Research Article

Performance of Various Metaheuristic Techniques for Economic Dispatch Problem with Valve Point Loading Effects and Multiple Fueling Options

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The paper presents the metaheuristic approaches based on pattern search and simulated annealing hybridized with sequential quadratic programming, a powerful nonlinear adaptive technique for estimation of the finest combination of generated power in a given system at lowest operating cost while sustaining the operating condition of system efficiently. The fuel cost is minimized by satisfying the nonlinear operating conditions of thermal units mainly based on generation capacity constraints, generator ramp limit, power balance constraints, and valve point loading effect and by keeping in view the prohibited operating zones, respectively. About the optimization, a comparative study is made for pattern search (PS) and simulated annealing (SA), as a viable global search technique and sequential quadratic programming, an efficient local optimizer and their hybrid versions. The applicability, stability, and reliability of the designed approaches are validated through comprehensive statistical analysis based on Monte Carlo simulations.

1. Introduction

The performance and designing of electric power system serve two purposes; one is to maintain the economy and the other is to include reliability of the system [1]. Economic dispatch problem (EDP) is one of the significant optimization problems in industrial independent power system and acquires increasing importance as the operating cost and electric energy demand is increasing globally around the planet [2, 3]. The procedure involves dividing the power demand of generating system for online thermal units in such manner that their operating cost is optimal while satisfying the power demand to the user and constraint adequately [3, 4]. Since the operating cost of generating unit is inflated an optimal EDP can lead to prominent cost saving with reducing the atmospheric emission of thermal power plant [4]. In the energy crises age, a number of engineers and scientists are taking interest in the optimized solution of this industrial problem [5] by keeping in view the constraints like

generator ramp limit, power balance, and valve point loading effect (VPLE) and prohibited operating zones [6, 7]. However the facts of EDP with multiple fueling option (MFO) and MFO combined with VPLE conditions is not well addressed simultaneously, which are the motivations for taking this study with metaheuristic computing approach.

Traditional numerical models and programming strategies have been developed, like lambda iteration method, quadratic programming, Lagrangian relaxation algorithm, base point participation factor, gradient method linear programming, reduced gradient method, piecewise linear cost function, Newton method, and dynamic programming [8, 9], but all these EDP tools are flexible and applicable if the function between operating cost and fuel or heat are piecewise linear and increasing categorically [10]. Modern approaches use artificial neural network models which show dominance on classical techniques [11], while its parameters can overwrite during the learning in complex constraints based EDP problems. In order to overcome this issue the

metaheuristic approaches based on tabu search, Hopfield, hill climbing, evolutionary computation, swarm intelligence, and ant/bee colony optimizations can outperform [12, 13].

In the recent development on optimization technique for EDP problems, multiperiod hydrothermal economic dispatch is employed with an efficient local search based on interior point method [14] that has an issue of getting stuck in the local minima. The solution of unit commitment problem is addressed by a hybrid scheme based on modified subgradient method and simulating annealing algorithm [15]; this combination of derivative based and derivative free method is good but with the constraints of high computational budget involved in its computations. The predictor corrector primal dual interior point algorithm is well presented in [16] in terms of computational cost for solving EDP problem; however the mean level of accuracy is not well matured. Parametric approach involved in parameter setting free harmony search and parallel adaptive algorithm schemes are established for economic dispatch and environmental power dispatch problems, respectively [17, 18]. With all this recent development, this paper presents the comprehensive analysis in terms of minimum fuel cost, level of accuracy in the optimizer, statistical analysis on computational budget needed, and convergence rate as well, on complex dynamical version of EDP problem involving MFO and VPLE conditions. The presented work is compared with [19–21], in which EDP problem has been exploited by evolutionary strategy, standard particle swarm optimization (PSO), Gaussian PSO, and chaotic PSO, respectively.

The remaining of this paper is organized as follows: Section 2 describes the formulation of an EDP along with the constraints. The proposed methodology along with the procedural hybrid step is revealed in Section 3. Section 4 discusses the computational procedure and analyzes the results when applied to the scenarios of EDPs with 13, 15, and 20 thermal units. The last section of the paper outlines the conclusions drawn from the simulation and results along with some directions for the future research.

2. Economic Dispatch Problem Formulation

The objective of EDP is to find the finest combination of generated power in a given system at lowest operating cost while sustaining the operating condition of system efficiently by satisfying the nonlinear operating conditions formulated as follow.

