Research Article

Splitting Groups with Basis Property

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A finite group G is called splitting or splittable if it is a union of some collections of its proper subgroups intersecting pairwise at the identity. A special kind of splitting is known to be normal splitting. Also, a group G is said to have the basis property if, for each subgroup $H \leq G$, H has a basis (minimal generating set), and any two bases have the same cardinality. In this work, I discuss a relation between classes of finite groups that possess both normal splitting and the basis property. This paper shows mainly that any non- p -group with basis property is normal splitting. However, the converse is not true in general. A counterexample is given. It is well known that any p -group has basis property. I demonstrate some types of p -groups which are splitting as well.

1. Introduction

Miller [1] was the first one who studied the concept of splitting groups; then it was discussed by many authors such as Young [2] who gave a precise definition of this concept. Hugher and Thompson [3] introduced the definition of HT-groups as follows: a finite nilpotent group G (not p -group) is called HT-group if there exists a prime number p and a subgroup H which is generated by all elements of order not equal to p with $H \neq G$ and $p \mid |H|$.

In Baer [4, 5], Kegel [6], and Kontorovich [7], it was given a classification of solvable non- p -groups that are splitting. Also, Isaacs [8] studied the so-called equally splitting. Zassenhaus [9] had a great role in studying splitting groups, and Suzuki [10] had a crucial contribution in this regard by finding a new class of simple splitting groups, and finishing the topic of classifying unsolvable splitting finite groups. Sozutov and Shlepkov [11] generalized Kegel's theorem [6] on finite HT-groups and Baer's theorem [5]. I use the same notation and terminology that appeared in the references, especially [5, 12]. In [13], it is shown that any finite group with basis property is either elementary p -group, nonelementary p -group, or a group that can be written as $P \rtimes H$, where P is of order p^a and H is a cyclic q -group of order q^b where p, q are distinct primes and $a, b \in \mathbb{Z}^+$. This work shows that all finite groups with basis property of the first or third kind are

splitting, while groups of the second kind may or may not be splitting.

Let G be a nontrivial finite group. A collection $\{G_i\}$ of nontrivial subgroups of G is said to be splitting of G if every element $g \neq 1$ of G is contained in one and only one subgroup of $\{G_i\}$, and $\{G_i\}$ is called nontrivial if $G_i \neq G$ for all i .

Let G be a group with splitting $\{G_i\}$; then the subgroup G_i is called a component of this splitting. So, one has

$$G_i \cap G_j = \{1\}, \quad \text{if } i \neq j. \quad (1)$$

Let H be a subgroup of G ; then the collection of subgroups H_i of H defined by $H_i = G_i \cap H$ is a splitting of H . This is called the induced splitting. The induced splitting is nontrivial if and only if H is contained in no component of the splitting $\{G_i\}$.

Lemma 1 (Young [2]). *If a torsion abelian group has a splitting, then there exists a prime number p so that each nonidentity element is of order p . Moreover, if $|G| > p$ (prime number), and the nonidentity elements of G have order p , then G has a nontrivial splitting.*

Lemma 2 (Miller [1]). *An abelian group of finite order has a nontrivial splitting if and only if it is of order p^m ($m > 1$), and it must be an elementary abelian p -group.*

2. Some Results on Splitting p -Groups

In 1960-1961, Baer [4, 5], Kegel [6], and Suzuki [10] obtained the classification of finite splitting groups that are not p -groups, such groups must be either simple or solvable.

Theorem 3 (the classification theorem). *Let G be a finite splitting group which is not p -group. Then G is isomorphic to one of the following groups:*

- (i) S_4 ;
- (ii) $\text{PGL}(2, q)$ where $q \geq 5$, q is odd;
- (iii) $\text{PSL}(2, q)$ where $q \geq 4$;
- (iv) Suzuki simple group, $S_Z(2^{2n+1})$, $n \in \mathbb{Z}^+$;
- (v) Frobenius group;
- (vi) HT-group (the Hughes and Thompson group).

