Research Article

Gain Incorporated Split-Ring Resonator Structures for Active Metamaterials

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We present a systematic study of split-ring resonator (SRR) structures that are used as the basic building blocks of active metamaterials with incorporated gain. The active split-ring resonator (aSRR) structures with gain elements can in theory have similar unusual electromagnetic responses such as negative effective permeability near their resonance of the artificial magnetic response just like their passive counterparts. At the same time aSRRs can have reversed imaginary part of the effective permeability and, therefore, mitigate the loss of passive SRRs. We explored in detail both passive and active SRRs through analytic theory, numerical simulations, and lab experimentation and demonstrated that aSRRs can have the similar negative effective permeability responses while reducing and even reversing the loss.

1. Introduction

Split-ring resonator- (SRR-) based structures are building blocks for metamaterials that can exhibit exotic effective electromagnetic responses including but not limited to negative or near-zero index of refraction [1], reverse Casimir effect [2], and perfect absorption [3]. Such exotic engineered electromagnetic responses can be extremely useful and may enable many potentially revolutionary applications such as superresolution imaging, cloak of invisibility, quantum levitation, and conforming and electrically small antennas [2–5]. However, one fundamental drawback of the SRR-based metamaterials is their intrinsic loss due to metals used for SRRs [6]. For most metamaterial applications such as superresolution imaging and cloak of invisibility, such loss is detrimental. For other applications that are more tolerant to loss, the intrinsic metal loss of the SRR-based metamaterials still tends to limit the practical applications to those of metasurfaces [7, 8], where only a very thin layer of metamaterials is required. For bulk metamaterials, attempts have been made to compensate the intrinsic metal loss of SRR structures by either adding layers of gain materials or embedding the SRRs in host materials that will provide gain [9, 10]. Dong et al., for example, have explored SSRs on active substrate in optical wavelengths up to 610 nm [9]. According

to their approach, the heavy metal loss of the SRR structures is compensated by the optical gain material as the substrate and left-hand resonance transmission is obtained at 20 dB reduced loss, essentially providing transparent transmission. In this approach, the absorption nature of the negative index material (NIM) is not changed; it is only compensated by the separate layer of optical gain. Other investigators also explored the similar concepts of compensating the metal loss of plasmonic structures by separate gain layers [10]. Experimentally such approaches so far only have had limited successes. On the other hand, there is still a hot debate whether or not such mixing formula based approaches would work in theory due to the spatial averaging nature of the underlying assumptions of the effective medium theory. More specifically, the separate layers of gain materials employed in these loss compensation approaches are all normal materials with, for example, positive indices of refraction. While the spatial average according to the effective medium theory approaches reduces the effective metamaterial losses in the spectral amplitude responses, which are determined by the imaginary part of the effective index of refraction, it also tends to reduce the negative real part of the effective index of refraction of the NIM at the same time, which defines the unusual effective electromagnetic responses that are related to the spectral phase responses. We propose here to