2.1. Fuel Cost Minimization. Assume we have a system where n thermal unit committed generating real power P_{gi} and total fuel cost is $F_{ci}(P_{gi})$; usually the fuel cost curve of committed units is assumed to be quadratic function of the real power output of generators. Consider

$$F_{TC}|_{\min} = \sum_{i=1}^{NG} F_{ci}(P_{gi}) = \sum_{i=1}^{NG} (A_i + B_i P_{gi} + C_i P_{gi}^2), \quad (1)$$

where NG is the number of committed generators and P_{gi} is real output power of generator in megawatt (MW) and can be

A: Primary valve
C: Tertiary valve
E: Quinary valve
B: Secondary valve
D: Quaternary valve

FIGURE 1: Cost function with primary, secondary, tertiary, quaternary, and quinary valves.

FIGURE 2: Flow diagram of metal of annealing algorithm.

represented as a real vector $P_{gi} = [p_{g1} + p_{g2} + p_{g3} + \dots + p_{gNG}]$, while F_{TC} and F_{ci} are the cost of all committed generators and cost of the i th generator in terms of rupees/hour (R/h), respectively. A_i , B_i , and C_i are the fuel-cost coefficient of i th generator.

FIGURE 3: Process of proposed solution (a) overall procedure and (b) optimization of fitness function using hybrid computation.

2.2. Inequality Constraint

2.2.1. Generation Capacity Constraint. For the stable operation of each thermal unit the power generated, that is, P_{gi} , must be restricted by minimum power that must be generated and maximum power that a unit can deliver as follows:

$$P_{gi}(\min) \leq P_{gi} \leq P_{gi}(\max), \quad i = 1, 2, 3, \dots, NG. \quad (2)$$

2.2.2. Generator Ramp Limit. With reference of [4] considering the ramp rate limit of generator, generation capacity constraint equation (2) can be modified as given below:

$$\begin{aligned} & \text{Max} (P_{gi}(\min), P_{gi}^0 - DR_i) \\ & \leq P_{gi} \leq \text{Min} (P_{gi}(\max), P_{gi}^0 + UR_i), \quad (3) \\ & i = 1, 2, 3, \dots, NG, \end{aligned}$$

where P_{gi}^0 , UR_i , and DR_i are the previous operating point, upper ramp limit, and down ramp limit of the i th generator, respectively.

2.2.3. Power Balance Constraint. Usually all the power plants are not working on same place they are spread geographically and also they must have some transmission power losses. The total power generated by committed thermal unit must be equal to the power demand of electric network plus the losses associated in power transmission [22]:

$$\sum_{i=1}^{NG} P_{gi} = P_D + P_{NL}, \quad (4a)$$

$$P_{NL} = \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_i B_{ij} P_j, \quad P_i = P_{gi} - P_D, \quad (4b)$$

$$\beta_{ij} = \frac{R_{ij} \cos(\theta_i - \theta_j)}{|V_i| |V_j| \cos \theta_i \cos \theta_j}, \quad (5)$$

$$\theta_j = \delta_i - \theta_i, \quad \theta_i = \tan\left(\frac{Q_i}{P_i}\right),$$

where P_D is the power demand, NG is number of generators, and P_{NL} is the transmission power loss collective equation that is also known as coordination equation. The most

TABLE 1: Parameter setting/values using PS and SA.

PS		SA	
Parameters	Settings	Parameters	Setting
Poll method	MADS positive basis 2N	Start point size	15
Poll order	Random	Population size	300
Mesh size	(1, 15)	Annealing function	Boltzmann
Expansion factor	2.5	Reannealing interval	150
Contraction factor	0.7	Temperature update	Exponential
Penalty factor	150	Initial temperature	100
Initial range	[0-1]	Bound	Problem specific
Maximum function Evaluations	30000	Maximum function evaluation	45000
Maximum iterations	1500	Maximum iterations	1500
Others	Default	Others	Default

TABLE 2: Parameter setting/values using SQP method.

Sequential quadratic programming	
Parameters	Settings
Start point	Final weights of PS or SA
Bounds	Specific lower and upper limit of generators
X-tolerance	10^{-12}
Function tolerance	10^{-12}
Hessian	BFGS
Scaling	Objective and constraints
Maximum iterations	1000
Maximum function evaluations	1500
Others	Default

approximate and important method for transmission network losses as a function of generator power is β coefficient method based on fact that, under normal operating conditions, the transmission loss is quadratic in the injected bus real power. Equation (4b) represents the general form of the loss formula using β coefficient method. Here NB, $P_i P_j$, $Q_i Q_j$, and Z_{ij} are the number of buses, the real power, reactive power of i th and j th buses, and elements of impedance matrix, respectively. However in this study β coefficient method will be used so β_{ij} are the loss constant of coefficient β .

