline{Q}_i + S_i,
\]

\[
\Pi_{ijm}^{12} = \overline{A}_{ijm} P_2 + P_2^T - \delta_i P_2^T R_i,
\]

\[
\Pi_{ijm}^{22} = -P_2^T - P_2 + \tau_i \overline{J}_i,
\]

\[
\Pi_{ijm}^{33} = E_{i1}, \quad \Pi_{ijm}^{34} = E_{i2}, \quad \Pi_{ijm}^{35} = E_{i3}, \quad \Pi_{ijm}^{44} = E_{i4}, \quad \Pi_{ijm}^{45} = E_{i5},
\]

\[
\Pi_{ijm}^{13} = E_{i1}, \quad \Pi_{ijm}^{23} = E_{i2}, \quad \Pi_{ijm}^{33} = E_{i3}, \quad \Pi_{ijm}^{43} = E_{i4}, \quad \Pi_{ijm}^{53} = E_{i5},
\]

\[
\overline{Q}_i = \text{diag} \left[ 0, Q_i + \left( K_i^T \right)^T R_i K_i \right],
\]

\[
\Pi'_{ijm} = \begin{bmatrix}
\Pi_{ijm}^{11} & \Pi_{ijm}^{12} & \Pi_{ijm}^{13} & \Pi_{ijm}^{14} & \Pi_{ijm}^{15} \\
* & \Pi_{ijm}^{22} & \Pi_{ijm}^{23} & \Pi_{ijm}^{24} & \Pi_{ijm}^{25} \\
* & * & \Pi_{ijm}^{33} & \Pi_{ijm}^{34} & \Pi_{ijm}^{35} \\
* & * & * & \Pi_{ijm}^{44} & \Pi_{ijm}^{45} \\
\end{bmatrix} - \frac{T_j}{\tau_{ij0}}.
\]

If \( \overline{w}_i = 0 \),
\[
\overline{V} \leq \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{M} \mu_l \mu_m e_i^T(t) R_i u_i(t)
\]

\[
- \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds - \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds < 0,
\]

then the whole nonlinear interconnected systems are asymptotically stable.

If \( \overline{w}_i \neq 0 \),
\[
\overline{V} + \sum_{i=1}^{I} \epsilon_i^T(t) Q \epsilon_i(t) + \sum_{i=1}^{I} u_i^T(t) R u_i(t)
\]

\[
- \sum_{i=1}^{I} \int_{\tau_{ij}(t)}^{t} \mathcal{J}_j(x_j(s)) ds < 0.
\]

With zero initial condition (\( \Phi(t) = 0, t \in [-\overline{T}, 0] \)), hence \( V(\Phi(t)) = 0, t \in [-\overline{T}, 0] \), and \( V(X_{\infty}) \geq 0 \), integrating both sides of (32) from 0 to \( \infty \), we have
\[
\int_{0}^{\infty} \overline{W}_i(t) \overline{v}_i(t) dt < 0.
\]

implying that \( \overline{V} \leq \sum_{i=1}^{I} \int_{0}^{\infty} \overline{W}_i(t) \overline{v}_i(t) dt \leq \sum_{i=1}^{I} \int_{0}^{\infty} \overline{W}_i(t) \overline{v}_i(t) dt \)
Let \( Y_{2i} = P_{2i}^{-1}, \tilde{E}_{i1} = Y_{2i}^T E_{i1} Y_{2i}, \tilde{E}_{i2} = Y_{2i}^T E_{i2} Y_{2i}, \tilde{E}_{ij3} = Y_{2i}^T E_{ij3} Y_{2i}, \tilde{E}_{ij4} = Y_{2i}^T E_{ij4} Y_{2i}, \tilde{E}_{ij5} = E_{ij5} Y_{2i}. \) By Schur complement and premultiplying and postmultiplying to (29) by positive-definite matrix \( \text{diag}(P_{2i}^T P_{2i}^{-1} P_{2i}) \), we have

\[
\dot{\Xi}(t) = \sum_{i=1}^{r} \sum_{m=1}^{r} \mu_m \text{diag} \left( \tilde{A}_{bm}(t) + \tilde{B}_5 \tilde{w}(t) \right),
\]

where \( \tilde{w}(t) = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \tilde{B}_5 = \begin{bmatrix} B_1^T & 0 \\ B_1^T & -B_r \end{bmatrix} \).

