Research Article

New Improvement of the Expansion Methods for Solving the Generalized Fitzhugh-Nagumo Equation with Time-Dependent Coefficients

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An improvement of the expansion methods, namely, the improved $\tan(\Phi(\xi)/2)$ -expansion method, for solving nonlinear second-order partial differential equation, is proposed. The implementation of the new approach is demonstrated by solving the generalized Fitzhugh-Nagumo equation with time-dependent coefficients. As a result, many new and more general exact travelling wave solutions are obtained including periodic function solutions, soliton-like solutions, and trigonometric function solutions. The exact particular solutions contain four types: hyperbolic function solution, trigonometric function solution, exponential solution, and rational solution. We obtained further solutions comparing this method with other methods. The results demonstrate that the new $\tan(\Phi(\xi)/2)$ -expansion method is more efficient than the Ansatz method and Tanh method applied by Triki and Wazwaz (2013). Recently, this method is developed for searching exact travelling wave solutions of nonlinear partial differential equations. Abundant exact travelling wave solutions including solitons, kink, and periodic and rational solutions have been found. These solutions might play an important role in engineering fields. It is shown that this method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool for solving the nonlinear physics.

1. Introduction

Nonlinear evolution equations (NLEEs) are very important model equations in mathematical physics, engineering, and applied mathematics for describing diverse types of physical mechanisms of natural phenomena in the field of applied sciences, biochemistry, and engineering. Nonlinear wave equations play a major role in various fields such as plasma physics, fluid mechanics, optical fibers, solid state physics, chemical kinetics, geochemistry, and nonlinear optics [1, 2]. Much work has been done over the years on the subject of obtaining the analytical solutions to the nonlinear partial differential equations (NPDEs). One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods to look for exact solutions for NPDEs. The advent of symbolic computation also enables performing some complicated and tedious algebraic processes coupled with differential calculations. With the

rapid development of nonlinear sciences based on computer algebraic system, many effective methods have been presented, such as the homotopy analysis method [3, 4], the variational iteration method [5–7], the homotopy perturbation method [8, 9], the sine-cosine method [10], the tanh-coth method [11–13], the modified extended tanh-function method [14, 15], the Exp-function method [16, 17], the $\exp(-\Phi(\xi))$ -expansion method [18, 19], the (G'/G) -expansion method [20, 21], the modified simple equation method [22], the novel (G'/G) -expansion method [23], the new approach of the generalized (G'/G) -expansion method [21], the Jacobi elliptic function method [24], and the homogeneous balance method [25]. Recently, Naher and Abdullah [21] presented an effective and straightforward method, called the new approach of generalized (G'/G) -expansion method, to obtain exact travelling wave solutions of nonlinear evolution equations. Here, we use an effective method, namely, the improved $\tan(\Phi(\xi)/2)$ -expansion

method, for constructing a range of exact solutions for the following ordinary partial differential equations that in this paper we developed solutions as well. In this paper, we put forth the new approach of improved $\tan(\Phi(\xi)/2)$ -expansion method to construct exact travelling wave solutions including solitons, kink, and periodic and rational solutions to the generalized Fitzhugh-Nagumo equation with time-dependent coefficients. In [22, 26], the standard form of the Fitzhugh-Nagumo equation is given:

$$u_t - u_{xx} + u(1 - u)(\rho - u) = 0, \quad (1)$$

where $0 < \rho < 1$ and $u(x, t)$ is of the unknown function depending on the temporal variable t and the spatial variable x . When $\rho = -1$, (1) reduces to the real Newell-Whitehead equation which describes the dynamical behavior near the bifurcation point for the Rayleigh-Bénard convection of binary fluid mixtures [27]. The FN equation has various applications in the fields of flame propagation, logistic population growth, neurophysiology, branching Brownian motion process, autocatalytic chemical reaction, and nuclear reactor theory [28]. This equation combines diffusion and nonlinearity which is controlled by the term $u(1 - u)(\rho - u)$. Many physicists and mathematicians have paid much attention to the Fitzhugh-Nagumo equation in recent years due to its importance in mathematical physics. Remarkably, this nonlinear evolution equation is an important nonlinear reaction-diffusion equation and is usually used to model the transmission of nerve impulses [29–31]. Shih et al. utilized the approximate conditional symmetry method to determine approximate solutions admitted by a perturbation of (1). Equation (1) has been presented by Hariharan and Kannan via Haar wavelet method [32]. Kawahara and Tanaka [33] have found new exact solutions of (1), by applying the nonclassical symmetry reductions approach by using Hirota method. In [34], Nucci and Clarkson have obtained some new exact solutions with Jacobi elliptic function. Li and Guo [35] have obtained the new exact solutions of the Fitzhugh-Nagumo equation by using first integral method. In this paper, we consider that the generalized Fitzhugh-Nagumo equation with time-dependent coefficients is given as

$$u_t + \alpha(t)u_x - \beta(t)u_{xx} + \gamma(t)\left(\rho u - (1 + \rho)u^2 + u^3\right) = 0, \quad (2)$$

where $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ are arbitrary functions of t . The nonlinear models with variable coefficients are needed to describe the propagation of pulses. Triki and Wazwaz [36] obtained a new variety of soliton solutions by means of specific solitary wave Ansatz and the tanh method for (2). Also, Jiwari et al. [37] applied polynomial differential quadrature method to find the numerical solution of the generalized Fitzhugh-Nagumo equation with time-dependent coefficients in one-dimensional space. In [38], the effect of diffusion on pattern formation in FitzHugh-Nagumo model has been surveyed. Van Gorder applied the method of homotopy analysis to study the Fitzhugh-Nagumo equation [39]. In [40], a FitzHugh-Nagumo monodomain model has been used to describe the propagation

of the electrical potential in heterogeneous cardiac tissue. Abbasian et al. have searched symmetric bursting behaviors in the generalized FitzHugh-Nagumo model [41]. Ray and Sahooz used the fractional subequation method for solving the space-time fractional Zakharov-Kuznetsov and modified Zakharov-Kuznetsov equations [42]. Also, the authors of [43] applied the fractional subequation method to the time fractional KdV-Zakharov-Kuznets and space-time fractional modified KdV-Zakharov-Kuznetsov equations. The purpose of this paper is to obtain exact solutions of the generalized Fitzhugh-Nagumo equation and to determine the accuracy of the improved $\tan(\Phi(\xi)/2)$ -expansion method in solving these kinds of problems. The paper is organized as follows. In Section 2, we describe the improved $\tan(\Phi(\xi)/2)$ -expansion method. In Section 3, we examine the generalized Fitzhugh-Nagumo equation with method introduced in Section 2. Also, conclusion and advantages are given in Section 4. Finally, some references are given at the end of this paper.

2. Description of Improved $\tan(\Phi(\xi)/2)$ -Expansion Technique

Step 1. We suppose the given nonlinear fractional partial differential equation for $u(x, t)$ to be in the form

$$\mathcal{N}(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \quad (3)$$

which can be converted to an ODE:

$$\mathcal{Q}(u, u', -\mu u', u'', \mu^2 u'', \dots) = 0 \quad (4)$$

the transformation, $\xi = x - \mu t$, is wave variable. Also, μ is constant to be determined later.

Step 2. Suppose the traveling wave solution of (4) can be expressed as follows:

$$\begin{aligned} u(\xi) &= S(\Phi) \\ &= \sum_{k=0}^m A_k \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^k \\ &\quad + \sum_{k=1}^m B_k \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-k}, \end{aligned} \quad (5)$$

where A_k ($0 \leq k \leq m$) and B_k ($1 \leq k \leq m$) are constants to be determined, such that $A_m \neq 0$, $B_m \neq 0$, and $\Phi = \Phi(\xi)$ satisfies the following ordinary differential equation:

$$\Phi'(\xi) = a \sin(\Phi(\xi)) + b \cos(\Phi(\xi)) + c. \quad (6)$$

We will consider the following special solutions of (6):

Family 1: When $a^2 + b^2 - c^2 < 0$ and $b - c \neq 0$, then $\Phi(\xi) = 2 \arctan[a/(b - c) - (\sqrt{c^2 - b^2 - a^2}/(b - c))\tan((\sqrt{c^2 - b^2 - a^2}/2)(\xi + C))]$.

Family 2: When $a^2 + b^2 - c^2 > 0$ and $b - c \neq 0$, then $\Phi(\xi) = 2 \arctan[a/(b - c) + (\sqrt{b^2 + a^2 - c^2}/(b - c)) \tanh((\sqrt{b^2 + a^2 - c^2}/2)(\xi + C))]$.

Family 3: When $a^2 + b^2 - c^2 > 0$, $b \neq 0$ and $c = 0$, then $\Phi(\xi) = 2 \arctan[a/b + (\sqrt{b^2 + a^2}/b) \tanh((\sqrt{b^2 + a^2}/2)(\xi + C))]$.

Family 4: When $a^2 + b^2 - c^2 < 0$, $c \neq 0$ and $b = 0$, then $\Phi(\xi) = 2 \arctan[-a/c + (\sqrt{c^2 - a^2}/c) \tanh((\sqrt{c^2 - a^2}/2)(\xi + C))]$.

Family 5: When $a^2 + b^2 - c^2 > 0$, $b - c \neq 0$ and $a = 0$, then $\Phi(\xi) = 2 \arctan[\sqrt{(b + c)/(b - c)} \tanh((\sqrt{b^2 - c^2}/2)(\xi + C))]$.

Family 6: When $a = 0$ and $c = 0$, then $\Phi(\xi) = \arctan[(e^{2b(\xi+C)} - 1)/(e^{2b(\xi+C)} + 1), 2e^{b(\xi+C)}/(e^{2b(\xi+C)} + 1)]$.

Family 7: When $b = 0$ and $c = 0$, then $\Phi(\xi) = \arctan[2e^{a(\xi+C)}/(e^{2a(\xi+C)} + 1), (e^{2a(\xi+C)} - 1)/(e^{2a(\xi+C)} + 1)]$.

Family 8: When $a^2 + b^2 = c^2$, then $\Phi(\xi) = -2 \arctan[((b + c)(a(\xi + C) + 2))/a^2(\xi + C)]$.

Family 9: When $a = b = c = ka$, then $\Phi(\xi) = 2 \arctan[e^{ka(\xi+C)} - 1]$.

Family 10: When $a = c = ka$ and $b = -ka$, then $\Phi(\xi) = -2 \arctan[e^{ka(\xi+C)}/(-1 + e^{ka(\xi+C)})]$.

Family 11: When $c = a$, then $\Phi(\xi) = -2 \arctan[((a + b)e^{b(\xi+C)} - 1)/((a - b)e^{b(\xi+C)} - 1)]$.

Family 12: When $a = c$, then $\Phi(\xi) = 2 \arctan[((b + c)e^{b(\xi+C)} + 1)/((b - c)e^{b(\xi+C)} - 1)]$.

Family 13: When $c = -a$, then $\Phi(\xi) = 2 \arctan[(e^{b(\xi+C)} + b - a)/(e^{b(\xi+C)} - b - a)]$.

Family 14: When $b = -c$, then $\Phi(\xi) = -2 \arctan[ae^{a(\xi+C)}/(ce^{a(\xi+C)} - 1)]$.

Family 15: When $b = 0$ and $a = c$, then $\Phi(\xi) = -2 \arctan[(c(\xi + C) + 2)/c(\xi + C)]$.