mitigate the intrinsic metal loss problem of the SRR-based metamaterials by making the underlying SRR structures active, namely, incorporating gain element in them while keeping the unusual spectral phase response characteristics similar to that of their passive counterparts. This is similar to the case where the population inversion is introduced in a normally absorptive atomic system, making the same spectral resonance feature now exhibiting gain instead of loss through the stimulated emission. In this way, the unusual spectral phase response governed by the real part of the effective index of refraction can be preserved but now potentially with gain. The successful demonstration of this novel concept presented in this paper will lead to a rich area of fundamental research of active metamaterials that are characteristically defined by its active composition of the meta-atoms. The key differentiator of our proposed active metamaterials approach as compared to previous metamaterial loss compensating approaches is that we focus on making the meta-atoms themselves active while most other groups were using active host materials in order to compensate the loss of the meta-atoms. In other words, our proposed approach fundamentally changes meta-atoms' spectral responses, notably from loss to transparent or even gain for the spectral amplitude response while it maintains the abnormal spectral phase response due to the resonance of the SRR structure that gives rise to its artificial magnetic responses, leading to the possibility of negative permeability. In contrast, other approaches try to compensate the passive SRR's loss by extra gain layers of normal materials, which, as pointed out above, may or may not succeed in achieving the goal of low loss metamaterials. Our proposed approach to active metamaterials is fundamentally different in this respect. We explore the single active SRR ("meta-atom") structures and take advantage that it is possible to maintain the abnormal dispersion relationship while making the meta-atoms a gain element rather than a loss element by directly incorporating gain in the SRR structure. The proposed approach also provides the benefit of the introduced active elements, which can in turn strongly interact with the stimulus electromagnetic fields and therefore provide means for tunable as well as active metamaterials. Such active metamaterials with gain can also be incorporated with passive metamaterials to make transparent materials without reducing the unusual effective electromagnetic responses that are related to the spectral phase responses, a.k.a. the real part of the effective index of refraction. In this paper, we present a systemic study including analytical theory, numerical simulations, and experimental investigations of incorporating gain element in single active SRR structures. We demonstrate that the spectral amplitude responses of the active SRR cells can in fact exhibit gain while preserving the unusual spectral phase responses similar to their passive counterparts.

2. Artificial Magnetic Response from Split-Ring Resonators

Split-ring resonators (SRRs) have been extensively studied and well understood [11]. As illustrated in Figure 1(a),

the metal ring structure provides the inductive response to the electromagnetic field, while the gap of the split ring provides the capacitive response. Depending on the design, double split rings can also be used to increase the capacitive response so that the resonant frequency of the SRR can be further reduced, making SRR suitable for electrically small antenna applications. In our study, we used the simplistic single ring design. Generally speaking, SRRs gain their special electromagnetic responses, that is, the artificial magnetic response (AMR), through the resonance of the SRR structures that is determined by the artificially introduced inductive and capacitive responses. Figure 1 shows a simple SRR design (Figure 1(a)) and its lump element equivalent circuit model for the AMR resonance (Figure 1(b)). Not all electromagnetic resonances of SRRs are the AMR resonance. Many are higher-order electrical resonances that typically have more complicated surface current distributions. In order to facilitate an intuitive understanding of the SRR's AMR resonance as well as different SRR designs in general, we use the lump element equivalent circuit model as illustrated in Figure 1(b). The transformer is used to model the electromagnetic wave excitation, which has the sinusoidal varying magnetic field perpendicular to the ring. The excitation can be modeled as the induced electromotive force (emf) in the SRR. Analysis using this equivalent circuit model yields the effective relative permeability of the SRR as

$$\mu_{\text{eff}}^{\text{SRR}} = 1 - \frac{\pi}{2} \frac{s^2 \mu_0 r C}{1 + sRC + s^2 LC}, \quad (1)$$

where r is the inner radius of the metal ring, L is the self-inductance of the SRR, C is the capacitance of the SRR, and R is the total resistance. The self-inductance L is proportional to the length of the metal ring. The capacitance C , on the other hand, is due to the dielectric gap introduced in the metal ring. The total resistance R includes metal Ohmic resistance as well as the radiation resistance that can be used to model the radiation property of the SRR as an antenna. The AMR resonance frequency of the SRR is therefore $\omega_0 = 1/\sqrt{LC}$.

For the metamaterial built with the SRR as the building blocks and a filling ratio of F , one has approximately for the effective relative permeability of the metamaterial

$$\mu_{\text{eff}} = (1 - F) \mu_{\text{host}} + F \mu_{\text{eff}}^{\text{SRR}}. \quad (2)$$

These simplified theoretical analyses are in consistence with previous studies of SRRs in the literature.