Proposition 4. *Consider the HT-group, and keep the notation in its definition. If $p = 2$, then H is abelian.*

Proof. Right from the definition of HT-group, H is generated by all elements of order greater than 2. Let $g \in G \setminus H$, so $g^2 = 1$. For an arbitrary element $h \in H$, one has $hg \notin H$, since, otherwise, $h^{-1}(hg) = g$ would be in H , and this contradicts the assumption. Thus hg is of order 2 or $ghg^{-1} = h^{-1}$. Now, for $h, h' \in H$, we get $h^{-1}h'^{-1} = ghg^{-1}gh'g^{-1} = gh'h'g^{-1} = (hh')^{-1} = h'^{-1}h^{-1}$. Thus H is abelian. $\square$

Proposition 5. *Let G be a nonelementary 2-group. Then G is splitting if and only if $G = H \rtimes \langle b \rangle$ where H is abelian 2-group, containing an element of order 4, $b^2 = 1$ and $bhb^{-1} = h^{-1}$, $\forall h \in H$.*

Proof. Assume that $G = H \rtimes \langle b \rangle$; then G is a Frobenius group with abelian kernel, so by Theorem 3, G is splitting.

Conversely, assume that G is splitting; then by Kontorovich [7], all elements of G whose order is greater than 2 are contained in one component H of the splitting of G . If an element $b \in G \setminus H$, then for an arbitrary element $h \in H$, the element $hb \notin H$. Hence $(hb)^2 = 1$. Since $bhb^{-1} = 1$, then $bhb = h^{-1}$, $\forall h \in H$. Thus H is abelian as we have seen in Proposition 4. Since G is nonelementary 2-group, H must have an element x of order 2^k , $k \geq 2$. Thus the element $x^{2^{k-2}}$ is of order 4.

Finally, we will prove that $G = H \rtimes \langle b \rangle$. Suppose that $c \in G \setminus H$. Then, as mentioned before, $chc^{-1} = h^{-1}$, $\forall h \in H$. Hence, $(bc)h(bc)^{-1} = b(chc^{-1})b^{-1} = bh^{-1}b^{-1} = h$, so, bc commutes with every element in H . Since H contains elements with fourth order, then by Kontorovich [14], $bc \in H$. Hence, $c \in H \rtimes \langle b \rangle$ and $G = H \rtimes \langle b \rangle$. $\square$

A splitting is called abelian if it consists of two components, one of them (G_1 say) is abelian, containing all elements of nonprime order, and G_1 can be written as $G_1 = (\langle a_1 \rangle \times \langle a_2 \rangle \times \cdots \times \langle a_{p-1} \rangle) \rtimes \langle b \rangle$, where $a_1^{p^2} = a_2^p = a_3^p = \cdots = a_{p-1}^p = b^p = 1$ and $a_{i+1} = [b, a_i]$ for $i = 1, 2, \dots, p-2$, $[b, a_{p-1}] = a_1^{-p}$.

Proposition 6. *Let P be a p -group with minimal abelian splitting which contains elements of order p^2 . Let $N \triangleleft P$ be an elementary abelian group such that $[P : N] = p$. Then, P is a direct product of dihedral group D_4 of order 8 and elementary 2-group. In particular $p = 2$.*