Family 16: When $a = 0$ and $b = c$, then $\Phi(\xi) = 2 \arctan[c(\xi + C)]$.

Family 17: When $a = 0$ and $b = -c$, then $\Phi(\xi) = -2 \arctan[1/c(\xi + C)]$.

where A_k, B_k ($k = 1, 2, \dots, m$), a , b , and c are constants to be determined later. But the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (6). If m is not an integer, then a transformation formula should be used to overcome this difficulty.

Step 3. Substituting (5) into (4) with the value of m obtained in Step 2, collecting the coefficients of $\tan(\Phi(\xi)/2)^k$, $\cot(\Phi(\xi)/2)^k$ ($k = 0, 1, 2, \dots$), and then setting each coefficient to zero, we can get a set of overdetermined partial differential equations for A_0, A_k, B_k ($k = 1, 2, \dots, m$), a , b , c , and p with the aid of symbolic computation Maple.

Step 4. Solve the algebraic equations in Step 3, and then substitute $A_0, A_1, B_1, \dots, A_m, B_m, \mu, p$ in (5).

3. The Generalized Fitzhugh-Nagumo Equation

We consider the generalized Fitzhugh-Nagumo equation with time-dependent coefficients as follows:

$$u_t + \alpha(t)u_x - \beta(t)u_{xx} + \gamma(t)(\rho u - (1 + \rho)u^2 + u^3) = 0, \quad (7)$$

where $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ are arbitrary functions of t . By using the wave variable $\xi = \mu[x - c(t)t]$, reduce it to an ODE as follows:

$$\mu \left(\alpha(t) - t \frac{dc(t)}{dt} - c(t) \right) u' - \mu^2 \beta(t) u'' + \gamma(t) (\rho u - (1 + \rho)u^2 + u^3) = 0, \quad (8)$$

where $u' = du/d\xi$ and $u'' = d^2u/d\xi^2$. Balancing the linear term of the highest order u'' with the highest order nonlinear term u^3 by using homogenous principal, we have

$$M + 2 = 3M, \quad M = 1. \quad (9)$$

Then, the trial solutions are

$$u(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right] + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \quad (10)$$

Substituting (10) and (6) into (8) and collecting all terms with the same order of $\tan(\Phi(\xi)/2)$ together, the left-hand side of (10) is converted into polynomial in $\tan(\Phi(\xi)/2)$. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for a , b , c , μ , $c(t)$, p , A_0 , A_1 , and B_1 as follows:

$$\begin{aligned} & \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^0 : 2\gamma(t) A_1^3 p^6 - 2\gamma(t) A_1^2 p^5 + 6\gamma(t) \\ & \cdot A_0 A_1^2 p^5 - 2\gamma(t) \rho A_1^2 p^5 - 4\gamma(t) \rho A_0 A_1 p^4 \\ & + 6\gamma(t) A_1^2 B_1 p^4 + 6\gamma(t) A_0^2 A_1 p^4 + 2\gamma(t) \rho A_1 p^4 \\ & - 4\gamma(t) A_0 A_1 p^4 - \mu t \frac{dc(t)}{dt} A_1 b p^3 - 4\gamma(t) A_1 B_1 p^3 \end{aligned}$$

$$\begin{aligned}
& + 2\gamma(t) * A_0^3 p^3 - \mu t \frac{dc(t)}{dt} A_1 c p^3 - 2\gamma(t) A_0^2 p^3 \\
& + 2\gamma(t) \rho A_0 p^3 - 4\gamma(t) \rho A_1 B_1 p^3 - \mu c(t) A_1 b p^3 \\
& + 12\gamma(t) A_0 A_1 B_1 p^3 - \mu c(t) A_1 c p^3 - \mu^2 \beta(t) \\
& \cdot A_1 a b p^3 - 2\gamma(t) \rho A_0^2 p^3 + \mu \alpha(t) A_1 b p^3 - \mu^2 \beta(t) \\
& \cdot A_1 a c p^3 + \mu \alpha(t) A_1 c p^3 - 4\gamma(t) A_0 B_1 p^2 + 6\gamma(t) \\
& \cdot A_0^2 B_1 p^2 - 4\gamma(t) \rho A_0 B_1 p^2 + 6\gamma(t) A_1 B_1^2 p^2 \\
& + 2\gamma(t) \rho B_1 p^2 - 2\gamma(t) B_1^2 p - \mu \alpha(t) B_1 c p + \mu^2 \beta(t) \\
& \cdot B_1 a b p + \mu c(t) B_1 c p + \mu t \frac{dc(t)}{dt} B_1 b p - \mu \alpha(t) B_1 b p \\
& + \mu c(t) B_1 b p - 2\gamma(t) \rho B_1^2 p + 6\gamma(t) A_0 B_1^2 p + \mu t \\
& \cdot \frac{dc(t)}{dt} B_1 c p + \mu^2 \beta(t) B_1 a c p - \mu^2 \beta(t) B_1 b^2 \\
& - \mu^2 \beta(t) B_1 c^2 - 2\mu^2 \beta(t) B_1 b c + 2\gamma(t) B_1^3, \\
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^1 : 12\gamma(t) A_1^3 p^5 - 10\gamma(t) \rho A_1^2 p^4 \\
& + 30\gamma(t) A_0 A_1^2 p^4 - 10\gamma(t) A_1^2 p^4 - 2\mu c(t) A_1 a p^3 \\
& - \mu^2 \beta(t) A_1 c^2 p^3 + 8\gamma(t) \rho A_1 p^3 + 2\mu \alpha(t) A_1 a p^3 \\
& + \mu^2 \beta(t) A_1 b^2 p^3 - 16\gamma(t) \rho A_0 A_1 p^3 - 16\gamma(t) \\
& \cdot A_0 A_1 p^3 - 2\mu^2 \beta(t) A_1 a^2 p^3 - 2\mu t \frac{dc(t)}{dt} A_1 a p^3 \\
& + 24\gamma(t) A_1^2 B_1 p^3 + 24\gamma(t) A_0^2 A_1 p^3 - 3\mu c(t) \\
& \cdot A_1 b p^2 - 3\mu^2 \beta(t) A_1 a b p^2 - 3\mu t \frac{dc(t)}{dt} A_1 c p^2 \\
& + 6\gamma(t) A_0^3 p^2 - 3\mu c(t) A_1 c p^2 + 3\mu \alpha(t) A_1 b p^2 \\
& + 3\mu \alpha(t) A_1 c p^2 - 12\gamma(t) A_1 B_1 p^2 + 6\gamma(t) \rho A_0 p^2 \\
& - 3\mu t \frac{dc(t)}{dt} A_1 b p^2 - 12\gamma(t) \rho A_1 B_1 p^2 - 3\mu^2 \beta(t) \\
& \cdot A_1 a c p^2 + 36\gamma(t) A_0 A_1 B_1 p^2 - 6\gamma(t) \rho A_0^2 p^2 \\
& - 6\gamma(t) A_0^2 p^2 + 12\gamma(t) A_0^2 B_1 p + 2\mu t \frac{dc(t)}{dt} B_1 a p \\
& - 8\gamma(t) \rho A_0 B_1 p + 2\mu c(t) B_1 a p + \mu^2 \beta(t) B_1 c^2 p \\
& + 2\mu^2 \beta(t) B_1 a^2 p + 4\gamma(t) \rho B_1 p - \mu^2 \beta(t) B_1 b^2 p \\
& - 2\mu \alpha(t) B_1 a p + 12\gamma(t) A_1 B_1^2 p - 8\gamma(t) A_0 B_1 p \\
& + \mu t 4\gamma(t) \rho B_1 p - \mu^2 \beta(t) B_1 b^2 p - B_1 b - \mu \alpha(t) B_1 c \\
& + 6\gamma(t) A_0 B_1^2 - 2\gamma(t) B_1^2 - 2\gamma(t) \rho B_1^2 + \mu c(t) B_1 b
\end{aligned}$$

$$\begin{aligned}
& - \mu \alpha(t) B_1 b + \mu t 4\gamma(t) \rho B_1 p - \mu^2 \beta(t) B_1 b^2 p - B_1 c \\
& - 3\mu^2 \beta(t) B_1 a c - 3\mu^2 \beta(t) B_1 a b + \mu c(t) B_1 c, \\
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^2 : 30\gamma(t) A_1^3 p^4 - \mu \alpha(t) A_1 b p^3 - \mu t \\
& \cdot \frac{dc(t)}{dt} A_1 c p^3 - 3\mu^2 \beta(t) A_1 a c p^3 + \mu t \frac{dc(t)}{dt} A_1 b p^3 \\
& - 20\gamma(t) \rho A_1^2 p^3 - \mu c(t) A_1 c p^3 - 20\gamma(t) A_1^2 p^3 \\
& + \mu c(t) A_1 b p^3 + \mu \alpha(t) A_1 c p^3 + 60\gamma(t) A_0 A_1^2 p^3 \\
& + 3\mu^2 \beta(t) A_1 a b p^3 - 24\gamma(t) \rho A_0 A_1 p^2 - 6\mu t 20\gamma(t) \\
& \cdot A_1^2 p^3 + \mu c(t) A_1 b p^3 + \mu \alpha(t) A_1 c p^3 + 60\gamma(t) \\
& \cdot A_0 A_1^2 p^3 + A_1 a p^2 - 6\mu c(t) A_1 a p^2 - 3\mu^2 \beta(t) \\
& \cdot A_1 c^2 p^2 + 36\gamma(t) A_1^2 B_1 p^2 - 6\mu^2 \beta(t) A_1 a^2 p^2 \\
& + 3\mu^2 \beta(t) A_1 b^2 p^2 - 24\gamma(t) A_0 A_1 p^2 + 12\gamma(t) \\
& \cdot \rho A_1 p^2 + 36\gamma(t) A_0^2 A_1 p^2 + 6\mu \alpha(t) A_1 a p^2 + \mu t \\
& \cdot \frac{dc(t)}{dt} B_1 c p - 3\mu^2 \beta(t) A_1 a b p + 36\gamma(t) A_0 A_1 B_1 p \\
& + \mu \alpha(t) B_1 b p - \mu t \frac{dc(t)}{dt} B_1 b p - \mu c(t) B_1 b p - 6\gamma(t) \\
& \cdot A_0^2 p + 3\mu^2 \beta(t) B_1 a c p - 3\mu^2 \beta(t) B_1 a b p \\
& - 3\mu^2 \beta(t) B_1 a c p - 12\gamma(t) A_1 B_1 p - \mu \alpha(t) B_1 c p \\
& + 6\gamma(t) A_0^3 p + \mu c(t) B_1 c p + 3\mu \alpha(t) A_1 c p + 3\mu \alpha(t) \\
& \cdot A_1 b p + 6\gamma(t) \rho A_0 p - 3\mu t \frac{dc(t)}{dt} A_1 b p - 3\mu c(t) \\
& \cdot A_1 b p - 3\mu c(t) A_1 c p - 12\gamma(t) \rho A_1 B_1 p - 6\gamma(t) \\
& \cdot \rho A_0^2 p - 3\mu t \frac{dc(t)}{dt} A_1 c p - 2\mu \alpha(t) B_1 a - 4\lambda(t) \\
& \cdot A_0 B_1 + 2\gamma(t) \rho B_1 + 6\gamma(t) A_0^2 B_1 + \mu^2 \beta(t) B_1 b^2 \\
& + 6\gamma(t) A_1 B_1^2 - 2\mu^2 \beta(t) B_1 a^2 + 2\mu t \frac{dc(t)}{dt} B_1 a \\
& - 4\gamma(t) \rho A_0 B_1 + 2\mu c(t) B_1 a - \mu^2 \beta(t) B_1 c^2, \\
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^3 : 2\mu^2 \beta(t) A_1 b c p^3 - \mu^2 \beta(t) A_1 c^2 p^3 \\
& - \mu^2 \beta(t) A_1 b^2 p^3 + 40\gamma(t) A_1^3 p^3 - 3\mu c(t) A_1 c p^2 \\
& + 60\gamma(t) A_0 A_1^2 p^2 + 3\mu c(t) A_1 b p^2 - 3\mu \alpha(t) A_1 b p^2 \\
& + 3\mu t \frac{dc(t)}{dt} A_1 b p^2 - 9\mu^2 \beta(t) A_1 a c p^2 + 3\mu \alpha(t)
\end{aligned}$$

