Research Article

SH-Wave at a Plane Interface between Homogeneous and Inhomogeneous Fibre-Reinforced Elastic Half-Spaces

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The problem of reflection and refraction of SH-waves at a plane interface between the homogeneous and inhomogeneous fibre-reinforced elastic half-spaces has been investigated. Amplitude and energy ratios corresponding to the reflected and refracted SH-waves are derived using appropriate boundary conditions. These ratios are computed numerically for a particular model and the results are depicted graphically.

1. Introduction

Fibre-reinforced composite material with the reinforcement distributed continuously in concentric circles is a locally transversely isotropic material, with the circumferential direction as the preferred direction coinciding with fibres directions. The fibres may be continuous, in which each fibre extends through a body from one boundary to another, or discontinuous, but in the discontinuous case the length of the fibre must be large compared to its diameter. The elastic fibre-reinforced composite materials, for example, typical carbon fibre-epoxy resin composites, are not just anisotropic but also strongly anisotropic in the sense that the modulus for extension in the fibre direction greatly exceeds the moduli for extension in the transverse direction and for shear in the fibre or transverse direction. Fibres have an excellent potential to improve the mechanical properties of rapid-setting materials and could be used effectively to improve the performance of repairs. The behavior of fibre-reinforced rapid-setting materials is similar to that of Portland cement fibre-reinforced concrete. Spencer [1] gave the concept of deformation in fibre-reinforced elastic materials. Belfield et al. [2] discussed the problem of stress in elastic plates reinforced by fibers lying in concentric circles and explained the anisotropic characters of fibre-reinforced materials. Sengupta and Nath [3] studied the problem of surface waves in fibre-reinforced anisotropic elastic media and derived the frequency equation for surface wave. Chattopadhyay et al. [4] investigated the problem of reflection of quasi-P and quasi-SV waves at the plane-free and rigid boundaries of a fibre-reinforced elastic medium and obtained the phase velocity of quasi-P and quasi-SV waves. B. Singh and S. J. Singh [5] discussed the problem of reflection of plane waves at the free surface of a fibre-reinforced elastic half-space and obtained the closed form expression of the amplitude ratios for reflected qSV and qP waves at the free surface of a fibre-reinforced, anisotropic, homogeneous, elastic half-space. Singh [6] studied the problem of propagation of plane waves in thermally conducting linear fibre-reinforced composite materials and derived the frequency equation. Abbas [7] discussed a two-dimensional problem for a fibre-reinforced anisotropic thermoelastic half-space with energy dissipation and the results with energy dissipation and without energy dissipation are compared. Singh and Zorammuana [8] solved the problem of incident longitudinal wave at a fibre-reinforced thermoelastic half-space and obtained the amplitude and energy ratios of the reflected waves.

A number of researchers have attempted the problem of SH-waves in elastic media. Gutenberg [9, 10] explored the existence of low velocity layer in the earth mantle. Chattopadhyay and Keshri [11] discussed the propagation of shear waves along the plane surface between two different elastic
media with initial stress. Singh and Tomar [12] investigated the problem of propagation of SH-waves at a corrugated interface between two dissimilar fiber-reinforced elastic half-spaces using Rayleigh's method of approximation. Chattopadhyay and Singh [13] studied the problem of G-type seismic waves in fibre-reinforced media and obtained the dispersion equation for the propagation of G-type seismic wave in fibre-reinforced layer lying over an inhomogeneous fibre-reinforced elastic half-space. Literatures showed many problems of propagation of SH-waves and notable among them are Bath and Arroyo [14], Gupta [15], Tomar et al. [16], Kumar et al. [17], Chaudhary et al. [18, 19], Emets et al. [20], Chattopadhyay and Michel [21], Tomar and Singh [22], Tomar and Kaur [23, 24], Abd-Alla and Alsheikh [25], Chattopadhyay et al. [26], Singh [27], Wang and Zhao [28], and Sahu et al. [29].

Fibre-reinforced composite materials are very attractive in many engineering applications due to their high strength and low weight. In this paper, the problem of reflection and refraction of SH-waves in the homogeneous/inhomogeneous fibre-reinforced elastic half-spaces has been studied. We have obtained the amplitude and energy ratios corresponding to the reflected and refracted SH-waves using appropriate boundary conditions. These ratios are computed numerically for a particular model.