Proof. Assume that P is not 2-group. Since P has abelian splitting and it contains a normal subgroup N of index p , then $P = H \rtimes \langle b \rangle$ where H is an abelian component containing all elements of composite order. And all cyclic subgroups that are not contained in H form abelian splitting, see [15]. Hence, by [7], the center $Z(P)$ of P is contained in the abelian components of P which is generated by all elements of composite order. If $h \in H$ is of order p^2 , then we prove that $Z(P) = \langle h^p \rangle$. If $x \in Z(P) \setminus \langle h \rangle$, then $P/\langle x \rangle$ is p -group with abelian splitting containing element of order p^2 and $|P/\langle x \rangle| < |P|$. This leads to a contradiction. So, $Z(P) \leq \langle h \rangle$. On the other hand, $\langle h \rangle \not\leq Z(P)$ because, otherwise, $\langle h, b \rangle$ would be an abelian splitting of P . Thus $Z(P) = \langle h^p \rangle$. In this case, by [15], P corresponds to the Wreath product of two cyclic subgroups of P with prime orders. Hence, P is not splitting if $p \neq 2$. But this forces p to be equal to 2. This means that $P = H \rtimes \langle b \rangle$ where $b^2 = 1$ and $bhb^{-1} = h^{-1}$, $\forall h \in H$, and H is abelian 2-group, containing an element of order 4. So, by Proposition 5, P is 2-group and $P = D_4 \times H_1$ where D_4 is dihedral group of order 8 and H_1 is an elementary 2-group. $\square$

3. Normal Splitting

Let $\{G_i\}$ be a splitting of G and $\{K_s\}$ another splitting of G ; then the collection of intersections $G_i \cap K_s$ is a splitting of G . A splitting $\{K_s\}$ is said to be a refinement of another splitting $\{G_i\}$ if every component K_s is contained in a component of G_i . Thus, a splitting $\{K_s\}$ is a trivial refinement of $\{G_i\}$, if every K_s of the splitting $\{K_s\}$ is either trivial group; that is, K_s is the identity or $K_s = G_i$ for some i . Hence if the splitting $\{G_i\}$ admits no nontrivial refinement, then it is called a minimal splitting.

Definition 7. A normal splitting $\{G_i\}$ is a splitting in which every conjugate subgroup $g^{-1}G_jg$ of a component G_j is again a component of $\{G_i\}$.

Remark 8. The sum of subgroups, which are conjugate of H , forms the following normal subgroup:

$$\overline{H} = H + H^{(g_2)} + \cdots + H^{(g_s)}. \quad (2)$$

Frobenius theorem gives the sufficient conditions to decompose Frobenius group (Frobenius group is a group G which has a subgroup H , such that $H = N_G(H)$ and $H \cap H^{(g)} = \{1\}$ for every $g \in G \setminus H$) into a direct sum. Then by Frobenius theorem, G is decomposable into a direct sum of subgroups; that is, $G = P \oplus \overline{H}$, such that P is normal subgroup of G and

$$\overline{H} = H + H^{(g_2)} + \cdots + H^{(g_j)}, \quad ([G : H] = j). \quad (3)$$

Example 9. The symmetric group $G = \langle \{\sigma, \tau \mid \sigma^2 = \tau^3 = 1, \tau\sigma = \sigma\tau^2\} \rangle$ is normal splitting group, since it is equal to $\langle \sigma \rangle \cup \langle \sigma\tau \rangle \cup \langle \sigma\tau^2 \rangle \cup \langle \tau \rangle$ and $G = \langle \tau \rangle \rtimes \langle \sigma \rangle$. Take $G_1 = \langle \sigma \rangle$, and verify that $\langle \sigma \rangle \cap \alpha \langle \sigma \rangle \alpha^{-1} = \{1\}$ for all $\alpha \in G \setminus \langle \sigma \rangle$.

Any splitting can be refined. A splitting is called minimal if it cannot be refined anymore. This has been shown in the following.

Lemma 10 (see [10]). *Every splitting has a refinement which is normal.*

Proposition 11. *Any minimal splitting is normal.*

Proof. It is straightforward that if $\{G_i\}$ is splitting, then $\{gG_i g^{-1}\}$ is again splitting for any $g \in G$. Now consider the refinement splitting $gG_i g^{-1} \cap G_j$ and notice that $\{G_i\}$ is minimal. Then, $gG_i g^{-1} \cap G_j$ is either 1 or G_j . In both cases $G_j \subseteq gG_i g^{-1}$. Similarly one has $G_s \subseteq gG_j g^{-1} \subseteq G_j$, so $s = j$ and thus $gG_i g^{-1} = G_j$. This shows that $\{G_i\}$ is normal. $\square$

Definition 12. A finite group G is said to have the basis property if any subgroup $H \leq G$ has a base (minimal generating set) and any two bases of H have the same cardinality.