**Corollary 9.** For the given constant \( \delta, \eta \), if there exist matrices \( Y_1, Y_2, Y_3, Y_4, P_2, P_3 \) and positive definite matrices \( W_{m}^m, P_1, P_3 \), satisfy (38), then the T-S fuzzy system (36) is asymptotically stable with the \( H_{\infty} \) tracking performance index \( \gamma \), and the gain of controllers \( K_m = W_{m}^{m-1} I, m = 1, \ldots, r, L = 1, \ldots, 2^{m-1}; \)

\[
\min_{\gamma} \text{subject to (39) and (40)} \quad \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0,
\]

\[
\begin{bmatrix}
\Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \delta B_1^T & 0 & Y_3 & \Pi_{18} \\
\Pi_{12} & \Pi_{22} & \Pi_{23} & \Pi_{24} & \delta B_2^T & \delta B_r & \eta Y_3 & \Pi_{28} \\
\Pi_{13} & \Pi_{23} & \Pi_{33} & \Pi_{34} & \delta B_1^T & -B_r & \eta Y_3 & \Pi_{38} \\
\Pi_{14} & \Pi_{24} & \Pi_{34} & \Pi_{44} & \delta B_2^T & -B_r & 0 & 0 \\
\Pi_{15} & \Pi_{25} & \Pi_{35} & \Pi_{45} & \delta B_1^T & -B_r & 0 & 0 \\
\Pi_{16} & \Pi_{26} & \Pi_{36} & \Pi_{46} & \delta B_2^T & 0 & 0 & 0 \\
\Pi_{17} & \Pi_{27} & \Pi_{37} & \Pi_{47} & -\gamma^2 I & 0 & 0 & 0 \\
\Pi_{18} & \Pi_{28} & \Pi_{38} & \Pi_{48} & -\gamma^2 I & 0 & 0 & 0 \\
\end{bmatrix} < 0,
\]

where

\[
\begin{bmatrix}
\text{diag} \left[ 0, Q_1, (K_1^T)^T K_1 \right], \\
\text{diag} \left[ 0, Q_1 \right], \\
Y_{2i} \\
\end{bmatrix} = \begin{bmatrix} Y_{2i} & Y_{2i} \end{bmatrix}, \text{ using (13), and by Schur complement, the proof is completed.}
\]

**Remark 8.** Firstly, utilizing the descriptor model transformation, LMIs-based conditions with some free variables are derived. Second, when \( \tilde{w}(t) = 0 \), compared with the stable conditions in the sense of Lyapunov in [13], we obtain the asymptotically stable conditions. Finally, compared with [16], the time-varying delay fuzzy large systems are considered, and a more practical \( H_{\infty} \) performance index is considered. Therefore, obtained results are new and less conservative.
\[ \dot{x}_{12}(t) = - \frac{Dd_i}{M_{ij}} x_{12}(t) + \frac{1}{M_{ij}} u_i(t) \]
\[ + \sum_{j=1, j \neq i}^{2} \frac{E_{ij} E_{ji} Y_{ij}}{M_j} \times \left( \cos(\delta_{ij}^0 - \theta_{ij}) - \cos(x_{ij}(t) - x_{ij}(t - \tau_{ij}(t))) + \delta_{ij}^0 - \theta_{ij} \right) + B_{ij} w_{12}, \quad i = 1, 2. \] 

(42)

We assume the two-machine interconnected systems’ parameters as follows:

\[ E_{11} = 1.017, \quad E_{12} = 1.005, \quad M_{11} = 1.03, \]
\[ M_{12} = 1.25, \quad Dd_1 = 0.8, \quad Dd_2 = 1.2, \]
\[ Y_{12} = Y_{21} = 1.98, \quad \theta_{12} = \theta_{21} = 1.5, \]
\[ \delta_{12}^0 = -\delta_{21}^0 = 1.2, \quad B_{11} = B_{12} = \text{diag}[1, 1], \]
\[ \tau_{12}(t) = \tau_{21}(t) = 0.6 \sin^2(t) + 1.2. \]

The systems are approximated by the following nine-rule fuzzy model which is the same as [13], and the details of the rules are omitted here.