2. Basic Equations

The constitutive relation for a linearly fibre-reinforced elastic medium is given by Belfield et al. [2] as

\[ \tau_{ij} = \lambda \delta_{ij} \delta_{ij} + 2 \mu \epsilon_{ij} + \alpha (\epsilon_{ik} \epsilon_{kj} + \epsilon_{ik} \epsilon_{kj}) + 2 (\mu_L - \mu_T) (\epsilon_{ik} \epsilon_{kj} + \epsilon_{ik} \epsilon_{kj}) + \beta \epsilon_{ik} \epsilon_{kj} \],

where \( \tau_{ij} \) is stress tensor, \( \epsilon_{ij} \) is strain tensor and is defined by

\[ \epsilon_{ij} = \frac{1}{2} (u_{ij} + u_{ji}), \]

\( \lambda, \mu, \alpha, \beta, \) and \( \mu_L \) are elastic constants in which \( \mu_T \) is identified as the shear modulus in transverse shear across the preferred direction and \( \mu_L \) as the shear modulus in longitudinal shear in the preferred direction, \( \alpha \) and \( \beta \) are specific stress components depending upon the concrete part of the composite materials, \( \delta_{ij} \) is Kronecker delta, and \( a_i \) is the component of a unit vector \( \mathbf{a} = (a_1, a_2, a_3) \) which gives the preferred direction of fibre-reinforcement and \( a_1^2 + a_2^2 + a_3^2 = 1 \).

The equation of motion for the fibre-reinforced elastic material without body forces may be written as

\[ \tau_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = 1, 2, 3), \]

where \( u_i = (u_1, u_2, u_3) \) and \( x_1 = x, x_2 = y, x_3 = z \).

Let us take two-dimensional problem of SH-wave propagation in \( xz \)-plane and the preferred direction of fibre-reinforcement is taken as \((a_1, 0, a_3) \). We may take \( u_1 = u_3 = 0 \), \( u_2 = u_2(x, z, t) \). With the help of (1) and (3), we have

\[ \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial z^2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \]

where

\[ \tau_{12} = \mu_T \frac{\partial u_2}{\partial x} + (\mu_L - \mu_T) a_1 \left( a_1 \frac{\partial u_2}{\partial x} + a_3 \frac{\partial u_2}{\partial z} \right), \]

\[ \tau_{23} = \mu_T \frac{\partial u_2}{\partial z} + (\mu_L - \mu_T) a_3 \left( a_1 \frac{\partial u_2}{\partial x} + a_3 \frac{\partial u_2}{\partial z} \right). \]

It may be noted that the second term in the expression of the stress tensors \( \tau_{12} \) and \( \tau_{23} \) contributes due to the effect of direction of reinforcement, that is, fibre orientation in the self-reinforced material.

Using the stress tensors into (4), we obtain the equation of motion for SH-wave propagation in the fibre-reinforced elastic medium as

\[ \rho \frac{\partial^2 u_2}{\partial x^2} + Q \frac{\partial^2 u_2}{\partial x \partial z} + R \frac{\partial^2 u_2}{\partial x \partial z} = \rho \frac{\partial^2 u_2}{\partial t^2}, \]

where

\[ P = \mu_T + a_1^2 (\mu_L - \mu_T), \]

\[ Q = \mu_T + a_1^2 (\mu_L - \mu_T), \]

\[ R = 2a_1a_3 (\mu_L - \mu_T). \]

3. Problem Formulation

Let \( x \)- and \( y \)-axes of the Cartesian coordinate system be on the horizontal plane and let \( z \)-axis be pointing vertically downward. Consider a homogeneous fibre-reinforced elastic half-space, \( \{H; 0 \leq z < \infty \} \) with \( \rho \) as density and \( \mu_L, \mu_T \) as elastic constants and a nonhomogeneous fibre-reinforced elastic half-space, \( \{H'; -\infty < z \leq 0 \} \) with \( \rho' \) as density and \( \mu_L', \mu_T' \) as elastic constants. The elastic constants and density in the nonhomogeneous fibre-reinforced elastic half-space \( H' \) are defined as (see [13])

\[ \mu_L' = \mu_L^{(0)} (1 - \epsilon \cos sz), \]

\[ \mu_T' = \mu_T^{(0)} (1 - \epsilon \cos sz), \]

\[ \rho' = \rho_0 (1 - \epsilon \cos sz), \]

where \( \epsilon \) is small positive constant and \( s \) is real depth parameter.

