Research Article

Hilbert Transforms along Convex Curves for l^q -Valued Functions

Honghai Liu

School of Mathematics and Information Science, Henan Polytechnic University, Jiaozuo, Henan 454003, China

Correspondence should be addressed to Honghai Liu; hhliu@hpu.edu.cn

Received 15 September 2013; Accepted 22 October 2013; Published 6 February 2014

Academic Editors: B. Djafari Rouhani, G. Lu, G. Schimperna, and I. Straškraba

Copyright © 2014 Honghai Liu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We show that Hilbert transforms along a large class of convex curves are bounded on $L^p(\mathbb{R}^2, l^q)$, where $5/3 < p, q < 5/2$.

1. Introduction

Let $\Gamma : \mathbb{R} \rightarrow \mathbb{R}^n$ be a curve in $\mathbb{R}^n$ with $\Gamma(0) = 0$, $n \geq 2$. For $f \in C_0^\infty(\mathbb{R}^n)$, the Hilbert transform along curve Γ is defined by the following principal-valued integral:

$$\mathcal{H}f(x) = \text{p.v.} \int_{\mathbb{R}} f(x - \Gamma(t)) \frac{dt}{t}. \quad (1)$$

There has been considerable interest to determine for which curves Γ , and which indices p , one has

$$\|\mathcal{H}f\|_{L^p} \leq A_p \|f\|_{L^p}, \quad (2)$$

for a constant A_p depending only on Γ and p . This problem has been extensively studied by a large number of authors. More results are found in [1–6].

The question of whether these results could be extended to the Lebesgue-Bôchner spaces $L^p(\mathbb{R}^n; X)$ of vector-valued functions was taken up by several authors recently, where X is some suitable Banach space. But the Banach space of most interest to us will be l^q for $1 < q < \infty$. Let us state some previous theorems which establish the background for our current work. The first is the work done in 1986 by Rubio de Francia et al. [7]. They dealt with well-curved curves and obtained the following result.

Theorem 1. Let $\Gamma : \mathbb{R} \rightarrow \mathbb{R}^n$ be a well-curved curve in $\mathbb{R}^n$ with $\Gamma(0) = 0$. Then the l^q -valued inequality

$$\left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}f_j|^q \right)^{1/q} \right\|_{L^p} \leq C_{p,q} \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p} \quad (3)$$

holds for all p, q with $1 < p, q < \infty$, and all $f_j \in L^p(\mathbb{R}^n)$, $n \geq 2$.

Recently, the author considered the submanifold of finite type which means that the image of the mapping $t \rightarrow \Gamma(t)$ is of finite type at 0. We first restrict our attention to $\Gamma(t)$ that are given as polynomial functions, because it is a model problem in this situation. We obtained the following.

Theorem 2 (see [8, 9]). (1) Let $\Gamma : \mathbb{R} \rightarrow \mathbb{R}^n$ be a polynomial function. Then the l^q -valued inequality

$$\left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}f_j|^q \right)^{1/q} \right\|_{L^p} \leq C_{p,q} \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p} \quad (4)$$

holds for all p, q with $1 < p, q < \infty$, and all $f_j \in L^p(\mathbb{R}^n)$, $n \geq 2$.

(2) Let the image of $\Gamma : \mathbb{R} \rightarrow \mathbb{R}^n$ be of finite type at 0. Then the l^q -valued inequality

$$\left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}f_j|^q \right)^{1/q} \right\|_{L^p} \leq C_{p,q} \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p} \quad (5)$$

holds for all p, q with $1 < p, q < \infty$, and all $f_j \in L^p(\mathbb{R}^n)$, $n \geq 2$.

An important feature of the well-curved curve is that it is in a sense approximated by the model curves; the models are homogeneous with respect to some nonisotropic dilations. This means that corresponding scaling arguments are essential for well-curved curves.

“The image of $\Gamma : \mathbb{R} \rightarrow \mathbb{R}^n$ is of finite type at 0” is a crucial condition imposed on Γ , this condition bears on finitely many derivatives of Γ at the origin, this property is captured by a Taylor polynomial of Γ at origin of sufficiently high degree and should be modeled by polynomial behavior. For polynomial function Γ , in order to utilize the scaling arguments, the situation need be simplified further. This is accomplished by using the lifting technique.

All these results have common roots in that both depend in a key way on the use of scaling arguments or nonisotropic dilations, although these dilations are given implicitly for above curves in different way. More precisely, an analytic family of convolution operators $\mathcal{H}_z$ are defined such that $\mathcal{H}_0 = \mathcal{H}$. The $L^2(\mathbb{R}^n, l^2)$ result of $\mathcal{H}_z$ is obtained with the help of the Fourier transform and Plancherel theorem for all z with $\operatorname{Re}(z) < a$, where a is some positive constant. $\mathcal{H}_z$ is closer to a more traditional singular integral when $\operatorname{Re}(z) < 0$, because there is some slack in the estimate of the kernel. Therefore, by the vector-valued Calderón-Zygmund theory of Benedek et al. in [10], one can obtain the $L^p(\mathbb{R}^n, l^q)$ result for all p, q with $1 < p, q < \infty$, $\operatorname{Re}(z) < 0$. By generalized analytic interpolation of operators (see Lemma 12), a combination of above two results gives the desired $L^p(\mathbb{R}^n, l^q)$ estimate for $\mathcal{H}$.