2.3. Valve-Point Loading Effect (VPLE). For a power plant having NG committed units have multivalve opening sequence to meet the fluctuating loads demand. VPLE introduce ripples in the input-output characteristics of generating units resultantly making the heat curve rate nonconvex with increase in unit loading. Heat rate curve decreases between the opening points of two valves as shown in Figure 1 [23].

VPLE introduces ripples in the heat rate curve in multi valve opening turbine as shown in Figure 1, and for a perfect modeling of EDP a filter in the form of sinusoidal function

must be added in objective function in this paper as given below:

$$F_{ci}(P_{gi}) = A_i + B_i P_{gi} + C_i P_{gi}^2 + |e_i \times \sin(f_i \times (P_{gi}^{\min} - P_{gi}))|, \quad (6)$$

where e_i and f_i are the coefficient of i th unit that represents valve-point effect.

2.4. Prohibited Operating Zones. According to [3] practical generating units contain forbidden operating regions also known as “prohibited operating zones” due to machinery errors such as boiler pumps motors faults which ultimately leads input-output curve nonlinear and this can be modeled as follows:

$$P_{gi} \in \begin{cases} P_{gi}^{\min} \leq P_{gi} \leq P_{gi1}^{NL} \\ P_{gik-1}^U \leq P_{gi} \leq P_{gik}^{NL} \\ P_{gizI}^U \leq P_{gi} \leq P_{gi}^{\max} \end{cases}, \quad (7)$$

where P_i is the number of prohibited zones in the i th generator curve, k is the index of prohibited zone, and P_{gik}^{NL} and P_{gik}^U are lower and upper limit of k th prohibited zone of the i th generator.

2.5. EDP with Multiple-Fueling Option (MFO). Another significant consideration involved in designing of EDP optimal cost is unit multiple fueling. A thermal unit has multiple cost rate curves because unit is fueled by different sources and needs to operate the unit on lower curve which result in producing hybrid cost function (HCF) and HCF contains information about unit operation or type of fuel used in generating operation. Practically multiple fuel sources are supplied to the thermal units and a single thermal unit can be represented by multiple quadratic cost functions; each cost function represents the type of fuel used and each unit must

TABLE 3: Fitness evaluation values and their respective computational time for proposed schemes.

Scenarios	PS		SA		SQP		PS-SQP		SA-SQP	
	F_{val}	τ	F_{val}	τ	F_{val}	τ	F_{val}	τ	F_{val}	τ
Case 1	$2.01E-13$	0.29975	$1.25E-15$	26.9359	$5.24E-19$	0.09749	$8.95E-21$	0.22732	$8.40E-19$	0.05562
Case 2	$1.84E-11$	1.68357	$1.20E-16$	29.8246	$1.22E-11$	2.98696	$2.90E-13$	1.10707	$1.00E-19$	0.24536
Case 3	$2.01E-13$	0.29975	$1.25E-15$	26.9359	$5.24E-19$	0.09749	$8.95E-21$	0.22732	$8.40E-19$	0.05562

TABLE 4: Reliability based on 100 independent runs for fifteen units system with $P_D = 2630$ MW.

Method	Min _{cost}	Mean _{cost}	STD _{cost}	Max _{cost}	Mean _{CPU} time
Evolutionary strategy [19]	32568.54	32620.00	—	32710.00	0.48
PSO [20]	33858.00	33039.00	—	3331.000	26.59
Genetic algorithm [20]	33113.00	33228.00	—	3337.000	49.31
Gaussian PSO [21]	32508.12	35122.79	1918.62	38044.42	0.69
Chaotic PSO [21]	32775.68	35340.70	1942.84	38104.54	0.59
Proposed					
PS	33154.10	34908.12	1712.34	36.001.14	0.640587
SA	33124.24	35012.54	1612.35	36312.12	22.78067
SQP	32567.45	33467.14	1712.12	35123.90	0.418991
PS-SQP	32005.00	32145.23	1735.12	32568.28	1.059578
SA-SQP	32000.14	32112.12	1659.38	32412.02	23.199661

pin point the economic fuel to burn and fuel cost function formulation is as follows [23]:

$$F_{ci}(P_{gi}) = \begin{cases} A_{i1} + B_{i1}P_{gi} + C_{i1}P_{gi}^2, & \text{fuel 1, } P_{gi}^{\min} \leq P_{gi} \leq P_{gi1} \\ A_{i2} + B_{i2}P_{gi} + C_{i2}P_{gi}^2, & \text{fuel 2, } P_{gi} \leq P_{gi} \leq P_{gi2} \\ \vdots & \vdots \\ A_{ik} + B_{ik}P_{gi} + C_{ik}P_{gi}^2, & \text{fuel } k, P_{gik-1} \leq P_{gi} \leq P_{gi}^{\max}, \end{cases} \quad (8)$$

where A_{ik} , B_{ik} , and C_{ik} are the cost coefficient of generator i th using k fuels.