Figure 2(a) shows a simulation of the real and imaginary parts of the effective permeability of a hypothetical case of $r = 12$ mm, $L = 24$ nH, $C = 1$ pF, and $R = 10$ Ohm. The AMR resonance frequency is around 1 GHz. The effective permeability of the SRR determines the magnetic field inside the SRR given a fixed electromagnetic wave excitation. The induced surface current in the SRR will give rise to the induced electromagnetic waves and the far-field scattered wave from the SRR is the superposition of the induced electromagnetic wave and the original excitation wave. For example, in the direction of the original wave propagation, the complex number μ_{eff} means that the induced wave and

FIGURE 1: (a) A simple design of the split-ring resonator (SRR). The metal ring provides the magnetic field response while the dielectric gap provides the capacitive response and together SRR demonstrates a distinct resonance response referred to as the artificial magnetic response. (b) The lump element equivalent circuit model for the SRR's artificial magnetic response.

FIGURE 2: AMR resonance of the SRR. (a) Effective permeability. Solid curve shows the real part of the effective relative permeability. Dashed curve shows the imaginary part. (b) The far-field scattered wave response that clearly exhibits the AMR resonance.

the original wave will add up with a phase shift and this may result in attenuation in the propagation in that particular direction. In addition, in the spectral range where the real part of the effective permeability is negative, it is possible to make a negative index of refraction (NIM) material if similar negative electrical responses can be incorporated. In the research of the metamaterials built from SRRs, it is always desired to locate the actual AMR resonant frequency induced by the SRR. It is, however, not always an easy task to do so since the far-field transmission responses of the SRR, for example, may exhibit many resonance features that may not be related to the desired AMR resonance. Identifying without ambiguity the actual AMR resonance is therefore a critical task in design and optimization of SRR-based metamaterials.

Consider the special case of the propagation of scattered waves in the orthogonal direction where there is no propagation of excitation waves (e.g., in the direction along which

the electric field of the excitation wave oscillates or polarized); the scattered wave is solely determined by the induced magnetic field inside the SRR and proportional to the magnitude of $|\mu_{\text{eff}}^{\text{SRR}} - 1|$, which has the characteristics of a second-order high-pass filter response. This configuration provides a convenient far-field characterization method for SRRs AMR and its far-field effects. The characterization requires only to take far-field measurements perpendicular to the direction of propagation of the excitation wave on the SRR plane. One should be expecting a second-order high-pass filter type response for the scattered wave and the AMR frequency, for example, can be identified from the characteristic response. This special far-field scattered wave response is shown in Figure 2(b) for the same hypothetical SRR as in Figure 2(a). If different far-field configurations are used, AMR is typically blended with other higher-order electrical and/or magnetic

(a)

Curve info

— dB(rEX)
Setup1: sweep1
 $\phi = "10 \text{ deg.}" \theta = "90 \text{ deg.}"$
— dB(rEX)
Setup1: sweep1
 $\phi = "30 \text{ deg.}" \theta = "90 \text{ deg.}"$
— dB(rEX)
Setup1: sweep1
 $\phi = "50 \text{ deg.}" \theta = "90 \text{ deg.}"$

— dB(rEX)
Setup1: sweep1
 $\phi = "70 \text{ deg.}" \theta = "90 \text{ deg.}"$
— dB(rEX)
Setup1: sweep1
 $\phi = "90 \text{ deg.}" \theta = "90 \text{ deg.}"$

(b)

FIGURE 3: 3D HFSS simulation of single SRR. (a) HFSS 3D SRR simulation setup. (b) Far-field scattered field responses at different locations on the xy -plane exhibiting the characteristic resonance that give rise to the artificial magnetic response (AMR) of the SRR structure along with other higher-order electrical resonance responses.

responses, adding to the difficulty to single out the far-field AMR effects.