What about the situation when the curves Γ do not have appropriate homogeneity as well-curved curves or finite type curves? This kind of curves has been extensively considered in [1–5]. The Calderón-Zygmund type argument cannot be used to show that $\mathcal{H}_z$ is bounded in $L^p(\mathbb{R}^n, l^q)$ for $1 < p, q < \infty$ and $\operatorname{Re}(z) < 0$. New techniques should be utilized. In this paper, we deal with plane curves of the form $\Gamma(t) = (t, \gamma(t))$, where $\gamma(t)$ is a convex function for $t \geq 0$. We have the following main result.

Theorem 3. *Suppose that $\gamma(t)$ is a continuous odd function, twice continuously differentiable, increasing and convex for $t \geq 0$. Suppose also that γ'' is monotone for $t > 0$ and that there exists $C > 0$ so that $\gamma'(t) \leq C t \gamma''(t)$ for $t > 0$. Then $\mathcal{H}$ is bounded on $L^p(\mathbb{R}^2, l^q)$ for all p, q with $5/3 < p, q < 5/2$.*

The proof of Theorem 3 is given in several steps, which are similar to that in [4]. An analytic family of operators $\mathcal{H}_z$ are introduced so that $\mathcal{H}_0 = \mathcal{H}$. The $L^2(\mathbb{R}^2, l^2)$ result is equivalent to the boundedness of the multiplier associated to $\mathcal{H}_z$ with $\operatorname{Re}(z) < a$, where a is some positive constant. Van der Corput lemma and integration by part are used to obtain the boundedness. The kernels of $\mathcal{H}_z$ do not have appropriate homogeneity, we have to use the vector-valued Fourier multiplier theorem (see Lemma 11) to show that $\mathcal{H}_z$ is bounded on $L^p(\mathbb{R}^2, l^q)$ for p, q with $1 < p, q < \infty$, and $\operatorname{Re}(z) < b$, where b is some negative constant. Finally, the

theorem follows from above two facts by using generalized analytic interpolation of operators.

Remark 4. (1) Theorem 3 covers a large class of functions $\gamma(t)$ such as

$$\begin{aligned} \gamma(t) &= \operatorname{sgn}(t) |t|^\alpha, \quad (\alpha \geq 2), \\ \gamma(t) &= t e^{-1/|t|}. \end{aligned} \quad (6)$$

The first one is homogeneous, while the other one is without homogeneity. We extend the class of curves for which the $L^p(\mathbb{R}^2, l^q)$ -result is known for $5/3 < p, q < 5/2$. On the other hand, we also generalized Theorem 3.1 of Nagel and Wainger in [4] to the l^q -valued setting for $5/3 < q < 5/2$.

(2) In the present work, the kernels must be smoothed a definite amount, we are unable to take $a \geq 1/4$ or $b \geq -1$. Hence we obtain bounds for $\mathcal{H}$ on $L^p(\mathbb{R}^2, l^q)$ only when $5/3 < p, q < 5/2$.

2. Proof of the Theorem

For $z \in \mathbb{C}$, we define an analytic family of operators $\mathcal{H}_z$ by

$$\widehat{\mathcal{H}_z f}(\xi, \eta) = m_z(\xi, \eta) \widehat{f}(\xi, \eta), \quad (7)$$

where m_z are given by

$$m_z(\xi, \eta) = \text{p.v.} \int_{\mathbb{R}} e^{-2\pi i [\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t}. \quad (8)$$

Obviously, $\mathcal{H}_0$ is our original operator $\mathcal{H}$.

2.1. The Boundedness of $\mathcal{H}_z$ on $L^2(\mathbb{R}^2, l^2)$. In this subsection, we prove that

$$\left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}_z f_j|^2 \right)^{1/2} \right\|_{L^2} \leq C_\delta [1 + |\operatorname{Im}(z)|] \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^2 \right)^{1/2} \right\|_{L^2}, \quad (9)$$

where $\operatorname{Re}(z) = (1/4) - \delta$ for some $\delta > 0$.

Clearly, the boundedness of $\mathcal{H}_z$ on $L^2(\mathbb{R}^2, l^2)$ is equivalent to the uniform boundedness of the corresponding multiplier $m_z(\xi, \eta)$. Thus, we just need to show that

$$|m_z(\xi, \eta)| \leq C_\delta [1 + |\operatorname{Im}(z)|], \quad (10)$$

where the constant C_δ is independent of $\operatorname{Im}(z)$.

In proving (10), some of our estimates will require the following lemmas. The first one is detailed in [4, Lemma 2.1].

Lemma 5. *Let $A < B$, let $h \in L^\infty((A, B)) \cap C^1((A, B))$, and suppose that $(h(t)/t)' = 0$ has at most k roots in $(A, B) \setminus (-1, 1)$. Then*

$$\left| \int_A^B \sin(t) h(t) \frac{dt}{t} \right| \leq (2 + 4k) \|h\|_\infty. \quad (11)$$

The second one is Van der Corput lemma which plays an important role in estimating related multipliers. This lemma appears in several books or papers, compare, for example, Stein [11, P.332].

Lemma 6. Suppose ϕ is real-valued and smooth in (a, b) , and that $|\phi^{(k)}(x)| \geq 1$ for all $x \in (a, b)$. Then

$$\left| \int_a^b e^{i\lambda\phi(x)} dx \right| \leq C_k \lambda^{-1/k} \quad (12)$$

holds when $k \geq 2$; the bound C_k is independent of ϕ and λ .