$$\begin{aligned}
& \cdot A_1 c p^2 - 3\mu t \frac{dc(t)}{dt} A_1 c p^2 + 9\mu^2 \beta(t) A_1 a b p^2 \\
& - 20\gamma(t) A_1^2 p^2 - 20\gamma(t) \rho A_1^2 p^2 - 3\mu^2 \beta(t) A_1 c^2 p \\
& - 6\mu^2 \beta(t) A_1 a^2 p + 8A_1 \gamma(t) \rho p + 6\mu \alpha(t) A_1 a p \\
& - 2\mu^2 \beta(t) B_1 b c p - 16\gamma(t) \rho A_0 A_1 p + 3\mu^2 \beta(t) \\
& \cdot A_1 b^2 p - 6\mu c(t) A_1 a p - 6\mu t \frac{dc(t)}{dt} A_1 a p \\
& - 16A_1 \gamma(t) A_0 p + \mu^2 \beta(t) B_1 c^2 p + \mu^2 \beta(t) B_1 b^2 p \\
& + 24\gamma(t) A_0^2 A_1 p + 24\gamma(t) A_1^2 B_1 p - 4\gamma(t) \rho A_1 B_1 \\
& - \mu t \frac{dc(t)}{dt} A_1 c - \mu^2 \beta(t) B_1 a c - \mu c(t) B_1 b - \mu c(t) \\
& \cdot A_1 b + \mu t \frac{dc(t)}{dt} B_1 c + 2\gamma(t) A_0^3 + \mu \alpha(t) A_1 c \\
& + 12\gamma(t) A_0 A_1 B_1 - \mu c(t) A_1 c + \mu \alpha(t) A_1 b \\
& + 2\gamma(t) \rho A_0 - 2\gamma(t) A_0^2 - \mu \alpha(t) B_1 c - \mu^2 \beta(t) A_1 a c \\
& - 4\gamma(t) A_1 B_1 - \mu t \frac{dc(t)}{dt} B_1 b + \mu^2 \beta(t) B_1 a b \\
& - \mu^2 \beta(t) A_1 a b - \mu t \frac{dc(t)}{dt} A_1 b + \mu \alpha(t) B_1 b + \mu c(t) \\
& \cdot B_1 c - 2\gamma(t) \rho A_0^2, \\
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^4 : A_1 \left(3\mu^2 \beta(t) b^2 p^2 + 3\mu^2 \beta(t) c^2 p^2 \right. \\
& - 6\mu^2 \beta(t) b c p^2 - 30\gamma(t) A_1^2 p^2 + 3\mu t \frac{dc(t)}{dt} c p \\
& + 3\mu c(t) c p - 3\mu \alpha(t) c p - 3\mu t \frac{dc(t)}{dt} b p \\
& + 10A_1 \gamma(t) p - 30A_1 \gamma(t) A_0 p + 3\mu \alpha(t) b p \\
& + 9\mu^2 \beta(t) a c p - 9\mu^2 \beta(t) a b p + 10A_1 \gamma(t) \rho p \\
& - 3\mu c(t) b p - 6\gamma(t) A_0^2 - 2\gamma(t) \rho + 4\gamma(t) \rho A_0 \\
& + 2\mu t \frac{dc(t)}{dt} a - 2\mu \alpha(t) a - 6\gamma(t) A_1 B_1 + 2\mu c(t) a \\
& + 2\mu^2 \beta(t) a^2 - \mu^2 \beta(t) b^2 + \mu^2 \beta(t) c^2 + 4\gamma(t) A_0 \Big), \\
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^5 : A_1 \left(12\gamma(t) A_1^2 p - 3\mu^2 \beta(t) c^2 p \right. \\
& - 3\mu^2 \beta(t) b^2 p + 6\mu^2 \beta(t) b c p - 2\gamma(t) A_1 \\
& + 6\gamma(t) A_0 A_1 - 2\gamma(t) \rho A_1 + \mu c(t) b + 3\mu^2 \beta(t) a b \\
& + \mu t \frac{dc(t)}{dt} b - \mu \alpha(t) b - 3\mu^2 \beta(t) a c - \mu t \frac{dc(t)}{dt} c
\end{aligned}$$

$$+ \mu \alpha(t) c - \mu c(t) c \Big),$$

$$\begin{aligned}
& \left(\tan \left(\frac{\Phi(\xi)}{2} \right) \right)^6 : -\mu^2 \beta(t) A_1 c^2 + 2\mu^2 \beta(t) A_1 b c \\
& + 2\gamma(t) A_1^3 - \mu^2 \beta(t) A_1 b^2.
\end{aligned}$$

(11)

Solving the set of algebraic equations using Maple, we get the following results.

Set 1. We have the following

$$a = a,$$

$$b = c,$$

$$c = c,$$

$$A_0 = 0,$$

$$A_1 = 0,$$

(12)

$$B_1 = p - \frac{c}{a},$$

$$\mu = \frac{1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$p = p,$$

$$\begin{aligned}
c(t) = \frac{1}{t} \int \left(\alpha(t) \right. \\
\left. + (1 - 2\rho) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt
\end{aligned}$$

(13)

$$+ C_1,$$

$$u(\xi) = B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (13), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
u_1(\xi) = & \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \right. \right. \\
& \cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \Bigg]^{-1},
\end{aligned}$$

(14)

$$\begin{aligned}
 u_2(\xi) &= \left[1 + \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1}, \\
 u_3(\xi) &= \left[1 - \frac{2a}{p-1} e^{a(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{15}$$

where $\xi = (1/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (1 - 2\rho)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 2. We have the following:

$$\begin{aligned}
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= 0, \\
 A_1 &= 0,
 \end{aligned}
 \tag{16}$$

$$\begin{aligned}
 B_1 &= \left(p - \frac{c}{a} \right) \rho, \\
 \mu &= \frac{\rho}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1,
 \end{aligned}
 \tag{17}$$

$$u(\xi) = B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (17), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
 u_4(\xi) &= \rho \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1} \right. \\
 &\left. \left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right) \right] \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
 u_5(\xi) &= \rho \left[1 + \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1}, \\
 u_6(\xi) &= \rho \left[1 - \frac{2a}{p+1} e^{a(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{18}$$

where $\xi = (\rho/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 3. We have the following:

$$\begin{aligned}
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= 1, \\
 A_1 &= 0,
 \end{aligned}
 \tag{19}$$

$$\begin{aligned}
 B_1 &= \left(-p + \frac{c}{a} \right), \\
 \mu &= \frac{1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
 &\left. + (1 - 2\rho) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1,
 \end{aligned}
 \tag{20}$$

$$u(\xi) = 1 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (20), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
 u_7(\xi) &= 1 - p \left[p + \tan \left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1} \right. \\
 &\left. \left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right) \right] \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
 u_8(\xi) &= 1 + \left[1 + \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1}, \\
 u_9(\xi) &= 1 - \left[1 - \frac{2a}{p+1} e^{a(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{21}$$

where $\xi = (1/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (1 - 2\rho)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 4. We have the following:

$$\begin{aligned}
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= 1, \\
 A_1 &= 0,
 \end{aligned}
 \tag{22}$$

$$\begin{aligned}
 B_1 &= (\rho - 1) \left(p - \frac{c}{a} \right), \\
 \mu &= \frac{\rho - 1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) + (\rho + 1) \sqrt{2\gamma(t)\beta(t)} \right) dt \\
 &+ C_1, \\
 u(\xi) &= 1 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},
 \end{aligned}
 \tag{23}$$

where a , b , and c are arbitrary constants. By using (23), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
 u_{10}(\xi) &= 1 - (\rho - 1) \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \right. \\
 &\left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right) \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
 u_{11}(\xi) &= 1 + (\rho - 1) \left[1 + \frac{1}{p-1} \right. \\
 &\cdot e^{ka(\xi+C)} \left. \right]^{-1}, \\
 u_{12}(\xi) &= 1 + (\rho - 1) \left[1 - \frac{2a}{p+1} \right. \\
 &\cdot e^{a(\xi+C)} \left. \right]^{-1},
 \end{aligned}
 \tag{24}$$

where $\xi = ((\rho - 1)/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho + 1)\sqrt{2\gamma(t)\beta(t)}dt + C_1)$.

Set 5. We have the following:

$$\begin{aligned}
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= \rho, \\
 A_1 &= 0,
 \end{aligned}
 \tag{25}$$

$$\begin{aligned}
 B_1 &= \rho \left(-p + \frac{c}{a} \right), \\
 \mu &= \frac{\rho}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) + (2 - \rho) \sqrt{2\gamma(t)\beta(t)} \right) dt \\
 &+ C_1, \\
 u(\xi) &= \rho + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},
 \end{aligned}
 \tag{26}$$

where a , b , and c are arbitrary constants. By using (26), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
 u_{13}(\xi) &= \rho - p \left[p + \tan \left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \right. \\
 &\left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right) \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
 u_{14}(\xi) &= \rho - \rho \left[1 + \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1}, \\
 u_{15}(\xi) &= \rho - \rho \left[1 - \frac{2a}{p+1} e^{a(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{27}$$

where $\xi = (\rho/a) \sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (2 - \rho) \sqrt{2\gamma(t)\beta(t)})dt + C_1)$.

Set 6. We have the following:

$$\begin{aligned}
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= \rho, \\
 A_1 &= 0, \\
 B_1 &= (\rho - 1) \left(-p + \frac{c}{a} \right),
 \end{aligned}
 \tag{28}$$

$$\begin{aligned}
 \mu &= \frac{\rho - 1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) - (\rho + 1) \sqrt{2\gamma(t)\beta(t)} \right) dt \\
 &+ C_1,
 \end{aligned}
 \tag{29}$$

$$u(\xi) = \rho + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (29), Families 7, 9, and 13 can be written, respectively, as

$$\begin{aligned}
 u_{16}(\xi) &= \rho - (\rho - 1) \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \right. \\
 &\left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right) \right]^{-1}, \\
 u_{17}(\xi) &= \rho - (\rho - 1) \left[1 + \frac{1}{p-1} \right. \\
 &\cdot e^{ka(\xi+C)} \left. \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
 u_{18}(\xi) &= \rho - (\rho - 1) \left[1 - \frac{2a}{p+1} \right. \\
 &\cdot e^{a(\xi+C)} \left. \right]^{-1},
 \end{aligned}
 \tag{30}$$

where $\xi = ((\rho - 1)/a) \sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) - (\rho + 1) \sqrt{2\gamma(t)\beta(t)})dt + C_1)$.