2.6. EDP Considering Both VPLe and MFO. To obtain the realistic and transparent EDP solution in this paper both MFO and VPLe model are considered. Mathematical modeling for this consideration is combination of both cost functions given in (6) and (8). Consider

$$F_{ci}(P_{gi}) = \begin{cases} A_{i1} + B_{i1}P_{gi} + C_{i1}P_{gi}^2 + |e_{i1} \times \sin(f_{i1} \times (P_{gi1}^{\min} - P_{gi1}))|, & \text{fuel 1, } P_{gi}^{\min} \leq P_{gi} \leq P_{gi1} \\ A_{i2} + B_{i2}P_{gi} + C_{i2}P_{gi}^2 + |e_{i2} \times \sin(f_{i2} \times (P_{gi2}^{\min} - P_{gi2}))|, & \text{fuel 2, } P_{gi} \leq P_{gi} \leq P_{gi2} \\ \vdots & \vdots \\ A_{ik} + B_{ik}P_{gi} + C_{ik}P_{gi}^2 + |e_{ik} \times \sin(f_{ik} \times (P_{gik}^{\min} - P_{gik}))|, & \text{fuel } k, P_{gik-1} \leq P_{gi} \leq P_{gi}^{\max}. \end{cases} \quad (9)$$

3. Proposed Methodology

The brief introduction of optimizers used for optimization of EDP is narrated along with the necessary detail of hybrid algorithm in the form of procedural steps.

3.1. Simulated Annealing. Simulated annealing is heuristic as well as probabilistic method proposed by Scott Kirkpatrick, C. Daniel Gelatt, and Mario P. Vecchi in 1983 for localizing better approximation to find global minimum of given cost function that may be surrounded with several local minima in a large search space [24]. SA is more efficient as compared with hill climbing and is a good choice for engineers and

researchers to handle complex nonlinear systems especially problems like EDP due to its coding leniency to get guarantee finest optimal solutions. The method works on the principle of randomly initializing of finite solution space of specific size N on the basis of scattered neighborhood points that mimic from the principle of thermal annealing of solids [25]. The decreasing of temperature at a specific rate in each iteration resembles the optimization of the merit function of EDP subject to varying the heat values as adaptive parameters of a specific problem. The candidate solution is updated based on the values of the fitness in terms of mean square error of the previous possible solution along with the index term; on this basis the iterative process matures towards

TABLE 5: Reliability based on 100 independent runs for twenty units system with $P_D = 2500$ MW.

Method	Min _{cost}	Mean _{cost}	STD _{cost}	Max _{cost}	Mean _{CPU} time (s)
Lambda iteration method [38]	62456.6391	—	—	—	0.033
Hopfield neural networks [38]	62456.6341	—	—	—	0.006
Gaussian PSO [21]	60526.15	61123.36	330.89	62757.19	0.50
Chaotic PSO [21]	60803.51	61223.95	507.56	62982.19	0.37
Proposed					
PS	59829.15	61256.21	1342.56	63125.56	1.054175
SA	59803.45	61002.34	1423.89	62340.16	32.31666
SQP	57231.43	59012.45	1712.35	60123.23	0.294808
PS-SQP	58123.98	58563.65	976.140	58901.26	1.348983
SA-SQP	57120.21	58011.78	658.341	58764.38	33.643791

TABLE 6: Data for thirteen thermal units of generating unit capacity and coefficient [37].

Units	$P_i^{\min}$ (MW)	$P_i^{\max}$ (MW)	a (\$/MW ²)	b (\$/MW ²)	c (\$/MW ²)
1	0	680	0.00028	8.10	550
2	0	360	0.00056	8.10	309
3	0	360	0.00056	8.10	307
4	60	180	0.00028	7.74	240
5	60	180	0.00324	7.74	240
6	60	180	0.00324	7.74	240
7	60	180	0.00324	7.74	240
8	60	180	0.00324	7.74	240
9	60	180	0.00324	7.74	240
10	40	120	0.00284	8.60	126
11	40	120	0.00284	8.60	126
12	55	120	0.00284	8.60	126
13	55	120	0.00284	8.60	126

the region of convergence of the fitness function. SA belongs to the family of global search techniques so it has various start points in the start of the algorithm to avoid local minimum. Usually the number of iterations of SA is very large while the computational budget is quite reasonable for most of the test bench problem. Although, it has been applied in various areas of engineering, some key applications are found in crystallographic refinement, neural computing [26], optimal reconfiguration of distribution networks, wireless communication, phase balancing, van der pol oscillators involved in mechanical engineering [27], and combinatorial optimization problem [28]. The generic flow diagram used for the simulated annealing algorithm is provided in Figure 2.