Definition 13. A subgroup H of G is said to be isolated if for every cyclic subgroup C one has $H \cap C \neq \{1\}$ implies that $C \leq H$.

In [12, 13], the sufficient and necessary conditions for a finite group to have the basis property were stated and proved. A special attention was given to groups that have abelian fitting subgroups. The following two theorems summarize such results.

Theorem 14. *If a finite group G possesses the basis property, then it is a p -group or $G = P \rtimes \langle y \rangle$ where P is a p -group and $\langle y \rangle$ is a cyclic group of order q^b , $p \neq q$ are primes and $b \in \mathbb{Z}^+$.*

I remind the reader with some basic definitions. The isotopic representation is the representation which can be written as a direct sum of equivalent irreducible representation.

The Frattini subgroup $\Phi(P)$ of a group P is the intersection of all maximal proper subgroups of P . Frattini of higher order is defined inductively as $\Phi^i(P) = \Phi(\Phi^{i-1}(P))$. The fitting subgroup of a finite group is the unique maximal nilpotent normal subgroup. So we have the following theorem.

Theorem 15. *Let a finite group G be a semidirect product of p -group $P = \text{Fit}(G)$ by a cyclic q -group $\langle y \rangle$ of order q^b , $p \neq q$ are primes and $b \in \mathbb{Z}^+$. Then, the group G has the basis property if and only if for any element $u \in \langle y \rangle \setminus \{1\}$ and for any normal subgroup $N \trianglelefteq P$, the induced automorphism φ_u must define an isotopic representation on every quotient Frattini subgroup.*

As in [13], finite groups with basis property are either p -groups which are elementary, that is, the order of any element is less than or equal to p , or p -groups which are not elementary, or of mixed order $p^a q^b$ where p, q are distinct primes and $a, b \in \mathbb{Z}^+$.

Theorem 16. *Let G be a finite non- p -group with basis property such that $G = P \rtimes \langle y \rangle$, where $P \trianglelefteq G$ is the maximal nilpotent normal subgroup of G of order p^a ($P = \text{Fit}(G)$), and $\langle y \rangle$ is a cyclic q -group of order q^b where p, q are primes, $p \neq q$, and $a, b \in \mathbb{Z}^+$. Then G has a normal splitting.*

Proof. Since $P \trianglelefteq G$, then P is a p -Sylow subgroup of G , which contains all p -elements of G . Therefore, the order of every element in G is either power of p or q . So, we know that by [13], p -elements do not commute with q -elements. Thus for every element $x \in P$, $x \neq 1$, we have $C_G(x) \leq P$. $C_G(x)$ is the centralizer of x in G . Hence, P is isolated subgroup in G , so by [14], G is splitting group. That is, $G = P + \langle \bar{y} \rangle$ such that $\langle \bar{y} \rangle = \langle y \rangle + \langle y \rangle^{(x_1)} + \cdots + \langle y \rangle^{(x_r)}$ where $[G : \langle y \rangle] = r$.

Now, we will show that $\langle y \rangle \cap g \langle y \rangle g^{-1} = \{1\}$ for any $g \in G \setminus \langle y \rangle$. If $g = y^j c$ for some $c \in P \setminus \{1\}$ and $1 \leq j \leq q^b$, then $g^{-1} y g = c^{-1} y^{-j} y y^j c = c^{-1} y c$. So, we may assume that $g \in P \setminus \{1\}$. If $t \in c^{-1} \langle y \rangle c \cap \langle y \rangle$ where $t \neq 1$, one can find $1 \leq k, l \leq q^b$ such that $t = y^l = c^{-1} y^k c$ or