To estimate $|m_z(\xi, \eta)|$, for fixed η , we choose $t_0 > 0$ so that $|\eta|\gamma(t_0) = 1$ and decompose m_z as

$$\begin{aligned} |m_z(\xi, \eta)| &\leq \left| \text{p.v.} \int_{|t| < t_0} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right| \\ &\quad + \left| \int_{|t| \geq t_0} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right|. \end{aligned} \quad (13)$$

First of all, we consider the first integral. For any $0 < \varepsilon < t_0$,

$$\begin{aligned} &\left| \int_{\varepsilon < |t| < t_0} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right| \\ &\leq \left| \int_{\varepsilon < |t| < t_0} e^{-2\pi i \xi t} (e^{-2\pi i \eta \gamma(t)} - 1) [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right| \\ &\quad + \left| \int_{\varepsilon < |t| < t_0} e^{-2\pi i \xi t} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right| := \text{I} + \text{II}. \end{aligned} \quad (14)$$

Notice that $1 \leq 1 + \eta^2 \gamma^2(t) \leq 2$ for $|t| \leq t_0$. Because of the convexity of $\gamma(t)$, we have $\gamma(t) \leq t\gamma'(t)$ for $t > 0$. Thus,

$$\begin{aligned} \text{I} &\leq \int_{\varepsilon < |t| < t_0} |e^{-2\pi i \eta \gamma(t)} - 1| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \frac{dt}{|t|} \\ &\leq C \int_{\varepsilon < |t| < t_0} |\eta \gamma(t)| [1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \frac{dt}{|t|} \\ &\leq C 2^{5/4-\delta} |\eta| \int_{\varepsilon}^{t_0} \gamma(t) \frac{dt}{t} \leq C 2^{5/4-\delta} |\eta| \int_{\varepsilon}^{t_0} \gamma'(t) dt \\ &\leq C_{\delta} |\eta| \gamma(t_0) \leq C_{\delta}. \end{aligned} \quad (15)$$

For the integral II, note that $\gamma(t)$ is odd. An easy calculation shows that

$$\begin{aligned} \text{II} &= 2 \left| \int_{\varepsilon}^{t_0} \sin(2\pi \xi t) [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \right| \\ &\leq 2 \left| \int_{\varepsilon}^{t_0} \sin(2\pi \xi t) [1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \right. \\ &\quad \times \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \frac{dt}{t} \Big| \\ &\quad + 2 \left| \int_{\varepsilon}^{t_0} \sin(2\pi \xi t) [1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \right. \\ &\quad \times \sin[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \frac{dt}{t} \Big|. \end{aligned} \quad (16)$$

It is trivial that $[1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))]$ is piecewise monotone in $[\varepsilon, t_0]$. Further, it has at most $C[1 + |\text{Im}(z)|]$ monotone intervals in $[\varepsilon, t_0]$. Indeed, the derivative of $[1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))]$ is

$$\begin{aligned} &\frac{2\eta^2 \gamma(t) \gamma'(t)}{[1 + \eta^2 \gamma^2(t)]^{3/4+\delta}} \\ &\times \left\{ \left(\frac{1}{4} - \delta \right) \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \right. \\ &\quad \left. - \text{Im}(z) \sin[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \right\}. \end{aligned} \quad (17)$$

$\gamma(t)\gamma'(t)$ is nonzero for $t \in [\varepsilon, t_0]$. So, we consider the following equation:

$$\begin{aligned} &\left(\frac{1}{4} - \delta \right) \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \\ &\quad - \text{Im}(z) \sin[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] = 0. \end{aligned} \quad (18)$$

We suppose $\text{Im}(z) \neq 0$; otherwise, it is obvious. Then, above equation is

$$\tan[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] = \frac{(1/4 - \delta)}{\text{Im}(z)}. \quad (19)$$

For $\text{Im}(z) > 0$ and $t \in [\varepsilon, t_0]$, we have $\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t)) \in (0, \text{Im}(z) \ln 2)$. There are about $(\text{Im}(z) \ln 2/\pi) + 1$ periods in interval $(0, \text{Im}(z) \ln 2)$, only one root exists in one period. So, (19) has at most $(\text{Im}(z) \ln 2/\pi) + 1$ roots. That mean that $[1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))]$ has at most $C[1 + |\text{Im}(z)|]$ stationary points in $[\varepsilon, t_0]$, that is there are at most $C[1 + |\text{Im}(z)|]$ monotone intervals. The case $\text{Im}(z) < 0$ can be treated in the same way. We suppose it is monotone, otherwise, we deal with the integral on every monotone interval. By the second mean value theorem and Lemma 5, there exists $t_1 \in (\varepsilon, t_0)$ such that

$$\begin{aligned} &\left| \int_{\varepsilon}^{t_0} \sin(2\pi \xi t) [1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \right. \\ &\quad \times \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t))] \frac{dt}{t} \Big| \\ &\leq [1 + \eta^2 \gamma^2(\varepsilon)]^{1/4-\delta} \left| \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(\varepsilon))] \right| \\ &\quad \times \left| \int_{\varepsilon}^{t_1} \sin(2\pi \xi t) \frac{dt}{t} \right| \\ &\quad + [1 + \eta^2 \gamma^2(t_0)]^{1/4-\delta} \left| \cos[\text{Im}(z) \ln(1 + \eta^2 \gamma^2(t_0))] \right| \\ &\quad \times \left| \int_{t_1}^{t_0} \sin(2\pi \xi t) \frac{dt}{t} \right| \leq C_{\delta} [1 + |\text{Im}(z)|]. \end{aligned} \quad (20)$$

In the same way, we can prove that

$$\begin{aligned} & \left| \int_{\varepsilon}^{t_0} \sin(2\pi\xi t) [1 + \eta^2 \gamma^2(t)]^{1/4-\delta} \right. \\ & \quad \times \sin \left[\operatorname{Im}(z) \ln(1 + \eta^2 \gamma^2(t)) \right] \frac{dt}{t} \Big| \\ & \leq C_{\delta} [1 + |\operatorname{Im}(z)|]. \end{aligned} \quad (21)$$