3.2. Pattern Search. Pattern search (PS) is numerical optimization method proposed by Hooke and Jeeves [29] for statistical problems and main advantages of PS are that it is efficient and computational structure is simple to understand and also it does not required gradient of cost function so it can be used in functions that are not constant or not differentiable and method is also known as direct-search, derivative-free

approaches. Pattern search algorithm searches for a sequence of points to reach an optimal solution by initializing a set of points called mesh. The mesh formation procedure is involved by adding a pattern vector to the initial point and if PS acquires a point in mesh that refines the objective function then the new point becomes the current point at the next step of the algorithm [30].

Usually the practical nonlinear optimization problem of power system lies outside the reach of the standard optimization techniques. The practical EDP problems can be optimized by PS which is computationally efficient and easy to implement as compared to other heuristic techniques [31]. PS initializes by computing sequence of point that may or may not be reached optimally and established a set point called mesh around these given points. This current point could be a starting point provided by user or extracted from previous step of algorithm. Current points are then added to a pattern vector which is scalar multiple of a set of vectors for mesh formation. If an objective function is improved by point in mesh then the new point becomes the current point at the next iteration [32].

Pattern search is efficient optimization techniques applied in areas of parallel computing, pattern character recognition, graphic user interference thermal control systems, nonlinear and optimal control theory [33], and so forth.

3.3. Sequential Quadratic Programming. The SQP algorithm is one of the nonlinear programming techniques used extensively by the research community for constraints optimization problems. The strength of the scheme is well-established in terms of efficiency, accuracy, and percentage of successful solutions over a large number of test problems. Thum and Schilling [34] have provided excellently a brief introduction and description of SQP algorithms. As the physics of SQP is based on local search techniques so it is more probable to get stuck in the local minimum so some of the iterations during Monte Carlo simulation can show divergence or least accuracy. However the rate of convergence and level of accuracy for successful runs is dramatically high for SQP algorithms. The detailed history, mathematical formulation, importance, and application related with SQP methods can be seen in [35, 36]. The SQP methods have been applied on broad disciplines of engineering and applied science from

TABLE 7: (a) Data for fifteen thermal units of generating unit capacity and coefficient [19–21]. (b) Data for fifteen thermal units of generating units of ramp-rate limits and prohibited zones of generating units.

Units	$P_i^{\min}$ (MW)	$P_i^{\max}$ (MW)	a (\$/MW ²)	b (\$/MW ²)	c (\$/MW ²)
1	150	455	0.000299	10.1	671
2	150	455	0.000183	10.2	574
3	20	130	0.001126	8.8	374
4	20	130	0.001126	8.8	374
5	150	470	0.000205	10.4	461
6	135	460	0.000301	10.1	630
7	135	465	0.000364	9.8	548
8	60	300	0.000338	11.2	227
9	25	162	0.000807	11.2	173
10	25	160	0.001203	10.7	175
11	20	80	0.003586	10.2	186
12	20	80	0.005513	9.9	230
13	25	85	0.000371	13.1	225
14	15	55	0.001929	12.1	309
15	15	55	0.004447	12.4	323

Units	P_i^0 (MW)	UR _i (MW/h)	DR _i (MW/h)	Zone 1	Prohibited zones Zone 2	Zone 3
1	400	80	120			
2	300	80	120	[185 255]	[305 335]	[420 450]
3	105	130	130			
4	100	130	130			
5	90	80	120	[180 200]	[305 335]	[390 420]
6	400	80	120	[230 255]	[365 395]	[430 455]
7	350	80	120			
8	95	65	100			
9	105	60	100			
10	110	60	100			
11	60	80	80			
12	40	80	80	[30 40]	[55 65]	
13	30	80	80			
14	20	55	55			
15	20	55	55			

the day of their birth to today; for instance, few recently published articles addressing SQP technique are referred to for interested readers in [37, 38].

Now the intention is to formulate the overall procedure and the optimization of the unknown designed parameters of the units generation along with the fuel cost; the process of the proposed scheme is provided in Figure 3. The modeling is performed based on relations provided in (1) and relations of constraints, respectively. The optimization is performed using global versions of simulated annealing and pattern search hybrid with sequential quadratic programming, an efficient local search technique. In Figure 3(b) the designed parameters obtained from global optimizer are provided as a start point to SQP for further tuning of the design. The algorithm used for the optimization of EDP in the form of

the logical steps based on hybrid computational technique is as follows.