Other parameters are shown as follows:

\[ A_{ri} = \begin{bmatrix} 0 & 1 \\ -100 & -101 \end{bmatrix}, \]
\[ B_{ri} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \]
\[ v_1(t) = \begin{bmatrix} 0 \\ 100 \cos(0.5t) \end{bmatrix}, \]
\[ v_2(t) = \begin{bmatrix} 0 \\ 100 \sin(0.5t) \end{bmatrix}. \]

(44)

Here, we assume that there exists the actuator failure only in the actuator of the first subsystem. Let \( \alpha_{11} = 0.2, \alpha_1 = 1, \alpha_2 = 1; \) that is \( \lambda_{11} = 0.2, \lambda_{21} = 1, \lambda_{12} = 1. \) The actuator failure parameter \( \alpha_i \) is shown in Figure 1.

Let \( \delta_1 = \delta_2 = 1, \eta_1 = \eta_2 = 1, Q_1 = Q_2 = \text{diag}(1, 1), \]
\[ R_1 = R_2 = 0.02, \] according to the conditions proposed in Theorem 7; we have \( K^1_l = [-42.6186 - 18.7306], \]
\( K^2_l = [-13.7741 - 8.6137], l = 1, \ldots, 9. \) For the initial condition \( x_1(0) = [-1 3]^T, x_2(0) = [1 - 2]^T \) and the disturbances \( w_{12}(t) = \sin(2\pi t)e^{-0.5t}, w_{22}(t) = \cos(2\pi t)e^{-0.5t}, \) the tracking trajectories are shown in Figures 2 and 3, in which \( x_{rij}, i = \)
1, 2, \( j = 1, 2 \), denote the states of the reference model, \( x_{ij}, i = 1, 2, j = 1, 2 \), denote the states when there are no failure actuators, and \( x_{fij}, i = 1, 2, j = 1, 2 \), denote the states when there exist partial failure actuators for \( t \in [0.2, 5] \). It can be seen that there are nearly no differences between \( x_{fij} \) and \( x_{ij} \). Therefore, when there exist the actuator failures, the performances of the system are not influenced. Compared with the result \( K_1^f = [-140 - 67] \), \( K_2^f = [-183 - 85] \),

\[ l = 1, \ldots, 9 \] in [13], the gains of controllers are much smaller, and the control performances are nearly the same.

**Example 2.** Consider the following tracking system [16]:

\[
\dot{x}(t) = \begin{bmatrix} -1.4 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w(t),
\]

\[
\dot{x}_r(t) = \begin{bmatrix} -1 & 1 & 0.6 \\ -0.5 & -1 & 0 \\ -1 & -1 & -3 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 8 \sin(t) \\ 0 \end{bmatrix}.
\]  

(45)
Let $\alpha = 0.2$, $\alpha \bar{=} 1$, the actuator failure parameter $\alpha$ is the same as Example 1, $\delta = 1$, $\eta = 0.8$, $Q = \text{diag}(1, 1, 1)$, $R = 0.01$, according to the conditions proposed in Corollary 9; we have $\gamma^2 = 1.1291$, $K = [-2.9191 - 12.6548 - 3.3239]$. For the initial condition $x(0) = [2 \ 2]^T$, $x_r(0) = [0 \ 0]^T$, and the disturbance $w(t) = 0.1 \cos(t)$, the tracking trajectories are shown in Figures 4, 5, and 6. Compared with the result $\gamma^2 = 1.0751$, $K = 10^3 \ast [-4.1688 - 9.1600 - 1.3698]$ in [16], the gain of controller is much smaller, and the control performances are nearly the same. For the different $\delta$, $\eta$, the trajectories of $\gamma^2$ are shown in Figure 7.

5. Conclusion

This paper investigates the robust and reliable decentralized $H_{\infty}$ fuzzy tracking control issue for the fuzzy interconnected systems with time-varying delay, which consist of a number of T-S fuzzy subsystems with interconnections. Firstly, the ordinary fuzzy interconnected systems are equivalently transformed to the fuzzy descriptor systems; then the new LMIs-based conditions with some free variables are derived to guarantee the $H_{\infty}$ tracking performance not only when all control components are operating well, but also in the presence of some possible actuator failures. Finally, two simulation examples are provided to illustrate the effectiveness of the proposed method.

Conflict of Interests

The authors declared that they have no conflicts of interest to this work.

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