We next deal with the the part of the integral where $|t| \geq t_0$. If $\gamma''(t)$ is monotone increasing, we set

$$\varphi(t) = - \int_t^{\infty} e^{-2\pi i[\xi s + \eta \gamma(s)]} ds, \quad (22)$$

while if $\gamma''(t)$ is monotone decreasing, put

$$\varphi(t) = \int_{t_0}^t e^{-2\pi i[\xi s + \eta \gamma(s)]} ds. \quad (23)$$

Obviously, $\varphi'(t) = e^{-2\pi i[\xi t + \eta \gamma(t)]}$. By integration by part, we have

$$\begin{aligned} & \int_{t_0}^{\infty} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \\ & = \int_{t_0}^{\infty} \varphi'(t) [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \\ & = \varphi(t) \frac{[1 + \eta^2 \gamma^2(t)]^z}{t} \Big|_{t_0}^{\infty} \\ & \quad - \int_{t_0}^{\infty} \varphi(t) [1 + \eta^2 \gamma^2(t)]^{z-1} \\ & \quad \times \frac{2z\eta^2 t \gamma(t) \gamma'(t) - 1 - \eta^2 \gamma^2(t)}{t^2} dt. \end{aligned} \quad (24)$$

Notice that $|\xi s + \eta \gamma(s)|'' = |\eta| \gamma''(s) \geq |\eta| \gamma''(t)$ for $s \in [t, \infty)$ or $s \in [t_0, t]$. Van der Corput lemma shows that $|\varphi(t)| \leq C[|\eta| \gamma''(t)]^{-1/2}$ in either case. A combination of $\gamma(t) \leq t \gamma'(t)$ with the hypothesis $\gamma'(t) \leq C t \gamma''(t)$ implies that $\gamma(t) \leq C t^2 \gamma''(t)$. So, for the boundary terms, $t \in [t_0, \infty)$, we have

$$\begin{aligned} & \left| \varphi(t) \frac{[1 + \eta^2 \gamma^2(t)]^z}{t} \right| \\ & \leq C [|\eta| \gamma''(t)]^{-1/2} \frac{[2\eta^2 \gamma^2(t)]^{1/4-\delta}}{|t|} \\ & \leq C [|\eta| \gamma''(t) t^2]^{-1/2} [\eta \gamma(t)]^{1/2-2\delta} \\ & \leq C [\eta \gamma(t)]^{-2\delta} \leq C. \end{aligned} \quad (25)$$

Finally, we consider the integrated terms; it is bounded by

$$\begin{aligned} & \left| \int_{t_0}^{\infty} \varphi(t) [1 + \eta^2 \gamma^2(t)]^{z-1} \right. \\ & \quad \times \frac{2z\eta^2 t \gamma(t) \gamma'(t) - 1 - \eta^2 \gamma^2(t)}{t^2} dt \Big| \\ & \leq C \int_{t_0}^{\infty} [|\eta| \gamma''(t)]^{-1/2} [|\eta| \gamma(t)]^{-3/2-2\delta} \\ & \quad \times (2[|\operatorname{Re}(z)| + |\operatorname{Im}(z)|] \eta^2 t \gamma(t) \\ & \quad \times \gamma'(t) + 2\eta^2 \gamma^2(t)) \times (t^2)^{-1} dt \\ & \leq C [|\operatorname{Re}(z)| + |\operatorname{Im}(z)|] |\eta|^{-2\delta} \\ & \quad \times \int_{t_0}^{\infty} [t^2 \gamma''(t)]^{-1/2} [\gamma(t)]^{-1/2-2\delta} \gamma'(t) dt \\ & \quad + C |\eta|^{-2\delta} \int_{t_0}^{\infty} [t^2 \gamma''(t)]^{-1/2} [\gamma(t)]^{-1/2-2\delta} \frac{\gamma(t)}{t} dt \\ & \leq C [1 + |\operatorname{Im}(z)|] |\eta|^{-2\delta} \\ & \quad \times \int_{t_0}^{\infty} [\gamma(t)]^{-1-2\delta} \gamma'(t) dt \leq C_{\delta} [1 + |\operatorname{Im}(z)|]. \end{aligned} \quad (26)$$

The integral

$$\int_{-\infty}^{-t_0} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \quad (27)$$

can be handled similarly. This completes the proof of (10).

2.2. The Boundedness of $\mathcal{H}_z$ on $L^p(\mathbb{R}^2, l^q)$. In this subsection, we show that

$$\left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}_z f_j|^q \right)^{1/q} \right\|_{L^p} \leq C [1 + |\operatorname{Im}(z)|]^2 \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p}, \quad (28)$$

where $\operatorname{Re}(z) < -1$, $1 < p, q < \infty$, the constant C depends on $\operatorname{Re}(z)$, and is independent of $\operatorname{Im}(z)$.

To prove (28), we need a vector-valued Fourier multiplier theorem. We have to recall some definitions before we present the theorem.

Definition 7. Let X be a Banach space; X is said to be an UMD-space, if the Hilbert transform is bounded on $L^p(\mathbb{R}, X)$ for some (and then all) $p \in (1, \infty)$.

Definition 8. A basis $\{e_j\}_j$ in a Banach space X is called unconditional, if there is a constant C such that for every $x = \sum_j t_j e_j \in X$ one has $\|\sum_j \epsilon_j t_j e_j\| \leq C \|x\|$, for all $\epsilon_j = \pm 1$ for $j \in \mathbb{N}$.