Step 1 (population generation). A random and well scattered population of size M is generated with agents of length exactly equal to the number of power generating units N .

In general the i th agent of the candidate solution is represented by $\mathbf{c}_i = [c_{i1}, c_{i2}, \dots, c_{iN}]$, $c_{ij} \in \mathfrak{R} : L_b \leq c_{ij} \leq U_b \ \forall i = 1, 2, 3, \dots, N, j = 1, 2, 3, \dots, M$, where L_b and U_b are the lower and upper possible limits to generate power.

Step 2 (initialization). Initialize and assign the general and specific values for parameter setting involved in pattern search and simulated annealing algorithm as provided in Table 1.

TABLE 8: Data for twenty thermal generating units capacity and coefficients (20 units) [19–21].

Units	$P_i^{\min}$ (MW)	$P_i^{\max}$ (MW)	a (\$/MW ²)	b (\$/MW ²)	c (\$/MW ²)
1	150	600	0.00068	18.19	1000
2	50	200	0.00071	19.26	970
3	50	200	0.00650	19.80	600
4	50	200	0.00500	19.10	700
5	50	160	0.00738	18.10	420
6	20	100	0.00612	19.26	360
7	25	125	0.00790	17.14	490
8	50	150	0.00813	18.92	660
9	50	200	0.00522	18.27	765
10	30	150	0.00573	18.92	770
11	100	300	0.00480	16.69	800
12	150	500	0.00310	16.76	970
13	40	160	0.00850	17.36	900
14	20	130	0.00511	18.70	700
15	25	185	0.00398	18.70	450
16	20	80	0.07120	14.26	370
17	30	85	0.00890	19.14	480
18	30	120	0.00713	18.92	680
19	40	120	0.00622	18.47	700
20	30	100	0.00773	19.79	850

Step 3 (fitness evaluation). Find the fitness of each agent using the fitness evaluation function ξ as given below in the form of mean square error (MSE):

$$\xi = \left(\frac{\sum_{j=1}^N (|C_{op}^j - \widehat{C}_{op}^j|^2)}{N} \right), \quad (10)$$

where $\widehat{C}_{op}^j$ is the approximated operating cost for j th generator.

Step 4 (termination criteria). The algorithm is terminated on the basis of the following criteria.

- (a) The predefined fitness value is achieved.
- (b) The total number of generations is executed.
- (c) Predefined value of the function tolerance is achieved.
- (d) The desired function evaluations are performed.

If any of the above criteria are met then go to Step 6 otherwise simulate Step 5.

Step 5 (reproduction). New population is generated using the operations evolved in PS and SA as per the setting given in Table 1 and then go to Step 3.

Step 6 (refinement via SQP method). One of the best results obtained by PS and SA is provided to SP as a start point for further refinement, and the parameter setting and values used for SQP algorithm are provided in Table 2.

Step 7 (storage). Store the global best of this cycle and repeat the procedure from Steps 3 to 7, to get sufficient numbers of independent runs to have sufficient data for performing analysis.

Step 8 (statistical analysis). The statistical analysis is made on the basis of four fundamental performance modes like best, worst, mean, and standard deviation and skewness.

4. Simulation and Results

In this section, a scenario based simulation is performed based on three case studies composed of thirteen, fifteen, and twenty generating units, respectively, for EDP problem in which the objective functions were ramp rate limit, prohibited operating zones in the power system operation; MFO and VPLe and transmission losses are also employed to demonstrate efficiency of proposed schemes using MATLAB built in subroutines of the optimizers like PS, SA, SQP, PS-SQP, and SA-SQP, respectively, whose parameters setting and values are provided in Tables 1 and 2. The reference data sheet for thirteen thermal units [37] with load demand of 3080 MW is provided in Appendix A. The data set used for fifteen thermal units [19–21] along with generating unit capacity, coefficients, ramp rate limits, and prohibited zones is provided in Appendix B while the load demand expected to be determined is 2630 MW. However, the data sheet for twenty thermal units of generating unit capacity and coefficients [38] are provided in Appendix C, where system supplies a total load demand of 2500 MW.

One of the best designed parameters is in fact the power generating values of generators and is presented in Figure 4 at semilog scale in order to widen the small effects in the power values. The lowest cost of the power generation can be obtained by taking the generating values from the hybrid schemes based on PS-SQP and SA-SQP, respectively, from Figures 4(a), 4(b), and 4(c) in fitness evaluation function provided in (10) along with the subsequent relations given in (1) and expressions of EDP constraints. The x -axis in Figure 4 defines the number of thermal units taken in the study and y -axis demonstrates the solvers PS, SA, SQP, PS-SQP, and SA-SQP, respectively, while z -axis provides the optimized values of the power for thermal units.