Set 7. We have the following:

$$\begin{aligned}
 A_0 &= \frac{1 \pm ip}{2}, \\
 A_1 &= 0, \\
 B_1 &= \pm \frac{1 + p^2}{2} i,
 \end{aligned}
 \tag{31}$$

$$\begin{aligned}
 \mu &= \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[(pA_0 + B_1)^2 + A_0(1 + p^2) \right. \\
 &\quad \left. + A_0^2 - pB_1 \right], \\
 b &\neq c, \\
 c &= c, \\
 a^2 + b^2 - c^2 &= \frac{c^2 B_1^2}{((pA_0 + B_1)^2 + A_0(1 + p^2) + A_0^2 - pB_1)^2}, \\
 p &= p,
 \end{aligned}
 \tag{32}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \int \left(\alpha(t) + (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1,
 \end{aligned}$$

$$u(\xi) = A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (32), Family 2 can be written as

$$\begin{aligned}
 u_{19}(\xi) &= \frac{1 \pm ip}{2} \pm \frac{1 + p^2}{2} i \left[p + \frac{a}{b - c} \right. \\
 &\quad \left. + \frac{\sqrt{b^2 + a^2 - c^2}}{b - c} \right. \\
 &\quad \left. \cdot \tanh \left(\frac{\sqrt{b^2 + a^2 - c^2}}{2} (\xi + C) \right) \right]^{-1},
 \end{aligned}
 \tag{33}$$

where $\xi = \pm(1/cB_1)\sqrt{\gamma(t)/2\beta(t)}[(pA_0 + B_1)^2 + A_0(1 + p^2) + A_0^2 - pB_1](x - \int(\alpha(t) + (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1).$

Set 8. We have the following:

$$\begin{aligned} A_0 &= \frac{1 \pm ip}{2}\rho, \\ A_1 &= 0, \\ B_1 &= \pm \frac{1 + p^2}{2}\rho i, \end{aligned} \quad (34)$$

$$\mu = \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[(pA_0 + B_1)^2 + \rho A_0 (1 + p^2) + A_0^2 - \rho p B_1 \right],$$

$$b \neq c,$$

$$c = c,$$

$$a^2 + b^2 - c^2 = \frac{c^2 B_1^2}{\left[(pA_0 + B_1)^2 + \rho A_0 (1 + p^2) + A_0^2 - \rho p B_1 \right]^2},$$

$$p = p,$$

(35)

$$c(t) = \frac{1}{t} \int \left(\alpha(t) + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1,$$

$$u(\xi) = A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (35), Family 2 can be written as

$$\cdot \tanh \left(\frac{\sqrt{b^2 + a^2 - c^2}}{2} (\xi + C) \right) \right]^{-1}, \quad (36)$$

$$\begin{aligned} u_{19}(\xi) &= \frac{1 \pm ip}{2}\rho \pm \frac{1 + p^2}{2}\rho i \left[p \right. \\ &\quad \left. + \frac{a}{b - c} + \frac{\sqrt{b^2 + a^2 - c^2}}{b - c} \right] \end{aligned}$$

where $\xi = \pm(1/cB_1)\sqrt{\gamma(t)/2\beta(t)}[(pA_0 + B_1)^2 + \rho A_0(1 + p^2) + A_0^2 - \rho p B_1](x - \int(\alpha(t) + (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1).$

Set 9. We have the following:

$$A_0 = \frac{(\rho + 1) \pm ip(\rho - 1)}{2},$$

$$A_1 = 0,$$

$$B_1 = \pm \frac{(1 + p^2)(\rho - 1)}{p}i, \quad (37)$$

$$a^2 + b^2 - c^2 = \frac{c^2 B_1^2}{\left[(pA_0 + B_1)^2 + \rho A_0 (1 + p^2) + A_0^2 - \rho p B_1 \right]^2},$$

$$\begin{aligned}
b &\neq c, \\
c &= c, \\
\mu &= \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[(\rho+1) (A_0 p^2 + pB_1 + A_0) - (pA_0 + B_1)^2 - A_0^2 - \rho \right], \\
p &= p, \\
c(t) &= \frac{1}{t} \int \left(\alpha(t) + (\rho+1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \\
u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},
\end{aligned} \tag{38}$$

where a , b , and c are arbitrary constants. By using (38), Family 2 can be written as

$$\begin{aligned}
u_{19}(\xi) &= \frac{(\rho+1) \pm ip(\rho-1)}{2} \\
&\pm \frac{(1+p^2)(\rho-1)}{p} i \left[p + \frac{a}{b-c} \right. \\
&+ \frac{\sqrt{b^2+a^2-c^2}}{b-c} \\
&\cdot \left. \tanh \left(\frac{\sqrt{b^2+a^2-c^2}}{2} (\xi+C) \right) \right]^{-1},
\end{aligned} \tag{39}$$

where $\xi = \pm(1/cB_1) \sqrt{\gamma(t)/2\beta(t)} [(\rho+1)(A_0 p^2 + pB_1 + A_0) - (pA_0 + B_1)^2 - A_0^2 - \rho] (x - \int (\alpha(t) + (\rho+1) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 10. We have the following:

$$\begin{aligned}
A_0 &= 0, \\
A_1 &= 0, \\
B_1 &= \pm \frac{2\rho}{\rho-1} i, \\
a^2 + b^2 - c^2 & \\
&= \frac{c^2 B_1^2 (\rho-1)^2}{[B_1^2 - pB_1 + \rho(1-pB_1 + p^2)]^2},
\end{aligned} \tag{40}$$

$$p = \pm \frac{\rho+1}{\rho-1} i,$$

$$b \neq c,$$

$$c = c,$$

$$\begin{aligned}
\mu &= \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[B_1^2 - pB_1 + \rho(1 \right. \\
&\quad \left. - pB_1 + p^2) \right], \\
c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
&\quad \left. - (\rho+1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \\
u(\xi) &= B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},
\end{aligned} \tag{41}$$

where a , b , and c are arbitrary constants. By using (41), Family 2 can be written as

$$\begin{aligned}
u_{20}(\xi) &= \pm \frac{2\rho}{\rho-1} i \left[\pm \frac{\rho+1}{\rho-1} i + \frac{a}{b-c} \right. \\
&+ \frac{\sqrt{b^2+a^2-c^2}}{b-c} \\
&\cdot \left. \tanh \left(\frac{\sqrt{b^2+a^2-c^2}}{2} (\xi+C) \right) \right]^{-1},
\end{aligned} \tag{42}$$

where $\xi = \pm(1/cB_1) \sqrt{\gamma(t)/2\beta(t)} [B_1^2 - pB_1 + \rho(1-pB_1 + p^2)] (x - \int (\alpha(t) - (\rho+1) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 11. We have the following:

$$A_0 = 1,$$

$$A_1 = 0,$$

$$B_1 = B_1,$$

$$\begin{aligned} a^2 + b^2 - c^2 = & -\frac{c^2}{[\rho(1 + pB_1 + p^2) - 1 - (p + B_1)^2]^2} \left[B_1 \left(-12p - 6\rho p^3 - 6\rho^2 p + 2\rho^2 p^3 \right. \right. \\ & + p^2 \rho^2 + 18p\rho + 4p^3 \Big) + B_1^2 \left(-6\rho p^2 - 6 + 6\rho + 6p^2 \right) + B_1^3 (-2\rho p + 4p) + B_1^4 + 1 + \rho^2 \\ & \left. \left. + 12\rho p^2 - 6p^2 - 2\rho + p^4 - 2\rho^2 p^2 + \rho^2 p^4 - 2\rho p^4 \right] \right] \end{aligned} \quad (43)$$

$$p = -\frac{\rho - 2}{2(\rho - 1)} B_1,$$

$$b \neq c,$$

$$c = c,$$

$$\mu = \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[\rho \left(1 + pB_1 + p^2 \right) - 1 - (p + B_1)^2 \right], \quad (44)$$

$$c(t) = \frac{1}{t} \int \left(\alpha(t) + (\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1,$$

$$u(\xi) = 1 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (44), Family 2 can be written as

$$\cdot \tan \left(\frac{\sqrt{c^2 - b^2 - a^2}}{2} (\xi + C) \right) \Bigg]^{-1}, \quad (45)$$

$$\begin{aligned} u_{20}(\xi) = & 1 + B_1 \left[-\frac{\rho - 2}{2(\rho - 1)} B_1 + \frac{a}{b - c} \right. \\ & \left. - \frac{\sqrt{c^2 - b^2 - a^2}}{b - c} \right] \end{aligned}$$

where $\xi = \pm(1/cB_1)\sqrt{\gamma(t)/2\beta(t)}[\rho(1 + pB_1 + p^2) - 1 - (p + B_1)^2](x - \int(\alpha(t) + (\rho - 1)\sqrt{\gamma(t)\beta(t)/2})dt + C_1)$.

Set 12. We have the following:

$$A_0 = \rho,$$

$$A_1 = 0,$$

$$B_1 = \pm 2\rho(\rho - 1)i,$$

$$a^2 + b^2 - c^2 = \frac{c^2}{[(p\rho + B_1)^2 - pb_1 + \rho^2 - \rho - \rho p^2]^2} \left[B_1 \left(2p^3 \rho + 18p\rho^2 - 6p^3 \rho^2 - 6p\rho \right. \right.$$

$$\begin{aligned}
& -12p\rho^3 + 4p^3\rho^3) + B_1^2(-6\rho p^2 + 6\rho + 6\rho^2 p^2 - 6\rho^2 + p^2) + B_1^3(4p\rho - 2p) + B_1^4 + (p^4 \\
& - 6p^2)(\rho^4 - 2\rho^3 + \rho^2) + \rho^2(\rho - 1)^2],
\end{aligned}
\tag{46}$$

$$p = \mp(2\rho - 1)i,$$

$$b \neq c,$$

$$c = c,$$

$$\mu = \pm \frac{1}{cB_1} \sqrt{\frac{\gamma(t)}{2\beta(t)}} \left[(p\rho + B_1)^2 - pb_1 + \rho^2 - \rho - \rho p^2 \right],
\tag{47}$$

$$c(t) = \frac{1}{t} \int \left(\alpha(t) + (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1,$$

$$u(\xi) = \rho + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (47), Family 2 can be written as

$$\begin{aligned}
u_{20}(\xi) &= \rho \pm 2\rho(\rho - 1)i \left[\mp(2\rho - 1)i \right. \\
&+ \frac{a}{b - c} + \frac{\sqrt{b^2 + a^2 - c^2}}{b - c} \\
&\cdot \left. \tanh\left(\frac{\sqrt{b^2 + a^2 - c^2}}{2}(\xi + C)\right) \right]^{-1},
\end{aligned}
\tag{48}$$

where $\xi = \pm(1/cB_1)\sqrt{\gamma(t)/2\beta(t)}[(p\rho + B_1)^2 - pb_1 + \rho^2 - \rho - \rho p^2](x - \int(\alpha(t) + (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 13. We have the following:

$$a = \frac{1}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$b = -\frac{A_0 - 1}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$c = \frac{A_0 - 1}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$A_0 = A_0,$$

$$A_1 = 0,$$

$$B_1 = -pA_0,$$

$$p = p$$

(49)

$$\mu = \mu,$$

$$\begin{aligned}
c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
&- (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \Bigg) dt \\
&+ C_1,
\end{aligned}
\tag{50}$$

$$u(\xi) = A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1}.$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$u_{21}(\xi) = A_0 - pA_0 \left[p - \frac{p}{2(A_0 - 1)} + \frac{p}{2(A_0 - 1)} i \tan\left(\frac{1}{2\mu} \sqrt{-\frac{\gamma(t)}{2\beta(t)}}(\xi + C)\right) \right]^{-1},$$

$$\begin{aligned}
u_{22}(\xi) &= A_0 - pA_0 \left[p + \frac{p}{2(A_0 - 1)} - \frac{p}{2(A_0 - 1)} \tanh \left(\frac{1}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
u_{23}(\xi) &= A_0 \\
&\quad - pA_0 \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1}, \\
u_{24}(\xi) &= A_0 - pA_0 \left[p - \left[\frac{(1/\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{((A_0 - 1)/p\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1},
\end{aligned} \tag{51}$$

where $\xi = \mu(x - \int (\alpha(t) - (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2})dt + C_1)$.