Definition 9. A Banach X is said to have local unconditional structure if there exists a constant C such that for any finite-dimensional subspace X_0 of X there exists a Banach space F

with an unconditional basis and operators $u_0 : X_0 \rightarrow F$ and $w_0 : F \rightarrow X$ such that the natural embedding $j : X_0 \rightarrow X$ admits a factorization $j = w_0 u_0$ and $\|u_0\| \|w_0\| \leq C$.

Remark 10. Well-known examples of UMD-spaces are $L^p(\mathbb{R}^n)$, l^q and $L^{p,q}(\mathbb{R}^n)$, $1 < p, q < \infty$, three examples are also Banach lattices for their usual norm and the pointwise order. Further, all Banach lattices have local unconditional structure.

Let $m : \mathbb{R}^n \rightarrow \mathbb{C}$ be a bounded function, we associate operators T_m defined on the test functions $f \in \mathcal{S}(\mathbb{R}^n, X)$ by

$$T_m f(x) = (m \hat{f})^\vee(x). \quad (29)$$

As a vector-valued Fourier multiplier theorem, we state the following vector-valued Mikhlin theorem which already was proved in [12].

Lemma 11. *Let X be an UMD-space with local unconditional structure. Then for any $p \in (1, \infty)$ there is a constant $C < \infty$ such that*

$$\begin{aligned} \|T_m\|_{L^p(\mathbb{R}^n, X)} \\ \leq C \sup \{ |x^\alpha \partial^\alpha m(x)| : x \in \mathbb{R}^n \setminus \{0\}, \alpha \leq (1, 1, \dots, 1) \}. \end{aligned} \quad (30)$$

For $n = 2$, in view of Lemma 11 and Remark 10, it suffices to show that the following functions

$$\begin{aligned} m_z(\xi, \eta), \quad \xi \frac{\partial m_z}{\partial \xi}(\xi, \eta), \quad \eta \frac{\partial m_z}{\partial \eta}(\xi, \eta), \\ \xi \eta \frac{\partial^2 m_z}{\partial \xi \partial \eta}(\xi, \eta) \end{aligned} \quad (31)$$

are uniformly bounded on $\mathbb{R}^2$ for $\text{Re}(z) < -1$.

The uniform boundedness of $m_z(\xi, \eta)$ is trivial, it can be established by minor modification of the proof of (10). Without repetition, we omit the proof.

2.2.1. The Boundedness of $\xi(\partial m_z / \partial \xi)(\xi, \eta)$. Integration by part implies that

$$\begin{aligned} \xi \frac{\partial m_z}{\partial \xi}(\xi, \eta) \\ = -2\pi i \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \xi [1 + \eta^2 \gamma^2(t)]^z dt \\ = \int_{\mathbb{R}} \frac{d}{dt} (e^{-2\pi i \xi t}) e^{-2\pi i \eta \gamma(t)} [1 + \eta^2 \gamma^2(t)]^z dt \\ = e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^z \Big|_{-\infty}^{\infty} \\ + 2\pi i \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \gamma'(t) [1 + \eta^2 \gamma^2(t)]^z dt \\ - 2z \eta^2 \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \gamma(t) \gamma'(t) dt. \end{aligned} \quad (32)$$

Note that $\text{Re}(z) < -1$, for $t \in \mathbb{R}$, we have $|[1 + \eta^2 \gamma^2(t)]^z| = [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \leq 1$. The boundary terms is bounded by $1/\pi$.

For $\text{Re}(z) < -1$, making the change of variables $u = |\eta| \gamma(t)$, we obtain

$$\begin{aligned} \left| \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \gamma'(t) [1 + \eta^2 \gamma^2(t)]^z dt \right| \\ \leq \int_{\mathbb{R}} \gamma'(t) |\eta| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} dt \\ \leq \int_{\mathbb{R}} (1 + u^2)^{\text{Re}(z)} du \leq \pi. \end{aligned} \quad (33)$$

In the similar way, the second integrated term can be dominated by

$$\begin{aligned} \left| z \eta^2 \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \gamma(t) \gamma'(t) dt \right| \\ \leq 2|z| \int_0^\infty [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)-1} \eta^2 \gamma(t) \gamma'(t) dt \\ \leq |z| \int_0^\infty (1 + u)^{\text{Re}(z)-1} du \\ \leq \frac{|z|}{|\text{Re}(z)|} \leq 1 + |\text{Im}(z)|. \end{aligned} \quad (34)$$

Therefore, for $\text{Re}(z) < -1$,

$$\left| \xi \frac{\partial m_z}{\partial \xi}(\xi, \eta) \right| \leq C [1 + |\text{Im}(z)|]. \quad (35)$$

2.2.2. The Boundedness of $\eta(\partial m_z / \partial \eta)(\xi, \eta)$. Integrating by parts, we obtain

$$\begin{aligned} \eta \frac{\partial m_z}{\partial \eta}(\xi, \eta) \\ = -2\pi i \text{p.v.} \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma(t) [1 + \eta^2 \gamma^2(t)]^z \frac{dt}{t} \\ + 2z \text{p.v.} \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} \frac{dt}{t}. \end{aligned} \quad (36)$$

To estimate above two integrals, we follow the argument similar to that in the proof of (10). For the first integral, for any $\varepsilon > 0$, it suffices to bound the following two parts

$$\begin{aligned} \int_{\varepsilon < |t| < t_0} |\eta| |\gamma(t)| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \frac{dt}{|t|}, \\ \int_{|t| \geq t_0} |\eta| |\gamma(t)| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \frac{dt}{|t|}. \end{aligned} \quad (37)$$