3. Numerical Simulations of Single SRR Far-Field FAMR Effects

Extensive numerical simulations of single SRR using the HFSS 3D solver have been performed for far-field effects of AMR. The basic simulation setup is shown in Figure 3(a). The excitation electromagnetic wave is propagating along the x -axis (the green axis in the figure). The electrical field of the excitation waves is polarized along the y -axis (blue). The magnetic field is along the z -axis (red) and is perpendicular to the SRR surface. The far-field scattered waves are measured in the xy -plane. Since the excitation waves have the electrical field that is parallel to the y -axis, the far-field scattered waves in y direction, for example, are solely determined by the induced magnetic field excited inside the SRR. As the result, the far-field radiation pattern measured at a location that is on the y -axis will exhibit a second-order high-pass (SOHP) filter characteristic spectral response at the SRR's AMR resonance frequency. The numerical simulation covers a wide frequency range from 100 MHz to 12 GHz. Figure 3(b) shows the simulation results of 10 mm SRR. As shown in Figure 3(b), the typical frequency responses can be characterized by two dominant features. The first dominant feature is a clear SOHP filter type characteristic with the resonance frequency in the 1.5 GHz range. This feature does not shift in frequency for different scattering directions, which is expected for the AMR resonance of the SRR. The second dominant feature is the periodic dips in the passband of the SOHP filter. These periodic dip features have the frequency period that increases with the scattered angle

and they are attributed to the SRR's higher-order electrical resonances that determine its radiation impedance. The higher-order electrical resonant responses also give rise to the direction dependent nature of the radiation impedance of the SRR. The qualitative distinction between the AMR resonance and other resonances related to the radiation characteristics of SRR is important as the optimal design of the SRR-based metamaterial structure will require a match between these two features. In comparison, Figure 4 shows a far-field scattered field spectral responses of a full ring (no dielectric gap) in the y direction and clearly there is lack of the second-order high-pass filter type response that is characteristic for the AMR resonance. Obviously, in theory, the AMR resonance is expected to disappear without the dielectric gap that introduces the loop capacitance. The response of the full ring changes to a first-order high-pass type as expected with no indication of the AMR resonance. The same size of full-ring resonator still exhibits similar impedance matching type of periodic spectral responses in the passband due to higher-order electrical resonances.

4. Fabrication and Measurement of Single SRR Structures

Based on the extensive numerical simulation and theoretical analysis results, we fabricated and measured several different SRR structures including the ones incorporating gain elements. The SRRs were fabricated in-house using the printed circuit board (PCB) router on FR4 dielectric boards. First, the passive SRRs with various sizes were measured and compared to the numerical simulations. Figure 5 shows the experimental setup and the dependence of the AMR resonance peak frequency on the size of the SRR

FIGURE 4: 3D HFSS simulations of the far-field responses in y direction of a full ring of the same size as in Figure 3(b). There are no AMR characteristic responses in this case as compared to the splitting cases.

and compares the measurement results with the numerical simulations. The experimental setup consists of a Vector Network Analyzer (Agilent N5230A) and a pair of matched antennas for the frequency band of interest. The identified AMR resonance peak frequencies of various SRRs are plotted versus the radius r of the SRR ring. The power law fittings yield close approximate relations of $f_0 \propto 1/r$, which is in agreement with the lump element equivalent circuit model discussed in Section 2 for both the numerical simulation and measurement results. The measured results are also in excellent agreement with the simulated results.

On the other hand, the active SRRs (aSRRs) need to incorporate gain elements in the passive SRR structures. We started with the externally driven SRRs where we demonstrated that the SRRs emit efficiently in their resonant frequency range and the emission adds coherently to the input. Consequently, we designed a self-driven active SRR consisting of a passive SRR connected to a circulator followed by an amplifier that provides the gain as shown in Figure 6(a). The circulator is necessary here in the single ring aSRR configuration in order to avoid the direct short circuit connection between the input and output of the amplifier. The amplified signal at the amplifier output is in turn fed to the SRR and re-emitted via the circulator. This configuration of aSRR is capable of capturing the input waves in the frequency range of SRR's resonance and then amplifying and reemitting at the same frequency coherently, changing the amplitude response of the SRR structure from loss to gain, while still keeping the abnormal phase response in the interested aSRR response frequency range. This configuration allows one to incorporate the gain element with a single SRR ring via the circulator and the design is conceptually closest to the ideal aSRR where a gain element is incorporated into a single SRR ring structure. Conceptually, the effect of the gain incorporated can be modeled by the negative resistance in the lump element circuit model similar to that presented in Section 2. Replacing the resistance R in (1) with an effective resistance R_{eff} that

models the combined effects of metal loss, radiation loss, and the gain, one can have the effective relative permeability of the aSRR as

$$\mu_{\text{eff}}^{\text{aSRR}} = 1 - \frac{\pi}{2} \frac{s^2 \mu_0 r C}{1 + s R_{\text{eff}} C + s^2 L C}. \quad (3)$$