The optimized final values along with computational time for PS, SA, SQP, PS-SQP, and SA-SQP are provided in Table 3. The time analysis carried out for this paper is on Intel(R) Core (TM) i5-3210M CPU @ 2.50 GHz, 2.5 GHz, 4.0 GB of RAM, and running with MATLAB version 2012a. It is quite evident from the table that level of accuracy is dominated in case of hybrid schemes as compared to global search and local search techniques separately. However the computational budget of hybrid approach is a little bit more as compared to other presented schemes but this effect can be over shed due to the high accuracy attained by hybrid schemes. The local search scheme seems to be outperformed in terms of both accuracy and computational cost; however one cannot rely on one of these best values of SQP as the local search techniques are more probable to get stuck in the local optima due to bad initial guess. Another observation is that as the number of

TABLE 9: Reliability based on 100 independent runs for fifteen units system with $P_D = 3080$ MW [37].

Method	Minimum cost	Mean cost	STD of cost	Maximum cost	Mean CPU time
Proposed					
PS	17524.78	17659.34	1543.72	17860.12	0.394373
SA	17598.19	17623.78	1872.83	17765.23	14.15776
SQP	17653.11	17723.14	1712.34	189023.1	0.148936
PS-SQP	17201.98	17543.76	1612.35	17678.91	0.543309
SA-SQP	17491.17	17502.12	1709.12	17233.24	14.306696

FIGURE 4: Analysis of the optimizers for 100 independent runs for (a) thirteen generating units, (b) fifteen generating units, and (c) twenty generating units.

generating units increases the level of accuracy reduces while computational time increases due to increase in the EDP complexity.

The reliability of the stochastic algorithms is being tested by a comprehensive statistical analysis in terms of minimum of cost (Min_{cost}), maximum value of cost (Max_{cost}), mean cost value ($\text{Mean}_{\text{cost}}$), and standard deviation of the cost (STD_{cost}). The analysis is not only performed on cost value but also performed on computational time needed for the optimization in term of mean CPU (Mean_{CPU}) time. The results of the Monte Carlo simulations for stochastic solvers are narrated in Tables 4 and 5 for fifteen and twenty units, respectively, while for thirteen units the same analysis is provided in Appendix D.

It is quite evident from Tables 4 and 5 that the results of the proposed schemes are in good agreement in terms of reliability, effectiveness, economical cost, and computational budget

required in terms of time complexity; however the hybrid approaches PS-SQP and SA-SQP show more supremacy than that of PS, SA, and SQP. The computational budget of hybrid schemes seems to be computationally heavy of order of just few seconds; however this aspect seems to be minimal at the cost of level of accuracy achieved.

As the proposed scheme is based on heuristic computation one of the best results cannot define the effectiveness and reliability of the schemes; therefore in order to see the effectiveness of given stochastic methods the results are obtained for hundred independent runs rather on a single successful run. The fitness given in (10) obtained on all independent iterations is plotted in Figure 5 for three variations of generating units.

The percentage of convergence is also calculated by fixing a certain level of the threshold ($\xi \leq 10^{-10}$) in the level of fitness obtained by all proposed algorithms and the results

FIGURE 5: Process of proposed solution (a) overall procedure and (b) optimization of fitness function using hybrid computation.

are plotted in Figure 6 at four hundred independent runs of the given algorithms. It is observed from Figure 6(a) that all the optimizers provide 100% convergence for EDP with thirteen thermal units while for the case of fifteen units the convergence for SA is 97% and 100% for rest of the algorithms. Scenario three that is relatively stiff does not show convergence for PS on a threshold of ($\xi \leq 10^{-10}$), while the other optimizers like SA, SQP, PS-SQP, and SA-SQP are 100% convergent as evident from Figure 6(c).

5. Conclusions

On the basis of the results obtained by various optimizers in the last sections, the following conclusion can be drawn.

The proposed scheme based on SA-SQP and PS-SQP hybrid algorithms can provide reliable, effective, and well

optimized cost of EDP for the scenarios of thirteen, fifteen, and twenty thermal units for a demand of 3080 M, 2630 MW, and 2500 MW, respectively.

The comparative study between the solvers is carried out in terms of absolute cost, computational complexity, and fitness value achieved by the PS, SA, SQP, PS-SP, and SA-SP algorithms. The hybrid algorithms are found to be better than that of global and local search techniques applied independently for all variants of ESPs.