Recall that $t_0 > 0$ was chosen so that $|\eta| \gamma(t_0) = 1$, and $\gamma(t) \leq t \gamma'(t)$ because of the convexity. Thus,

$$\begin{aligned} \int_{\varepsilon < |t| < t_0} |\eta| |\gamma(t)| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \frac{dt}{|t|} \\ \leq 2|\eta| \int_0^{t_0} \frac{\gamma(t)}{t} dt \leq 2|\eta| \int_0^{t_0} \gamma'(t) dt \leq 2. \end{aligned} \quad (38)$$

For $\text{Re}(z) < -1$, an elementary calculation shows that

$$\begin{aligned} & \int_{|t| \geq t_0} |\eta| |\gamma(t)| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \frac{dt}{|t|} \\ & \leq 2|\eta|^{2\text{Re}(z)+1} \int_{t_0}^{\infty} \gamma^{2\text{Re}(z)}(t) \frac{\gamma(t)}{t} dt \\ & \leq \frac{2}{|2\text{Re}(z) + 1|} \leq 2. \end{aligned} \quad (39)$$

Similarly, the second integral can be controlled by

$$\begin{aligned} & \left| z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} \frac{dt}{t} \right| \\ & \leq 2|z| \int_0^{t_0} \eta^2 \gamma^2(t) \frac{dt}{t} \\ & \quad + 2|z| \int_{t_0}^{\infty} \eta^2 \gamma^2(t) [\eta^2 \gamma^2(t)]^{\text{Re}(z)-1} \frac{dt}{t} \\ & \leq 2|z| \eta^2 \int_0^{t_0} \gamma(t) \gamma'(t) dt \\ & \quad + 2|z| \eta^{2\text{Re}(z)} \int_{t_0}^{\infty} \gamma^{2\text{Re}(z)-1}(t) \gamma'(t) dt \\ & \leq |z| + \frac{|z|}{|\text{Re}(z)|} \leq 2|\text{Re}(z)| [1 + |\text{Im}(z)|]. \end{aligned} \quad (40)$$

Therefore, for $\text{Re}(z) < -1$,

$$\left| \xi \frac{\partial m_z}{\partial \xi}(\xi, \eta) \right| \leq C [1 + |\text{Im}(z)|]. \quad (41)$$

2.2.3. The Boundedness of $\xi \eta (\partial^2 m_z / \partial \xi \partial \eta)(\xi, \eta)$. To take care of $\xi \eta (\partial^2 m_z / \partial \xi \partial \eta)(\xi, \eta)$, we note that it can be written as

$$\begin{aligned} & \xi \eta \frac{\partial^2 m_z}{\partial \xi \partial \eta}(\xi, \eta) \\ & = -4\pi^2 \xi \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \gamma(t) [1 + \eta^2 \gamma^2(t)]^z dt \\ & \quad - 4\pi i z \xi \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \eta \gamma^2(t) dt. \end{aligned} \quad (42)$$

For the first term, integrating by parts, we obtain

$$\begin{aligned} & 4\pi^2 \xi \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \gamma(t) [1 + \eta^2 \gamma^2(t)]^z dt \\ & = 2\pi i \int_{\mathbb{R}} \frac{d}{dt} (e^{-2\pi i \xi t}) e^{-2\pi i \eta \gamma(t)} [\eta \gamma(t)] [1 + \eta^2 \gamma^2(t)]^z dt \\ & = 2\pi i e^{-2\pi i[\xi t + \eta \gamma(t)]} [\eta \gamma(t)] [1 + \eta^2 \gamma^2(t)]^z \Big|_{-\infty}^{\infty} \\ & \quad - 4\pi^2 \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) [\eta \gamma(t)] [1 + \eta^2 \gamma^2(t)]^z dt \\ & \quad - 2\pi i \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) [1 + \eta^2 \gamma^2(t)]^z dt \\ & \quad - 4\pi i z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \eta^3 \gamma^2(t) \gamma'(t) dt. \end{aligned} \quad (43)$$

Obviously, for $\text{Re}(z) < -1$, $t \in \mathbb{R}$, $|2\pi i e^{-2\pi i[\xi t + \eta \gamma(t)]} [\eta \gamma(t)] [1 + \eta^2 \gamma^2(t)]^z| \leq 2\pi |\eta| |\gamma(t)| [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \leq 2\pi$. So, the boundary terms is bounded by 2π .

For the first integrated terms, making the change of variables $u = \eta^2 \gamma^2(t)$, one has

$$\begin{aligned} & \left| \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) [\eta \gamma(t)] [1 + \eta^2 \gamma^2(t)]^z dt \right| \\ & \leq 2 \int_0^{\infty} [1 + \eta^2 \gamma^2(t)]^{\text{Re}(z)} \eta^2 \gamma(t) \gamma'(t) dt \\ & \leq \int_0^{\infty} (1 + u)^{\text{Re}(z)} du \leq \frac{1}{|\text{Re}(z) + 1|}. \end{aligned} \quad (44)$$

The second integrated terms can be treated in the same way, let $u = \eta \gamma(t)$,

$$\begin{aligned} & \left| \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) [1 + \eta^2 \gamma^2(t)]^z dt \right| \\ & \leq \int_{\mathbb{R}} (1 + u^2)^{\text{Re}(z)} du \leq \pi. \end{aligned} \quad (45)$$

Similarly, a trivial calculation shows that

$$\begin{aligned} & \left| z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \eta^3 \gamma^2(t) \gamma'(t) dt \right| \\ & \leq 2|z| \int_0^{\infty} u^2 (1 + u^2)^{\text{Re}(z)-1} du \leq \pi |z|. \end{aligned} \quad (46)$$