Set 14. We have the following:

$$\begin{aligned}
a &= \frac{\rho}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
b &= -\frac{\rho - A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
c &= \frac{\rho - A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
A_0 &= A_0, \\
A_1 &= 0, \\
B_1 &= -pA_0,
\end{aligned}$$

$$\begin{aligned}
p &= p, \\
\mu &= \mu, \\
c(t) &= \frac{1}{t} \\
&\quad \cdot \int \left(\alpha(t) - (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
&\quad + C_1, \\
u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}.
\end{aligned} \tag{52}$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$\begin{aligned}
u_{25}(\xi) &= A_0 - pA_0 \left[p - \frac{p\rho}{2(\rho - A_0)} + \frac{p\rho}{2(\rho - A_0)} i \tan \left(\frac{\rho}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
u_{26}(\xi) &= A_0 - pA_0 \left[p + \frac{p\rho}{2(\rho - A_0)} - \frac{p\rho}{2(\rho - A_0)} \tanh \left(\frac{\rho}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
u_{27}(\xi) &= A_0 \\
&\quad - pA_0 \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1}, \\
u_{28}(\xi) &= A_0 - pA_0 \left[p - \left[\frac{(\rho/\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{((\rho - A_0)/p\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1},
\end{aligned} \tag{53}$$

where $\xi = \mu(x - \int (\alpha(t) - (\rho - 2)\sqrt{\gamma(t)\beta(t)/2})dt + C_1)$.

Set 15. We have the following:

$$a = \frac{1}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$b = -\frac{A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$c = \frac{A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$A_0 = A_0,$$

$$A_1 = 0,$$

$$B_1 = p - A_0 p,$$

$$p = p,$$

$$\mu = \mu,$$

$$\begin{aligned} c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\ &\quad \left. - (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1, \\ u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \end{aligned} \quad (54)$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$\begin{aligned} u_{29}(\xi) &= A_0 + (p - A_0 p) \left[p - \frac{p}{2A_0} + \frac{p}{2A_0} i \tan \left(\frac{1}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\ u_{30}(\xi) &= A_0 + (p - A_0 p) \left[p + \frac{p}{2A_0} - \frac{p}{2A_0} \tanh \left(\frac{1}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\ u_{31}(\xi) &= A_0 + (p - A_0 p) \\ &\quad \cdot \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1}, \\ u_{32}(\xi) &= A_0 + (p - A_0 p) \left[p - \left[\frac{(1/\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{(A_0/p\mu) \sqrt{\gamma(t)/2\beta(t)} e^{(1/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1}, \end{aligned} \quad (55)$$

where $\xi = \mu(x - \int (\alpha(t) - (2\rho - 1) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 16. We have the following:

$$a = \frac{\rho - 1}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$b = -\frac{\rho - A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$c = \frac{\rho - A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$A_0 = A_0,$$

$$A_1 = 0,$$

$$B_1 = p - A_0 p,$$

$$p = p,$$

$$\mu = \mu,$$

$$c(t) = \frac{1}{t}$$

$$\begin{aligned} &\cdot \int \left(\alpha(t) - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1, \end{aligned}$$

$$u(\xi) = A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \quad (56)$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{33}(\xi) &= A_0 + (p - A_0 p) \\
 &\cdot \left[p - \frac{p(\rho - 1)}{2(\rho - A_0)} + \frac{p(\rho - 1)}{2(\rho - A_0)} i \tan \left(\frac{\rho - 1}{2\mu} \sqrt{-\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
 u_{34}(\xi) &= A_0 + (p - A_0 p) \\
 &\cdot \left[p + \frac{p(\rho - 1)}{2(\rho - A_0)} - \frac{p(\rho - 1)}{2(\rho - A_0)} \tanh \left(\frac{\rho - 1}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
 u_{35}(\xi) &= A_0 + (p - A_0 p) \\
 &\cdot \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{((\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2(\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2(\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2(\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1}, \\
 u_{36}(\xi) &= A_0 + (p - A_0 p) \\
 &\cdot \left[p - \left[\frac{((\rho - 1)/\mu) \sqrt{\gamma(t)/2\beta(t)} e^{((\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{((\rho - A_0)/p\mu) \sqrt{\gamma(t)/2\beta(t)} e^{((\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1},
 \end{aligned} \tag{57}$$

where $\xi = \mu(x - \int (\alpha(t) - (\rho + 1) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 17. We have the following:

$$\begin{aligned}
 p &= p, \\
 a &= \frac{\rho}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 b &= -\frac{A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 c &= \frac{A_0}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 A_0 &= A_0, \\
 A_1 &= 0,
 \end{aligned}$$

$$B_1 = p(\rho - A_0),$$

$$\mu = \mu,$$

$$c(t) = \frac{1}{t}$$

$$\begin{aligned}
 &\cdot \int \left(\alpha(t) - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1,
 \end{aligned}$$

$$u(\xi) = A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \tag{58}$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{37}(\xi) &= A_0 + p(\rho - A_0) \left[p - \frac{p\rho}{2A_0} + \frac{p\rho}{2A_0} i \tan \left(\frac{\rho}{2\mu} \sqrt{-\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1} \\
 u_{38}(\xi) &= A_0 + p(\rho - A_0) \left[p + \frac{p\rho}{2A_0} - \frac{p\rho}{2A_0} \tanh \left(\frac{\rho}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1},
 \end{aligned}$$

$$\begin{aligned}
u_{39}(\xi) &= A_0 + p(\rho - A_0) \\
&\cdot \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1}, \\
u_{40}(\xi) &= A_0 + (p - A_0 p) \left[p - \left[\frac{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{(A_0/p\mu)\sqrt{\gamma(t)/2\beta(t)}e^{(\rho/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1},
\end{aligned} \tag{59}$$

where $\xi = \mu(x - \int(\alpha(t) - (\rho + 1)\sqrt{\gamma(t)\beta(t)/2})dt + C_1)$.

Set 18. We have the following:

$$p = p,$$

$$a = \frac{\rho - 1}{\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$b = -\frac{A_0 - 1}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$c = \frac{A_0 - 1}{p\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$A_0 = A_0,$$

$$A_1 = 0,$$

$$B_1 = p(\rho - A_0),$$

$$\mu = \mu,$$

$$c(t) = \frac{1}{t}$$

$$\cdot \int \left(\alpha(t) + (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt$$

$$+ C_1,$$

$$u(\xi) = A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \tag{60}$$

By using (50), Families 1, 2, 7, and 14 can be written, respectively, as

$$\begin{aligned}
u_{37}(\xi) &= A_0 + p(\rho - A_0) \\
&\cdot \left[p - \frac{p(\rho - 1)}{2(A_0 - 1)} + \frac{p(\rho - 1)}{2(A_0 - 1)} i \tan \left(\frac{\rho - 1}{2\mu} \sqrt{-\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
u_{38}(\xi) &= A_0 + p(\rho - A_0) \\
&\cdot \left[p + \frac{p(\rho - 1)}{2(A_0 - 1)} - \frac{p(\rho - 1)}{2(A_0 - 1)} \tanh \left(\frac{\rho - 1}{2\mu} \sqrt{\frac{\gamma(t)}{2\beta(t)}} (\xi + C) \right) \right]^{-1}, \\
u_{39}(\xi) &= A_0 + p(\rho - A_0) \\
&\cdot \left[p + \tan \left(\frac{1}{2} \arctan \left[\frac{2e^{((\rho-1)/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{e^{(2(\rho-1)/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1}, \frac{e^{(2(\rho-1)/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1}{e^{(2(\rho-1)/\mu)\sqrt{\gamma(t)/2\beta(t)}(\xi+C)} + 1} \right] \right) \right]^{-1},
\end{aligned}$$

$$u_{40}(\xi) = A_0 + (p - A_0 p) \cdot \left[p - \left[\frac{((\rho - 1)/\mu) \sqrt{\gamma(t)/2\beta(t)} e^{((\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)}}{((A_0 - 1)/p\mu) \sqrt{\gamma(t)/2\beta(t)} e^{((\rho-1)/\mu) \sqrt{\gamma(t)/2\beta(t)}(\xi+C)} - 1} \right] \right]^{-1}, \quad (61)$$

where $\xi = \mu(x - \int (\alpha(t) + (\rho + 1) \sqrt{\gamma(t)\beta(t)/2} dt + C_1)$.

Set 19. We have the following:

$$\begin{aligned} \mu &= \frac{1}{pc} \sqrt{\frac{\gamma(t)}{8\beta(t)}}, \\ p &= p, \\ a &= 2pc, \\ b &= -c, \\ c &= c, \\ A_0 &= \frac{1}{2}, \\ A_1 &= 0, \\ B_1 &= \frac{1}{2}p(2\rho - 1), \\ c(t) &= \frac{1}{t} \\ &\cdot \int \left(\alpha(t) + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{8}} \right) dt \\ &+ C_1, \\ u(\xi) &= A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1}. \end{aligned} \quad (62)$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned} u_{41}(\xi) &= \frac{1}{2} + (2\rho - 1)i \\ &\cdot \cot\left(\sqrt{-p^2c^2}(\xi + C)\right), \\ u_{42}(\xi) &= \frac{1}{2} - (2\rho - 1) \\ &\cdot \coth(pc(\xi + C)), \\ u_{43}(\xi) &= \frac{1}{2} + \frac{1}{2}p(2\rho - 1) \\ &\cdot \left[p - \frac{2pce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1}, \end{aligned} \quad (63)$$

where $\xi = (1/pc) \sqrt{\gamma(t)/8\beta(t)}(x - \int (\alpha(t) + (\rho - 2) \sqrt{\gamma(t)\beta(t)/8} dt + C_1)$.

Set 20. We have the following:

$$\begin{aligned} \mu &= \frac{1}{pc} \sqrt{\frac{\gamma(t)}{8\beta(t)}}, \\ p &= p, \\ a &= 2pc(1 - \rho), \\ b &= -c, \\ c &= c, \\ A_0 &= \frac{1}{2}, \\ A_1 &= 0, \\ B_1 &= \frac{1}{2}p(2\rho - 1), \\ c(t) &= \frac{1}{t} \\ &\cdot \int \left(\alpha(t) - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{8}} \right) dt \\ &+ C_1, \\ u(\xi) &= A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1}. \end{aligned} \quad (64)$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned} u_{44}(\xi) &= \frac{1}{2} + \frac{1}{2}p(2\rho - 1) \left[p\rho \right. \\ &\quad \left. + p(1 - \rho)i \right. \\ &\quad \left. \cdot \tan\left((1 - \rho) \sqrt{-p^2c^2}(\xi + C)\right) \right]^{-1}, \\ u_{45}(\xi) &= \frac{1}{2} + \frac{1}{2}(2\rho - 1) \left[\rho - (1 - \rho) \right. \\ &\quad \left. \cdot \tanh(pc(1 - \rho)(\xi + C)) \right]^{-1}, \end{aligned}$$

$$u_{46}(\xi) = \frac{1}{2} + \frac{1}{2}(2\rho - 1) \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1}, \quad (65)$$

where $\xi = (1/pc)\sqrt{\gamma(t)/8\beta(t)}(x - \int(\alpha(t) - (\rho + 1)\sqrt{\gamma(t)\beta(t)/8}dt + C_1)$.