$$y^{-k} c y^l = c. \quad (4)$$

Since $c \in P \setminus \{1\}$, there is a number $0 \leq i \leq j_p$ with $c \in \Phi^i(P) \setminus \Phi^{i+1}(P)$. If we apply the homomorphism $\phi : G \rightarrow G/P$ on (4), we get $1 = \phi(c) = \phi(y)^{-k} \phi(c) \phi(y)^l = \phi(y)^{l-k}$. Thus, $q^b \mid (l - k)$ or $l = k$, which leads to $c y^l = c y^l$. This later result, given that G possesses the basis property, contradicts, according to [13], the fact that p -elements do not commute with q -elements. So, c must be the identity. This means that G has a normal splitting. $\square$

For the sake of the converse of Theorem 16, one notices that in Theorem 3, S_4 does not have the basis property and cases (ii), (iii), and (iv) are simple, so they do not have basis property. The class of HT-group is nilpotent but the only nilpotent groups with the basis property are p -groups. The only possible true converse is Frobenius groups. In fact, this is not true in general. Here is a counterexample.

Example 17. The group $G = \mathbb{Z}_{19}^4 \rtimes Q_8$, where Q_8 is the quaternion with 8 elements, is normal splitting group that does not have the basis property. We demonstrate this situation.

The quaternion $Q_8 = \{x, y \mid x^4 = xyxy = 1, x^2 = y^2\}$, and the semidirect product $G = \mathbb{Z}_{19}^4 \rtimes Q_8$ is defined by an automorphism $\lambda : Q_8 \rightarrow \text{Aut}(\mathbb{Z}_{19}^4)$, which gives rise to a corresponding linear matrix representation for some base on

the field $\text{GF}(19)$. Such a representation can be given in terms of

$$\begin{aligned} A_x &= \begin{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{pmatrix}, \\ A_y &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix} \end{aligned} \quad (5)$$

by considering $\mathbf{Z}_{19}^4$ as a vector space on $\text{GF}(19)$. If ϕ_x, ϕ_y are operators represented by A_x, A_y for some base, respectively, then we get a linear representation ϕ for Q_8 whose kernel is 1. And we get that

$$\begin{aligned} \phi(1) &= \begin{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}, \\ \phi(x^2) &= \begin{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \end{pmatrix}, \\ \phi(y^3) &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}, \\ \phi(x^3) &= \begin{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{pmatrix}, \\ \phi(xy) &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}, \\ \phi(yx) &= \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}. \end{aligned} \quad (6)$$

So, we get all, linear representations for the elements of Q_8 which induce nonidentical minimal polynomials. So, we get a nonequivalent irreducible representation, and thus we do not have isotopic representation, but this violates one of the necessary conditions for finite groups with basis property as in the proof of Theorem 15 (see [13]). Thus $\mathbf{Z}_{19}^4 \rtimes Q_8$ does not have the basis property. From the other side, we show that this is a Frobenius group and so, it is a normal splitting group.

Consider the subgroup $H = \langle x, y \rangle$, and let $u \in G \setminus H$. If $v \in H \cap u^{-1}Hu$, $v \neq 1$, then we have

$$u = hc, \quad (7)$$

where $c \in \mathbf{Z}_{19}^4$ and $h \in H$. So, $v = u^{-1}su$, $v, s \in H$. That is to say

$$v = c^{-1}h^{-1}shc. \quad (8)$$

Applying the homomorphism $\theta : G \rightarrow G/\mathbf{Z}_{19}^4$ on (7) and (8) leads to $\theta(hc) = h$ for all $h \in H$, $c \in \mathbf{Z}_{19}^4$, and hence $v = \theta(v) = h^{-1}sh$. This implies that $h^{-1}sh = u^{-1}su$ or uh^{-1} commutes with s . That is $uh^{-1} \in H$ or $u \in H$ which contradicts the assumption. Henceforth, $H \cap u^{-1}Hu = \{1\}$ and G is a Frobenius group with abelian kernel.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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