The second term can be handled similarly. Integrating by parts, we decomposed it as

$$\begin{aligned}
& 4\pi iz \xi \eta \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-1} \eta \gamma^2(t) dt \\
&= 2\pi iz \int_{\mathbb{R}} \frac{d}{dt} (e^{-2\pi i \xi t}) e^{-2\pi i \eta \gamma(t)} \\
&\quad \times \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} dt \\
&= 2\pi iz e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} \Big|_{-\infty}^{\infty} \\
&\quad - 4\pi^2 z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) \\
&\quad \times \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} dt \\
&\quad - 4\pi iz \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma(t) \gamma'(t) [1 + \eta^2 \gamma^2(t)]^{z-1} dt \\
&\quad - 4\pi iz (z-1) \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-2} \\
&\quad \times \eta^2 \gamma(t) \gamma'(t) dt.
\end{aligned} \tag{47}$$

Obviously, for $\operatorname{Re}(z) < -1$, $t \in \mathbb{R}$, $|ze^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1}| \leq |z|$. The boundary term is dominated by $4\pi|z|$.

For the first integrated terms, by making the change of variables $u = \eta \gamma(t)$, we have the estimate

$$\begin{aligned}
& \left| z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta \gamma'(t) \eta^2 \gamma^2(t) [1 + \eta^2 \gamma^2(t)]^{z-1} dt \right| \\
& \leq |z| \int_{\mathbb{R}} u^2 (1 + u^2)^{\operatorname{Re}(z)-1} dt \leq \pi |z|.
\end{aligned} \tag{48}$$

To estimate the second integrated terms, we make the transformation $u = \eta^2 \gamma^2(t)$ and get

$$\begin{aligned}
& \left| z \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} \eta^2 \gamma(t) \gamma'(t) [1 + \eta^2 \gamma^2(t)]^{z-1} dt \right| \\
& \leq |z| \int_0^{\infty} (1 + u)^{\operatorname{Re}(z)-1} du \leq \frac{|z|}{|\operatorname{Re}(z)|}.
\end{aligned} \tag{49}$$

Similarly, the third integrated terms can be treated as

$$\begin{aligned}
& \left| z(z-1) \int_{\mathbb{R}} e^{-2\pi i[\xi t + \eta \gamma(t)]} [1 + \eta^2 \gamma^2(t)]^{z-2} \eta^4 \gamma^3(t) \gamma'(t) dt \right| \\
& \leq |z(z-1)| \int_0^{\infty} (1 + u)^{\operatorname{Re}(z)-1} du \leq \frac{|z(z-1)|}{|\operatorname{Re}(z)|}.
\end{aligned} \tag{50}$$

Note that for $\operatorname{Re}(z) < -1$, we have the following elementary estimates

$$\begin{aligned}
|z| &\leq |\operatorname{Re}(z)| [1 + |\operatorname{Im}(z)|], \\
|z-1| &\leq |\operatorname{Re}(z)-1| [1 + |\operatorname{Im}(z)|].
\end{aligned} \tag{51}$$

Finally, combining above eight estimates, we obtain

$$\left| \xi \eta \frac{\partial^2 m_z}{\partial \xi \partial \eta}(\xi, \eta) \right| \leq C[1 + \operatorname{Im}(z)]^2. \tag{52}$$

2.3. Conclusion. To complete the proof of our main result, we need a l^q -valued analytic interpolation theorem, which is a consequence of Theorem 1 in [13], a concrete proof can be found in [14].

Let S denote the closed strip $\{a \leq \operatorname{Re}(z) \leq b\} \subset C$. $\{U_z\}_{z \in S}$ is a family of uniformly bounded linear operators on $L^2(\mathbb{R}^n)$, that is, there is an $M > 0$ such that

$$\|U_z\|_{L^2 \rightarrow L^2} \leq M, \quad \forall z \in S. \tag{53}$$

Moreover, the function

$$z \mapsto \int_{\mathbb{R}^n} U_z f(x) g(x) dx \tag{54}$$

is continuous on S and analytic in the interior of S , whenever $f, g \in L^2(\mathbb{R}^n)$.

Lemma 12. *With the above assumptions, if for $1 < p_i, q_i < \infty$, $(i = 0, 1)$, $\{U_z\}$ satisfies the following conditions:*

- (i) $\|(\sum_{j \in \mathbb{Z}} |U_z f_j|^{q_0})^{1/q_0}\|_{L^{p_0}} \leq M_0 \|(\sum_{j \in \mathbb{Z}} |f_j|^{q_0})^{1/q_0}\|_{L^{p_0}}$
when $\operatorname{Re}(z) = a$;
- (ii) $\|(\sum_{j \in \mathbb{Z}} |U_z f_j|^{q_1})^{1/q_1}\|_{L^{p_1}} \leq M_1 \|(\sum_{j \in \mathbb{Z}} |f_j|^{q_1})^{1/q_1}\|_{L^{p_1}}$
when $\operatorname{Re}(z) = b$.

Then we have

$$\left\| \left(\sum_{j \in \mathbb{Z}} |U_{a(1-\theta)+\theta b} f_j|^q \right)^{1/q} \right\|_{L^p} \leq M_0^{1-\theta} M_1^{\theta} \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p}, \tag{55}$$

where $0 < \theta < 1$, $1/q = ((1-\theta)/q_0) + \theta/q_1$ and $1/p = ((1-\theta)/p_0) + \theta/p_1$.