The results of statistical analysis determined for 100 independent runs of the design approaches show that the values in terms of mean, standard deviation, and minimum of the total cost error associated with PS-SQP and SA-SQP are considerably more dominated than SA, PS, and SQP techniques. The value of the mean execution time for finding the optimal weight with proposed schemes shows

FIGURE 6: Process of proposed solution (a) overall procedure and (b) optimization of fitness function using hybrid computation.

that the hybrid approaches take relatively more time than the other three techniques but this exception can be overshadowed due to its utmost accuracy and consistently convergent performance.

At a fitness threshold of $\varepsilon \leq 10^{-10}$, the algorithms based on hybrid schemes on average provide 100% convergence while the convergences of SA and PS are observed to be in a good range of acceptance, although in some situations PS remains consistent due to the same creation of mesh in the polling and searching domain.

The main advantages of given schemes lie in their ease in implementation, simplicity in the concept, minimal computational complexity, less hardware implementation burden, and suitability for real-time applications. In future, one may look for biological inspired computational method to analyze synthesis and optimize the other dynamical constraint based EDPs problems.

Appendices

A. Generating Units Capacity and Coefficients (13 Units)

See Table 6.

B. (a) Generating Units Capacity and Coefficients (15 Units) (b) Ramp-Rate Limits and Prohibited Zones (15 Units)

See Table 7.

C. Generating Units Capacity and Coefficients (20 Units)

See Table 8.

TABLE 10: Optimal solution of the thirteen, fifteen, and twenty thermal units, respectively, using solvers involving PS, SA, SQP, PS-SQP, and SA-SQP.

Solver	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17	P18	P19	P20
PS	616.84	320.36	256.93	60.00	60.00	60.00	60.00	60.00	60.00	40.00	40.00	55.00	55.00							
SA	167.57	124.92	102.42	126.36	177.93	134.57	114.18	176.51	159.49	103.92	106.64	105.45	116.08							
SQP	389.86	213.26	213.10	77.79	77.79	77.79	77.79	77.79	77.79	101.77	101.77	120.00	120.00							
PS-SQP	615.89	319.52	256.55	60.20	60.20	60.20	60.20	60.20	60.20	40.26	40.26	55.22	55.22							
SA-SQP	53.56	155.96	83.33	117.24	152.98	178.22	179.87	143.53	179.96	111.76	117.59	116.14	117.38							
PS	362.30	406.00	49.38	20.00	470.00	391.00	391.00	188.00	153.00	25.00	20.00	20.00	25.00	15.00	15.00					
SA	288.19	282.66	103.84	118.81	289.05	298.57	325.19	232.56	149.19	139.45	66.08	75.58	79.88	40.72	44.98					
SQP	455.00	455.00	20.00	20.00	470.00	383.22	135.00	300.00	54.64	25.00	20.00	20.00	85.00	55.00	15.00					
PS-SQP	362.30	406.00	49.38	20.00	470.00	391.00	391.00	188.00	153.00	25.00	20.00	20.00	25.00	15.00	15.00					
SA-SQP	348.05	206.00	129.58	129.29	286.20	214.17	387.15	199.47	155.91	150.20	79.58	63.07	83.53	53.47	51.26					
PS	592.55	178.00	178.00	178.00	50.00	20.00	25.00	50.00	178.00	30.00	100.00	406.00	40.00	20.00	153.00	21.00	30.00	30.00	40.00	30.00
SA	198.51	579.0	79.62	126.63	100.81	66.24	93.72	131.98	199.39	131.02	174.43	260.56	99.35	71.17	90.09	71.07	82.59	101.27	107.38	96.09
SQP	600.00	199.98	56.60	50.12	65.91	82.33	43.48	74.14	50.05	111.65	100.00	150.00	61.65	115.64	179.77	79.99	73.71	75.72	99.82	72.93
PS-SQP	592.39	177.98	177.97	177.97	50.02	20.04	25.03	50.03	177.98	30.03	100.01	405.90	40.03	20.03	152.99	21.03	30.03	30.03	40.03	30.03
SA-SQP	200.92	98.28	181.61	85.93	137.73	76.53	76.42	113.69	94.98	88.16	243.88	216.69	57.41	125.65	81.96	78.04	56.07	111.10	107.88	97.10

D. Reliability Comparison Based upon 100 Independent Runs for 15 Units with $P_D = 3080$ MW

See Table 9.

E. Optimal Solution for 13, 15, and 20 Units Using Solvers Involving PS, SA, SQP, PS-SQP, and SA-SQP

See Table 10.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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