Figure 6(b) shows the real and imaginary parts of the effective permeability of a hypothetical case of $r = 12$ mm, $L = 24$ nH, $C = 1$ pF, and $R_{\text{eff}} = -10$ Ohm. The added gain does not affect the AMR frequency, which is determined by the self-inductance and capacitance of the split ring. Compared to Figure 2(a) which plots the effective relative permeability of the passive SRR, the real part stays the same, which, for example, still provides the opportunity for negative effective permeability. The imaginary part, on the other hand, has been flipped in sign. This means that, at the frequencies where the real part of the effective permeability can be negative, aSRRs will now exhibit gain rather than the loss as in cases for the passive SRR structures. Figure 7 shows the measurement results of this circulator based aSRR design. Figure 7(a) shows a picture of the measurement setup for this aSRR design configuration and the measured amplitude and phase spectral responses of the circulator based aSRR design with increasing applied voltages to the gain element (e.g., RF amplifier) are shown in Figures 7(b) and 7(c), respectively. The amplifier has a nominal gain of 23 dB around 10 GHz. As one can see from the amplitude spectral responses in Figure 7(b) loss of the resonance feature of the SRR in the vicinity of 10 GHz is significantly reduced by the increasing gain provided by the gain element. Figure 7(c) shows the phase spectral responses which is corresponding to the real part of the effective permeability.

The circulator based aSRR design, although conceptually the closest to an ideal aSRR that incorporates gain, is not suitable to be implemented as single board device. This is because when integrated on board, the bulky circulator disrupts the electromagnetic field that the aSRR is designed to respond. The circulator based design does not lend itself well to the active metamaterials implementation as the large array of aSRR unit cells would require large number of circulators, which are of high cost and difficult to be integrated in the metamaterials. Given this consideration, we experimented on a dual ring design with a small footprint surface mount amplifier in between. At 2.4 GHz targeted operation frequency, the relative size of the surface mount RF amplifier (SE2574L, $2 \text{ mm} \times 2 \text{ mm} \times 0.9 \text{ mm}$) as compared to the SRR is very small. A picture of this design is shown as the inset pictures in Figures 8(d) and 8(e). The small footprint surface mount amplifier minimizes the impact of additional components and introduces minimal disturbance to the electromagnetic fields surrounding the aSRRs. As a comparison, we also fabricated the same dual ring structure except for the fact that there is no amplifier in between (see insets in Figures 8(b) and 8(c)). A picture of the near-field measurement setup is shown in Figure 8(a). The spectral responses are measured as the calibrated S_{21} responses between two monopole antennas that are brought to the near-field range to the device. The measured results of dual ring SRR and aSRR are compared

FIGURE 5: (a) Passive single SRR measurement setup. VNA: Vector Network Analyzer; DUT: Device Under Test. (b) Measured AMR frequencies of single SRRs of different sizes as compared with the HFSS 3D simulation results. Solid curve: power law fitting for the measured data set. Dashed curve: power law fitting for the simulated data sets. Both data sets find the curve fitting to be closely inverse proportional to the ring radius. The measured results also agree very well with the simulated results.