Let $U_z f(x) = e^{z^2} \mathcal{H}_z f(x)$. Note that $|e^{z^2}| = e^{\operatorname{Re}(z)^2 - \operatorname{Im}(z)^2}$, then there exists a constant M_0 which is independent of $\operatorname{Im}(z)$ such that

$$\begin{aligned}
& \left\| \left(\sum_{j \in \mathbb{Z}} |U_z f_j|^2 \right)^{1/2} \right\|_{L^2} \\
& \leq C_{\delta} e^{-\operatorname{Im}(z)^2} [1 + |\operatorname{Im}(z)|] \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^2 \right)^{1/2} \right\|_{L^2} \\
& \leq M_0 \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^2 \right)^{1/2} \right\|_{L^2} \quad \text{when } \operatorname{Re}(z) = 1/4 - \delta.
\end{aligned} \tag{56}$$

Also, for $1 < p_1, q_1 < \infty$, there exists a constant M_1 which is independent of $\text{Im}(z)$ such that

$$\left\| \left(\sum_{j \in \mathbb{Z}} |U_z f_j|^{q_1} \right)^{1/q_1} \right\|_{L^{p_1}} \leq M_1 \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^{q_1} \right)^{1/q_1} \right\|_{L^{p_1}} \quad (57)$$

when $\text{Re}(z) < -1$.

For $2 \leq p, q < 5/2$, there exists $1 < p_1, q_1 < \infty$ so that

$$\frac{1}{p} = \frac{\theta}{2} + \frac{1-\theta}{p_1}, \quad \frac{1}{q} = \frac{\theta}{2} + \frac{1-\theta}{q_1}, \quad (58)$$

where $\theta \in (0, 1)$ and satisfies the equation

$$\left(\frac{1}{4} - \delta \right) \theta + (-1 - \varepsilon)(1 - \theta) = 0 \quad (59)$$

for some $\varepsilon > 0$ and $0 < \delta < 1/4$. Therefore, by Lemma 12, we obtain

$$\begin{aligned} r! \left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H} f_j|^q \right)^{1/q} \right\|_{L^p} &= \left\| \left(\sum_{j \in \mathbb{Z}} |\mathcal{H}_0 f_j|^q \right)^{1/q} \right\|_{L^p} \\ &= \left\| \left(\sum_{j \in \mathbb{Z}} |U_0 f_j|^q \right)^{1/q} \right\|_{L^p} \\ &\leq C \left\| \left(\sum_{j \in \mathbb{Z}} |f_j|^q \right)^{1/q} \right\|_{L^p} \end{aligned} \quad (60)$$

for $2 \leq p, q < 5/2$. The duality implies the result for $5/3 < p, q \leq 2$. This completes the proof of Theorem 3.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The author was supported by the National Natural Science Foundation of China (no. 11226103 and no. 11371057) and Doctor Foundation of Henan Polytechnic University (no. B2011-034).

References

- [1] A. Carbery, M. Christ, J. Vance, S. Wainger, and D. K. Watson, "Operators associated to flat plane curves: L^p estimates via dilation methods," *Duke Mathematical Journal*, vol. 59, no. 3, pp. 675–700, 1989.
- [2] H. Carlsson, M. Christ, A. Córdoba et al., " L^p estimates for maximal functions and Hilbert transforms along flat convex curves in $\mathbb{R}^2$," *Bulletin of the American Mathematical Society*, vol. 14, no. 2, pp. 263–267, 1986.
- [3] A. Córdoba, A. Nagel, J. Vance, S. Wainger, and D. Weinberg, " L^p bounds for Hilbert transforms along convex curves," *Inventiones Mathematicae*, vol. 83, no. 1, pp. 59–71, 1986.
- [4] A. Nagel and S. Wainger, "Hilbert transforms associated with plane curves," *Transactions of the American Mathematical Society*, vol. 223, pp. 235–252, 1976.
- [5] A. Nagel, J. Vance, S. Wainger, and D. Weinberg, "Hilbert transforms for convex curves," *Duke Mathematical Journal*, vol. 50, no. 3, pp. 735–744, 1983.
- [6] E. M. Stein and S. Wainger, "Problems in harmonic analysis related to curvature," *Bulletin of the American Mathematical Society*, vol. 84, no. 6, pp. 1239–1295, 1978.
- [7] J. L. Rubio de Francia, F. J. Ruiz, and J. L. Torrea, "Calderón-Zygmund theory for operator-valued kernels," *Advances in Mathematics*, vol. 62, no. 1, pp. 7–48, 1986.
- [8] H. H. Liu, "The boundedness of maximal operators and singular integrals via Fourier transform estimates," *Acta Mathematica Sinica*, vol. 28, no. 11, pp. 2227–2242, 2012.
- [9] H. H. Liu, "Rough singular integrals supported on submanifolds of finite type," *Acta Mathematica Sinica*, vol. 55, no. 2, pp. 311–320, 2012.
- [10] A. Benedek, A. P. Calderón, and R. Panzone, "Convolution operators on Banach space valued functions," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 48, pp. 356–365, 1962.
- [11] E. M. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, vol. 43, Princeton University Press, Princeton, NJ, USA, 1993.
- [12] F. Zimmermann, "On vector-valued Fourier multiplier theorems," *Studia Mathematica*, vol. 93, no. 3, pp. 201–222, 1989.
- [13] M. Cwikel and S. Janson, "Interpolation of analytic families of operators," *Studia Mathematica*, vol. 79, no. 1, pp. 61–71, 1984.
- [14] Y. Chen, Y. Ding, and H. Liu, "Rough singular integrals supported on submanifolds," *Journal of Mathematical Analysis and Applications*, vol. 368, no. 2, pp. 677–691, 2010.