Set 21. We have the following:

$$\begin{aligned} \mu &= \frac{\rho}{pc} \sqrt{\frac{\gamma(t)}{8\beta(t)}}, \\ p &= p, \\ a &= \frac{2pc}{\rho}, \\ b &= -c, \\ c &= c, \\ A_0 &= \frac{\rho}{2}, \\ A_1 &= 0, \\ B_1 &= -\frac{1}{2}p(\rho - 2), \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) - (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{8}} \right) dt + C_1, \\ u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \end{aligned} \quad (66)$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$u_{47}(\xi) = \frac{\rho}{2} - \frac{1}{2}p(\rho - 2) \left[p - \frac{p}{\rho} + \frac{p}{\rho} i \tan \left(\frac{\sqrt{-p^2 c^2}}{\rho} (\xi + C) \right) \right]^{-1},$$

$$\begin{aligned} u_{48}(\xi) &= \frac{\rho}{2} - \frac{1}{2}(\rho - 2) \left[1 - \frac{1}{\rho} - \frac{1}{\rho} \tanh \left(\frac{pc}{\rho} (\xi + C) \right) \right]^{-1}, \\ u_{49}(\xi) &= \frac{\rho}{2} - \frac{1}{2}(\rho - 2) \left[1 - \frac{(2c/\rho)e^{(2pc/\rho)(\xi+C)}}{ce^{(2pc/\rho)(\xi+C)} - 1} \right]^{-1}, \end{aligned} \quad (67)$$

where $\xi = (\rho/pc)\sqrt{\gamma(t)/8\beta(t)}(x - \int(\alpha(t) - (2\rho - 1)\sqrt{\gamma(t)\beta(t)/8}dt + C_1)$.

Set 22. We have the following:

$$\begin{aligned} \mu &= \frac{\rho}{pc} \sqrt{\frac{\gamma(t)}{8\beta(t)}}, \\ p &= p, \\ a &= \frac{2pc(\rho - 1)}{\rho}, \\ b &= -c, \\ c &= c, \\ A_0 &= \frac{\rho}{2}, \\ A_1 &= 0, \\ B_1 &= -\frac{1}{2}p(\rho - 2), \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{8}} \right) dt + C_1, \\ u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}. \end{aligned} \quad (68)$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{50}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. + \frac{p(\rho-1)}{\rho}i \right. \\
 &\quad \left. \cdot \tan \left(\frac{(\rho-1)\sqrt{-p^2c^2}}{\rho}(\xi+C) \right) \right]^{-1}, \\
 u_{51}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. \cdot \tanh \left((\rho-1)\frac{pc}{\rho}(\xi+C) \right) \right]^{-1}, \\
 u_{52}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p \right. \\
 &\quad \left. - \frac{(2pc(\rho-1)/\rho)e^{(2pc(\rho-1)/\rho)(\xi+C)}}{ce^{(2pc(\rho-1)/\rho)(\xi+C)} - 1} \right]^{-1},
 \end{aligned} \tag{69}$$

where $\xi = (\rho/pc)\sqrt{\gamma(t)/8\beta(t)}(x - \int(\alpha(t) - (\rho+1)\sqrt{\gamma(t)\beta(t)/8}dt + C_1)$.

Set 23. We have the following:

$$\begin{aligned}
 \mu &= \frac{\rho}{pc} \sqrt{\frac{\gamma(t)}{8\beta(t)}}, \\
 p &= p, \\
 a &= \frac{2pc(\rho-1)}{\rho}, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= \frac{\rho}{2}, \\
 A_1 &= 0, \\
 B_1 &= -\frac{1}{2}p(\rho-2),
 \end{aligned}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) - (\rho+1) \sqrt{\frac{\gamma(t)\beta(t)}{8}} \right) dt \\
 &+ C_1, \\
 u(\xi) &= A_0 + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1}.
 \end{aligned} \tag{70}$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{50}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. + \frac{p(\rho-1)}{\rho}i \right. \\
 &\quad \left. \cdot \tan \left(\frac{(\rho-1)\sqrt{-p^2c^2}}{\rho}(\xi+C) \right) \right]^{-1}, \\
 u_{51}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. - \frac{p(\rho-1)}{\rho} \right. \\
 &\quad \left. \cdot \tanh \left((\rho-1)\frac{pc}{\rho}(\xi+C) \right) \right]^{-1}, \\
 u_{52}(\xi) &= \frac{\rho}{2} - \frac{1}{2}p(\rho-2) \left[p \right. \\
 &\quad \left. - \frac{(2pc(\rho-1)/\rho)e^{(2pc(\rho-1)/\rho)(\xi+C)}}{ce^{(2pc(\rho-1)/\rho)(\xi+C)} - 1} \right]^{-1},
 \end{aligned} \tag{71}$$

where $\xi = (\rho/pc)\sqrt{\gamma(t)/8\beta(t)}(x - \int(\alpha(t) - (\rho+1)\sqrt{\gamma(t)\beta(t)/8}dt + C_1)$.

Set 24. We have the following:

$$\begin{aligned}
 \mu &= \frac{1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 a &= a,
 \end{aligned}$$

$$\begin{aligned}
b &= \frac{a(\rho-1)}{2p}, \\
c &= -\frac{a(\rho-1)}{2p}, \\
A_0 &= \frac{1}{2}(\rho+1), \\
A_1 &= 0, \\
B_1 &= -\frac{1}{2}p(\rho+1), \\
c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
&\quad \left. + (2\rho-1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
&\quad + C_1, \\
u(\xi) &= A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1}.
\end{aligned} \tag{72}$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
u_{53}(\xi) &= \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[p \right. \\
&\quad \left. + \frac{p}{\rho-1} \right. \\
&\quad \left. - \frac{p}{\rho-1} i \tan\left(\frac{\sqrt{-a^2}}{2}(\xi+C)\right) \right]^{-1}, \\
u_{54}(\xi) &= \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[1 \right. \\
&\quad \left. + \frac{1}{\rho-1} + \frac{1}{\rho-1} \tanh\left(\frac{a}{2}(\xi+C)\right) \right]^{-1}, \\
u_{55}(\xi) &= \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[p \right. \\
&\quad \left. + \frac{ae^{a(\xi+C)}}{(a(\rho-1)/2p)e^{a(\xi+C)}+1} \right]^{-1},
\end{aligned} \tag{73}$$

where $\xi = (1/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (2\rho-1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 25. We have the following:

$$\begin{aligned}
\mu &= \frac{\rho}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
p &= p, \\
a &= a, \\
b &= -\frac{a(\rho-1)}{2p\rho}, \\
c &= \frac{a(\rho-1)}{2p\rho}, \\
A_0 &= \frac{1}{2}(\rho+1), \\
A_1 &= 0, \\
B_1 &= -\frac{1}{2}p(\rho+1), \\
c(t) &= \frac{1}{t} \\
&\quad \cdot \int \left(\alpha(t) - (\rho-2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
&\quad + C_1, \\
u(\xi) &= A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1}.
\end{aligned} \tag{74}$$

By using (50), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
u_{56}(\xi) &= \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[p \right. \\
&\quad \left. + \frac{pp}{\rho-1} \right. \\
&\quad \left. - \frac{pp}{\rho-1} i \tan\left(\frac{\sqrt{-a^2}}{2}(\xi+C)\right) \right]^{-1}, \\
u_{57}(\xi) &= \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[p \right. \\
&\quad \left. + \frac{pp}{\rho-1} + \frac{pp}{\rho-1} \tanh\left(\frac{a}{2}(\xi+C)\right) \right]^{-1},
\end{aligned}$$

$$u_{58}(\xi) = \frac{1}{2}(\rho+1) - \frac{1}{2}p(\rho+1) \left[p - \frac{ae^{a(\xi+C)}}{(a(\rho-1)/2p\rho)e^{a(\xi+C)} - 1} \right]^{-1}, \quad (75)$$

where $\xi = (\rho/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) - (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 26. We have the following:

$$\begin{aligned} \mu &= \frac{1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\ p &= p, \\ a &= a, \\ b &= c, \\ c &= c, \end{aligned} \quad (76)$$

$$\begin{aligned} A_0 &= 0, \\ A_1 &= 0, \\ B_1 &= p - \frac{c}{a}, \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) - (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \end{aligned} \quad (77)$$

$$u(\xi) = B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (77), Families 7 and 9 can be written, respectively, as

$$\begin{aligned} u_{59}(\xi) &= \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right) \right]^{-1}, \end{aligned} \quad (78)$$

$$u_{60}(\xi) = \left[1 + \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1},$$

where $\xi = (1/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) - (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 27. We have the following:

$$\begin{aligned} \mu &= \frac{\rho}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\ p &= p, \\ a &= a, \\ b &= c, \\ c &= c, \end{aligned} \quad (79)$$

$$\begin{aligned} A_0 &= 0, \\ A_1 &= 0, \\ B_1 &= \left(p - \frac{c}{a} \right) \rho, \\ c(t) &= \frac{1}{t} \cdot \int \left(\alpha(t) + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \end{aligned} \quad (80)$$

$$u(\xi) = B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (80), Families 7 and 9 can be written, respectively, as

$$\begin{aligned} u_{61}(\xi) &= \rho \left[1 + \frac{1}{p} \tan \left(\frac{1}{2} \cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right) \right]^{-1}, \end{aligned} \quad (81)$$

$$u_{62}(\xi) = \rho \left[+ \frac{1}{p-1} e^{ka(\xi+C)} \right]^{-1},$$

where $\xi = (\rho/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 28. We have the following:

$$\begin{aligned}\mu &= \frac{1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\ p &= p, \\ a &= a, \\ b &= c, \\ c &= c,\end{aligned}\quad (82)$$

$$\begin{aligned}A_0 &= 1, \\ A_1 &= 0, \\ B_1 &= \left(-p + \frac{c}{a}\right), \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\ &\quad \left. + (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1,\end{aligned}\quad (83)$$

$$u(\xi) = A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (83), Families 7 and 9, respectively, can be written as

$$\begin{aligned}u_{63}(\xi) &= 1 - p \left[p + \tan\left(\frac{1}{2}\right. \right. \\ &\quad \left. \cdot \arctan\left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right) \right]^{-1}, \\ u_{64}(\xi) &= 1 + (1 - p) \left[p - 1 + e^{ka(\xi+C)} \right]^{-1},\end{aligned}\quad (84)$$

where $\xi = (1/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 29. We have the following:

$$\begin{aligned}\mu &= \frac{\rho - 1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\ p &= p, \\ a &= a,\end{aligned}$$

$$b = c,$$

$$c = c,$$

$$A_0 = 1,$$

$$A_1 = 0,$$

$$B_1 = \left(p - \frac{c}{a}\right)(\rho - 1), \quad (85)$$

$$\begin{aligned}c(t) &= \frac{1}{t} \\ &\cdot \int \left(\alpha(t) + (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1,\end{aligned}\quad (86)$$