FIGURE 6: Active SRR incorporating gain elements. (a) Conceptual design employing a circulator to incorporate a gain element with a single SRR ring. Top: a circuit board realization of the circulator and amplifier circuit. (b) Effective permeability of ideal active SRR with the incorporated gain modeled as negative resistance. Solid curve: the real part of the effective relative permeability. Dashed curve: the imaginary part.

in Figures 8(b)–8(e). With the passive dual ring SRR design, the AMR is shown around 2.4 GHz in Figures 8(b) and 8(c). Figure 8(b) shows the spectral response of the amplitude response that characterizes the imaginary part of the relative effective permeability, while Figure 8(c) shows the phase spectral responses that characterize the real part of the relative effective permeability. According to the spectral amplitude response shown in Figure 8(b), the AMR resonant loss of the single passive SRR is as deep as -50 dB, indicating a good AMR resonance feature. The aSRR design, on the other hand,

uses the narrow band RF amplifier with the gain frequency matched to around 2.4 GHz and a typical RF gain of 25 dB. Figures 8(d) and 8(e) show the same near-field measurements on the aSRR with the amplifier turned on. The near-field S-parameter measurement directly correlates to the effective permeability of the single SRR and aSRR. The spectral phase response of S_{21} corresponds to the real part of the relative effective permeability and the spectral amplitude response of S_{21} corresponds to the imaginary part of the relative effective permeability of the SRR/aSRR. It can be seen clearly that

FIGURE 7: Testing of the active SRR incorporating gain elements. (a) Testing setup for the circulator based aSRR designs. The passive SRR is connected to the circulator and amplifier block (Figure 6(a)) via a coax cable to minimize the impact on the effective testing region of the electromagnetic field between two probe antennas. (b) Amplitude spectral response shown as the relative spectral intensity of S_{21} versus the probing frequency. (c) Phase spectral response shown as the calibrated and unwrapped phase of S_{21} versus the probing frequency.

the imaginary part of the effective permeability due to AMR is now inversed and exhibiting gain, while the real part of the effective permeability due to AMR remains with the same characteristics as the passive SRR. This result is in good agreement with the theoretical expectation as shown in Figures 2(a) and 6(b) for the passive SRR and the active SRR, respectively. Considering the coupling loss between the radiating SRR and the monopole antenna used in the measurement setup, the actual gain block performance is estimated to be at least 10 dB or better. These results show that the SRR's AMR feature that plays critical role in metamaterials can be preserved while flipping the amplitude response from loss to gain using active SRR structures. Additionally, the close match between the responses for the passive SRR structures and aSRR structures but with no gain also confirms that the small surface mount RF gain block has minimum effects on the aSRR's AMR responses.

5. Discussion and Conclusions

We have studied in depth the artificial magnetic responses introduced by the SRR in theory and through numerical simulations, as well as experimentally. It is revealed that

the desired SRR resonances that give rise to the artificial magnetic responses should not be confused with other radiation resonances of the SRRs, although they need to be matched to obtain the optimized performance of the SRR-based metamaterials. The experimental results suggested that the active components of the proposed aSRR need to be closely matched to the passive SRR also. One problem encountered in the previous investigation is that the broadband RF amplifier used with the SRR will oscillate at frequencies that are different from the desired SRR resonance. This is understandable in the sense that at the SRR resonance the radiation loss is large for efficient reradiation SRR designs. As a result, while the RF amplifier gain control reduces the loss at the SRR resonance, we have not been able to reverse that loss into gain. The optimization of the single aSRR can be made, according to this observation, to use narrow bandwidth RF amplifiers that only have gain around the intended SRR resonance. This design optimization requires the accurate identification of the SRR resonance frequencies for an optimal match and the improved results have been demonstrated in the dual ring aSRR designs.

In conclusion, proof-of-principle investigation has shown that the active SRR structures with incorporated gain exhibit

(a)

(b)

(c)

(d)

(e)

FIGURE 8: Testing of the double-ring aSRR with incorporated gain elements. (a) the near-field measurement setup; ((b) and (c)) passive ring-to-ring; ((d) and (e)) with amplifier powered on.

significantly improved spectral amplitude responses while preserving their spectral phase responses. Therefore, it is very promising for metamaterials incorporating such active SRRs to have low loss and being electrically tunable at the same time while preserving its unusual responses similar to their passive counterparts.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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