$$u(\xi) = A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (86), Families 7 and 9 can be written, respectively, as

$$\begin{aligned}u_{65}(\xi) &= 1 + (\rho - 1) \left[1 + \frac{1}{p} \tan\left(\frac{1}{2}\right. \right. \\ &\quad \left. \cdot \arctan\left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right) \right]^{-1}, \\ u_{66}(\xi) &= 1 + (\rho - 1) \left[1 + \frac{1}{p - 1} e^{ka(\xi+C)} \right]^{-1},\end{aligned}\quad (87)$$

where $\xi = ((\rho - 1)/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho + 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 30. We have the following:

$$\mu = \frac{\rho}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$p = p,$$

$$a = a,$$

$$b = c,$$

$$c = c,$$

$$\begin{aligned}
 A_0 &= \rho, \\
 A_1 &= 0, \\
 B_1 &= \left(-p + \frac{c}{a}\right)\rho,
 \end{aligned}
 \tag{88}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) - (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1,
 \end{aligned}
 \tag{89}$$

$$u(\xi) = A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},$$

where a , b , and c are arbitrary constants. By using (89), Families 7 and 9 can be written, respectively, as

$$\begin{aligned}
 u_{67}(\xi) &= \rho - \rho \left[1 + \frac{1}{p} \tan\left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \left. \left. \right]^{-1}, \\
 u_{68}(\xi) &= \rho + \left(\frac{c}{a} - p\right) \rho \left[p - 1 + e^{ka(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{90}$$

where $\xi = (\rho/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) - (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 31. We have the following:

$$\begin{aligned}
 \mu &= \frac{\rho - 1}{a} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 a &= a, \\
 b &= c, \\
 c &= c, \\
 A_0 &= \rho, \\
 A_1 &= 0, \\
 B_1 &= \left(-p + \frac{c}{a}\right)(\rho - 1),
 \end{aligned}
 \tag{91}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \\
 &\cdot \int \left(\alpha(t) - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &+ C_1, \\
 u(\xi) &= A_0 + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},
 \end{aligned}
 \tag{92}$$

where a , b , and c are arbitrary constants. By using (92), Families 7 and 9 can be written, respectively, as

$$\begin{aligned}
 u_{69}(\xi) &= \rho - (\rho - 1) \left[1 + \frac{1}{p} \tan\left(\frac{1}{2} \right. \right. \\
 &\cdot \arctan \left[\frac{2e^{a(\xi+C)}}{e^{2a(\xi+C)} + 1}, \right. \\
 &\left. \left. \frac{e^{2a(\xi+C)} - 1}{e^{2a(\xi+C)} + 1} \right] \right]^{-1}, \\
 u_{70}(\xi) &= \rho + \left(\frac{c}{a} - p\right)(\rho - 1) \left[p - 1 \right. \\
 &\left. + e^{ka(\xi+C)} \right]^{-1},
 \end{aligned}
 \tag{93}$$

where $\xi = ((\rho - 1)/a)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) - (\rho + 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 32. We have the following:

$$\begin{aligned}
 \mu &= \frac{1}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}}, \\
 p &= p, \\
 a &= 2pc, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= \frac{1}{2}, \\
 A_1 &= -\frac{1}{4p}, \\
 B_1 &= -\frac{p}{4},
 \end{aligned}
 \tag{94}$$

$$c(t) = \frac{1}{t} \int \left(\alpha(t) + (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \quad (95)$$

$$u(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right] + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where c is an arbitrary constant. By using (95), Families 1, 2, and 14 can be written, respectively, as

$$u_{71}(\xi) = \frac{1}{2} - \frac{1}{4} \cdot i \left[\tan \left(\sqrt{-p^2 c^2} (\xi + C) \right) - \cot \left(\sqrt{-p^2 c^2} (\xi + C) \right) \right], \quad (96)$$

$$u_{72}(\xi) = \frac{1}{2} + \frac{1}{4} \left[\tanh \left(\sqrt{p^2 c^2} (\xi + C) \right) + \coth \left(\sqrt{p^2 c^2} (\xi + C) \right) \right],$$

$$u_{73}(\xi) = \frac{1}{2} - \frac{1}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right] - \frac{1}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1}, \quad (97)$$

where $\xi = (1/pc) \sqrt{\gamma(t)/32\beta(t)} (x - \int (\alpha(t) + (2\rho - 1) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 33. We have the following:

$$\mu = \frac{\rho}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}},$$

$$p = p,$$

$$a = 2pc,$$

$$b = -c,$$

$$c = c,$$

$$A_0 = \frac{\rho}{2},$$

$$A_1 = -\frac{\rho}{4p},$$

$$B_1 = -\frac{p\rho}{4},$$

(98)

$$c(t) = \frac{1}{t}$$

$$\cdot \int \left(\alpha(t) - (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \quad (99)$$

$$u(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right] + B_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right]^{-1},$$

where c is an arbitrary constant. By using (99), Families 1, 2, and 14 can be written, respectively, as

$$u_{74}(\xi) = \frac{\rho}{2} - \frac{\rho}{4} \cdot i \left[\tan \left(\sqrt{-p^2 c^2} (\xi + C) \right) - \cot \left(\sqrt{-p^2 c^2} (\xi + C) \right) \right],$$

$$u_{75}(\xi) = \frac{\rho}{2} + \frac{\rho}{4} \left[\tanh \left(\sqrt{p^2 c^2} (\xi + C) \right) + \coth \left(\sqrt{p^2 c^2} (\xi + C) \right) \right], \quad (100)$$

$$u_{76}(\xi) = \frac{\rho}{2} - \frac{\rho}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right] - \frac{\rho}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1},$$

where $\xi = (\rho/pc) \sqrt{\gamma(t)/32\beta(t)} (x - \int (\alpha(t) - (\rho - 2) \sqrt{\gamma(t)\beta(t)/2}) dt + C_1)$.

Set 34. We have the following:

$$\begin{aligned}\mu &= \frac{1}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}}, \\ p &= p, \\ a &= 2pc, \\ b &= -c, \\ c &= c,\end{aligned}\quad (101)$$

$$\begin{aligned}A_0 &= \frac{1}{2}, \\ A_1 &= \frac{1}{4p}, \\ B_1 &= \frac{p}{4}, \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\ &\quad \left. - (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1,\end{aligned}\quad (102)$$

$$\begin{aligned}u(\xi) &= A_0 + A_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right] \\ &\quad + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},\end{aligned}$$

where c is an arbitrary constant. By using (102), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}u_{77}(\xi) &= \frac{1}{2} + \frac{1}{4} \\ &\quad \cdot i \left[\tan\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right. \\ &\quad \left. - \cot\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right], \\ u_{78}(\xi) &= \frac{1}{2} \\ &\quad - \frac{1}{4} \left[\tanh\left(\sqrt{p^2 c^2}(\xi + C)\right) \right. \\ &\quad \left. + \coth\left(\sqrt{p^2 c^2}(\xi + C)\right) \right],\end{aligned}$$

$$\begin{aligned}u_{79}(\xi) &= \frac{1}{2} + \frac{1}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right] \\ &\quad + \frac{1}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1},\end{aligned}\quad (103)$$

where $\xi = (1/pc)\sqrt{\gamma(t)/32\beta(t)}(x - \int(\alpha(t) - (2\rho - 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 35. We have the following:

$$\begin{aligned}\mu &= \frac{\rho - 1}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}}, \\ p &= p, \\ a &= 2pc, \\ b &= -c, \\ c &= c,\end{aligned}\quad (104)$$

$$\begin{aligned}A_0 &= \frac{1}{2}(\rho + 1), \\ A_1 &= -\frac{\rho - 1}{4p}, \\ B_1 &= -\frac{p(\rho - 1)}{4}, \\ c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\ &\quad \left. - (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1,\end{aligned}\quad (105)$$

$$\begin{aligned}u(\xi) &= A_0 + A_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right] \\ &\quad + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},\end{aligned}$$

where c is an arbitrary constant. By using (105), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}u_{80}(\xi) &= \frac{1}{2}(\rho + 1) - \frac{\rho - 1}{4} \\ &\quad \cdot i \left[\tan\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right. \\ &\quad \left. - \cot\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right],\end{aligned}$$

FIGURE 1: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (14) are demonstrated at $C_1 = 2$, $p = 2$, $\rho = 2$, $a = 2$, $\beta(t) = \cos(t)$, $\gamma(t) = \sin(2t)$, and $\alpha(t) = \sin(t)$ and by considering the values ((a) and (c)) $-1000 < x < 1000$, $0 < t < 50$ and ((b) and (d)) $-1000 < x < 1000$, $t = 10$.

$$\begin{aligned}
 u_{81}(\xi) &= \frac{1}{2}(\rho + 1) \\
 &+ \frac{\rho - 1}{4} \left[\tanh \left(\sqrt{p^2 c^2} (\xi + C) \right) \right. \\
 &\left. + \coth \left(\sqrt{p^2 c^2} (\xi + C) \right) \right], \\
 u_{82}(\xi) &= \frac{1}{2}(\rho + 1) - \frac{\rho - 1}{4} \left[1 \right. \\
 &\left. - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right] - \frac{\rho - 1}{4} \left[1 \right. \\
 &\left. - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1},
 \end{aligned} \tag{106}$$

where $\xi = ((\rho - 1)/pc)\sqrt{\gamma(t)/32\beta(t)}(x - \int(\alpha(t) - (\rho + 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Set 36. We have the following:

$$\begin{aligned}
 \mu &= \frac{\rho}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}}, \\
 p &= p, \\
 a &= 2pc, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= \frac{\rho}{2}, \\
 A_1 &= \frac{\rho}{4p}, \\
 B_1 &= \frac{p\rho}{4},
 \end{aligned} \tag{107}$$

FIGURE 2: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (15) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $a = 2$, $\beta(t) = \cos(t)$, $\gamma(t) = \sin(2t)$, and $\alpha(t) = \sin(t)$ and by considering the values ((a) and (c)) $-1000 < x < 1000$, $0 < t < 50$ and ((b) and (d)) $-1000 < x < 1000$, $t = 10$.

$$\begin{aligned}
 c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
 &\quad \left. + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\
 &\quad + C_1, \\
 u(\xi) &= A_0 + A_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right] \\
 &\quad + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},
 \end{aligned} \tag{108}$$

where c is an arbitrary constant. By using (108), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{83}(\xi) &= \frac{\rho}{2} + \frac{\rho}{4} \\
 &\quad \cdot i \left[\tan\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right.
 \end{aligned}$$

$$\begin{aligned}
 &\quad \left. - \cot\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right], \\
 u_{84}(\xi) &= \frac{\rho}{2} \\
 &\quad - \frac{\rho}{4} \left[\tanh\left(\sqrt{p^2 c^2}(\xi + C)\right) \right. \\
 &\quad \left. + \coth\left(\sqrt{p^2 c^2}(\xi + C)\right) \right], \\
 u_{85}(\xi) &= \frac{\rho}{2} + \frac{\rho}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right] \\
 &\quad + \frac{\rho}{4} \left[1 - \frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)} - 1} \right]^{-1},
 \end{aligned} \tag{109}$$

where $\xi = (\rho/pc)\sqrt{\gamma(t)/32\beta(t)}(x - \int(\alpha(t) + (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

FIGURE 3: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (96) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $c = 2$, $\beta(t) = \cos(t)$, $\gamma(t) = \sin(2t)$, and $\alpha(t) = \sin(t)$ and by considering the values ((a) and (c)) $-1000 < x < 1000$, $0 < t < 50$ and ((b) and (d)) $-1000 < x < 1000$, $t = 10$.

Set 37. We have the following:

$$\begin{aligned}
 \mu &= \frac{\rho - 1}{pc} \sqrt{\frac{\gamma(t)}{32\beta(t)}}, \\
 p &= p, \\
 a &= 2pc, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= \frac{1}{2}(\rho + 1), \\
 A_1 &= \frac{\rho - 1}{4p}, \\
 B_1 &= \frac{p(\rho - 1)}{4},
 \end{aligned}
 \tag{110}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
 &\quad \left. + (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \\
 u(\xi) &= A_0 + A_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right] \\
 &\quad + B_1 \left[p + \tan\left(\frac{\Phi(\xi)}{2}\right) \right]^{-1},
 \end{aligned}
 \tag{111}$$

where c is an arbitrary constant. By using (111), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned}
 u_{86}(\xi) &= \frac{1}{2}(\rho + 1) + \frac{\rho - 1}{4} \\
 &\quad \cdot i \left[\tan\left(\sqrt{-p^2 c^2}(\xi + C)\right) \right]
 \end{aligned}$$

FIGURE 4: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (97) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $c = 2$, $\beta(t) = \cos(t)$, $\gamma(t) = \sin(2t)$, and $\alpha(t) = \sin(t)$ and by considering the values ((a) and (c)) $-1000 < x < 1000$, $0 < t < 50$ and ((b) and (d)) $-1000 < x < 1000$, $t = 10$.

$$\begin{aligned}
 & -\cot\left(\sqrt{-p^2c^2}(\xi+C)\right)\Bigg], \\
 u_{87}(\xi) &= \frac{1}{2}(\rho+1) \\
 & -\frac{\rho-1}{4}\left[\tanh\left(\sqrt{p^2c^2}(\xi+C)\right)\right. \\
 & \left.+\coth\left(\sqrt{p^2c^2}(\xi+C)\right)\right], \\
 u_{88}(\xi) &= \frac{1}{2}(\rho+1) + \frac{\rho-1}{4}\left[1\right. \\
 & \left.-\frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)}-1}\right] + \frac{\rho-1}{4}\left[1\right. \\
 & \left.-\frac{2ce^{2pc(\xi+C)}}{ce^{2pc(\xi+C)}-1}\right]^{-1},
 \end{aligned}
 \tag{112}$$

where $\xi = ((\rho-1)/pc)\sqrt{\gamma(t)/32\beta(t)}(x - \int(\alpha(t) + (\rho+1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1))$.

Set 38. We have the following:

$$\begin{aligned}
 \mu &= \frac{A_1}{c}\sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p, \\
 a &= -\frac{c(\rho-1)}{A_1}, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= 1 - pA_1, \\
 A_1 &= A_1, \\
 B_1 &= 0,
 \end{aligned}
 \tag{113}$$

FIGURE 5: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (14) are demonstrated at $C_1 = 2$, $p = 2$, $\rho = 2$, $a = 2$, $\beta(t) = t + 1$, $\gamma(t) = t - 1$, and $\alpha(t) = t^2$ and by considering the values ((a) and (c)) $-100 < x < 100$, $0 < t < 10$ and ((b) and (d)) $-100 < x < 100$, $t = 0.1$.

$$c(t) = \frac{1}{t} \int \left(\alpha(t) + (\rho + 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt + C_1, \quad (114)$$

$$u(\xi) = A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right],$$

where c is an arbitrary constant. By using (114), Families 1, 2, and 14 can be written, respectively, as

$$u_{89}(\xi) = 1 - pA_1 + A_1 \left[p + \frac{\rho - 1}{2A_1} - \frac{\rho - 1}{2A_1} i \right]$$

$$\begin{aligned} & \cdot \tan \left(\frac{\sqrt{-c^2}(\rho - 1)}{2A_1} (\xi + C) \right) \Bigg], \\ u_{90}(\xi) &= 1 - pA_1 + A_1 \left[p + \frac{\rho - 1}{2A_1} + \frac{\rho - 1}{2A_1} \tanh \left(\frac{\sqrt{c^2}(\rho - 1)}{2A_1} (\xi + C) \right) \right], \\ u_{91}(\xi) &= 1 - pA_1 + A_1 \left[p - \frac{(c(\rho - 1)/A_1) e^{-(c(\rho - 1)/A_1)(\xi + C)}}{ce^{-(c(\rho - 1)/A_1)(\xi + C)} - 1} \right], \end{aligned} \quad (115)$$

FIGURE 6: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (15) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $a = 2$, $\beta(t) = t + 1$, $\gamma(t) = t - 1$, and $\alpha(t) = t^2$ and by considering the values ((a) and (c)) $-100 < x < 100$, $0 < t < 10$ and ((b) and (d)) $-100 < x < 100$, $t = 0.1$.

where $\xi = (A_1/c)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho + 1)\sqrt{\gamma(t)\beta(t)/2}dt + C_1))$.

Set 39. We have the following:

$$\mu = \frac{A_1}{c} \sqrt{\frac{\gamma(t)}{2\beta(t)}},$$

$$p = p,$$

$$a = \frac{c}{A_1},$$

$$b = -c,$$

$$c = c,$$

$$A_0 = 1 - pA_1,$$

$$A_1 = A_1,$$

$$B_1 = 0,$$

(116)

$$\begin{aligned} c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\ &\quad \left. - (2\rho - 1) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt \\ &\quad + C_1, \\ u(\xi) &= A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right], \end{aligned} \quad (117)$$

where c is an arbitrary constant. By using (117), Families 1, 2, and 14 can be written, respectively, as

$$\begin{aligned} u_{91}(\xi) &= 1 - pA_1 + A_1 \left[p + \frac{1}{2A_1} \right. \\ &\quad \left. - \frac{1}{2A_1} i \tan \left(\frac{\sqrt{-c^2}}{2A_1} (\xi + C) \right) \right], \end{aligned}$$

FIGURE 7: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (96) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $c = 2$, $\beta(t) = t + 1$, $\gamma(t) = t - 1$, and $\alpha(t) = t^2$ and by considering the values ((a) and (c)) $-100 < x < 100$, $0 < t < 10$ and ((b) and (d)) $-100 < x < 100$, $t = 0.1$.

$$\begin{aligned}
 u_{92}(\xi) &= 1 - pA_1 + A_1 \left[p + \frac{1}{2A_1} \right. \\
 &\quad \left. + \frac{1}{2A_1} \tanh \left(\frac{\sqrt{c^2}}{2A_1} (\xi + C) \right) \right], \\
 u_{93}(\xi) &= 1 - pA_1 + A_1 \left[p \right. \\
 &\quad \left. - \frac{(c/A_1) e^{(c/A_1)(\xi+C)}}{ce^{(c/A_1)(\xi+C)} - 1} \right],
 \end{aligned}
 \tag{118}$$

where $\xi = (A_1/c) \sqrt{\gamma(t)/2\beta(t)} (x - \int (\alpha(t) - (2\rho - 1) \sqrt{\gamma(t)\beta(t)/2} dt + C_1))$.

Set 40. We have the following:

$$\begin{aligned}
 \mu &= \frac{A_1}{c} \sqrt{\frac{\gamma(t)}{2\beta(t)}}, \\
 p &= p,
 \end{aligned}$$

$$\begin{aligned}
 a &= -\frac{c\rho}{A_1}, \\
 b &= -c, \\
 c &= c, \\
 A_0 &= -pA_1, \\
 A_1 &= A_1, \\
 B_1 &= 0,
 \end{aligned}
 \tag{119}$$

$$\begin{aligned}
 c(t) &= \frac{1}{t} \int \left(\alpha(t) \right. \\
 &\quad \left. + (\rho - 2) \sqrt{\frac{\gamma(t)\beta(t)}{2}} \right) dt
 \end{aligned}
 \tag{120}$$

$$\begin{aligned}
 &+ C_1, \\
 u(\xi) &= A_0 + A_1 \left[p + \tan \left(\frac{\Phi(\xi)}{2} \right) \right],
 \end{aligned}$$

FIGURE 8: Graphs of ((a) and (b)) real values and ((c) and (d)) imaginary values of (97) are demonstrated at $C_1 = 2$, $p = 2$, $k = 2$, $\rho = 2$, $c = 2$, $\beta(t) = t + 1$, $\gamma(t) = t - 1$, and $\alpha(t) = t^2$ and by considering the values ((a) and (c)) $-100 < x < 100$, $0 < t < 10$ and ((b) and (d)) $-100 < x < 100$, $t = 0.1$.

where c is an arbitrary constant. By using (120), Families 1, 2, and 14 can be written, respectively, as

$$u_{94}(\xi) = -pA_1 + A_1 \left[p + \frac{\rho}{2A_1} - \frac{\rho}{2A_1} i \tan \left(\frac{\sqrt{-c^2}\rho}{2A_1} (\xi + C) \right) \right],$$

$$u_{95}(\xi) = -pA_1 + A_1 \left[p + \frac{\rho}{2A_1} + \frac{\rho}{2A_1} \tanh \left(\frac{\sqrt{c^2}\rho}{2A_1} (\xi + C) \right) \right],$$

$$u_{96}(\xi) = -pA_1 + A_1 \left[p - \frac{(c\rho/A_1) e^{(c\rho/A_1)(\xi+C)}}{ce^{(c\rho/A_1)(\xi+C)} - 1} \right], \quad (121)$$

where $\xi = (A_1/c)\sqrt{\gamma(t)/2\beta(t)}(x - \int(\alpha(t) + (\rho - 2)\sqrt{\gamma(t)\beta(t)/2}dt + C_1)$.

Remark 1. In Figure 1, we plot two- and three-dimensional graphics of imaginary and real values of (14), which denote the dynamics of solutions with appropriate parametric selections. Also, in Figure 2, we draw two- and three-dimensional graphics of imaginary and real values of (15), which demonstrate the dynamics of solutions with convenient parametric choices. In Figure 3, we plot two- and three-dimensional graphics of (96), which represent the dynamics of solutions with proper parametric values. Moreover, in Figure 4, we

draw two- and three-dimensional graphics of imaginary and real values of (97), which display the dynamics of solutions with suitable parametric selections. We plot two- and three-dimensional graphics of Figures 1–4, when $-1000 < x < 1000$, $0 < t < 50$ and for Figures 5–8 when $-100 < x < 100$, $0 < t < 10$. To the best of our knowledge, these complex exponential function solutions have not been submitted to the literature in advance. The analytical solutions and figures obtained in this paper give us a different physical interpretation for the nonlinear generalized Fitzhugh-Nagumo equation.

4. Conclusion

In this paper, the new approach of expansion method, namely, the improved $\tan(\Phi(\xi)/2)$ -expansion method, has successfully been implemented to investigate the generalized Fitzhugh-Nagumo equation with time-dependent coefficients. Abundant exact travelling wave solutions including solitons, kink, and periodic and rational solutions are attained. It is worth mentioning that some of the newly obtained solutions are identical to already published results. It has been shown that the applied method is effective and more wide ranging than the generalized and improved (G'/G) -expansion method because it gives many new solutions. Therefore, this method can be applied to study many other nonlinear partial differential equations which frequently arise in engineering, mathematical physics, and other scientific real time application fields.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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