The stress tensors in the nonhomogeneous fibre-reinforced elastic half-space (\( H' \)) are

\[ \tau_{12}' = \mu_T' \frac{\partial u_2'}{\partial x} + (\mu_L' - \mu_T') a_1 \left( a_1 \frac{\partial u_2'}{\partial x} + a_3 \frac{\partial u_2'}{\partial z} \right), \]

\[ \tau_{23}' = \mu_T' \frac{\partial u_2'}{\partial z} + (\mu_L' - \mu_T') a_3 \left( a_1 \frac{\partial u_2'}{\partial x} + a_3 \frac{\partial u_2'}{\partial z} \right). \]
The equation of motion for SH-wave in the half-space, $H'$, may be written as

$$\rho_0 c_2^2 = P^{(0)} \left\{ \left(1 - \epsilon \cos sz \right) \frac{\partial^2 u_2'}{\partial z^2} + \epsilon \sin sz \frac{\partial u_2'}{\partial z} \right\}$$

$$+ Q^{(0)} \left(1 - \epsilon \cos sz \right) \frac{\partial^2 u_2'}{\partial x^2}$$

$$+ R^{(0)} \left\{ 2 \left(1 - \epsilon \cos sz \right) \frac{\partial^2 u_2'}{\partial x \partial z} \right\}$$

$$+ \omega (\omega - k 0 (x \sin \theta 0 + z \cos \theta 0))$$

$$= \rho_0 \left(1 - \epsilon \cos sz \right) \frac{\partial^2 u_2'}{\partial t^2},$$

where

$$P^{(0)} = \mu^{(0)} + a_5 (\mu^{(0)} - \mu^{(0)}),$$

$$Q^{(0)} = \mu^{(0)} + a_1 (\mu^{(0)} - \mu^{(0)}),$$

$$R^{(0)} = a_1 a_3 (\mu^{(0)} - \mu^{(0)}).$$

The displacement components in the half-spaces, $H$ and $H'$, may be represented by

$$u_2 = A 0 \exp \left[ \frac{\omega}{\omega - k 0 (x \sin \theta 0 + z \cos \theta 0)} \right]$$

$$+ A \exp \left[ \frac{\omega}{\omega - k 2 (x \sin \theta 1 + z \cos \theta 1)} \right],$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

The expression of $c_2'$ depending on $\epsilon$ may be written as

(i) zeroth order, $O(\epsilon^0)$:

$$\rho_0 c_2'^{12} = P^{(0)} p_3^2 + Q^{(0)} p_3^2 + R^{(0)} (2 p_1 p_3),$$

(ii) first order, $O(\epsilon^1)$:

$$\rho_0 c_2'^{12} = P^{(0)} p_3^2 + \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

(iii) second order, $O(\epsilon^2)$:

$$\rho_0 c_2'^{12} = P^{(0)} p_3^2 + \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

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$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

$$+ \epsilon s p_3 \left\{ \frac{\omega}{\omega - \epsilon s p_3 (x \sin \theta 1 - z \cos \theta 1)} \right\}$$

and so on. We will use expression (15) of $c_2'$ for the numerical computation.

### 4. Wave Propagation

Suppose a plane SH-wave with amplitude, $A_0$, making an angle $\theta_0$ with the normal be incident at the plane interface ($z = 0$) between homogeneous and inhomogeneous fibre-reinforced elastic half-spaces. Such an incident SH-wave, due to the interface, give rises a reflected SH-wave in the half-space, $H$, and a refracted SH-wave in the half-space, $H'$. The geometry of the problem is given in Figure I.

The total displacement due to incident and reflected SH-waves in the half-space, $H$, is given by

$$u_2 = A_0 \exp \left[ i \left( \omega t - k_0 (x \sin \theta_0 + z \cos \theta_0) \right) \right]$$

$$+ A \exp \left[ i \left( \omega t - k_2 (x \sin \theta_1 - z \cos \theta_1) \right) \right],$$

where $k_0$ and $k_2$ are the wave numbers for incident and reflected waves, respectively.

$$k_0 = \sqrt{\mu_0 + \epsilon s p_3 (x \sin \theta_1 - z \cos \theta_1)},$$

$$k_2 = \sqrt{\mu_2 + \epsilon s p_3 (x \sin \theta_1 - z \cos \theta_1)}.$$
where $A$ is the amplitude constant of the reflected $SH$-wave at an angle $\theta_1$, $k_i = \omega/c_i$ $(i = 0, 2)$ is the wavenumber, $\omega$ is angular frequency, and $c_i$ is the phase speed.

The displacement due to refracted $SH$-wave in the half-space, $H^f$, is given by

$$u_i^f = B \exp \left\{ i \{ \omega t - k_i^f (x \sin \theta_2 + z \cos \theta_2) \} \right\}, \quad (19)$$

where $k_i^f = \omega/c_i^f$ is the wavenumber with $c_i^f$ as the phase speed and $B$ is the amplitude constant of the refracted $SH$-wave at an angle $\theta_2$. Snell’s law of this problem is given by

$$k_0 \sin \theta_0 = k_2 \sin \theta_1 = k_2^f \sin \theta_2. \quad (20)$$

We assume that the angle of incidence is equal to the angle of reflection.

5. Boundary Conditions

The boundary conditions are the continuities of the displacements and stress tensors at the interface. Mathematically, these conditions at $z = 0$ are

$$u_2 = u_2^r, \quad (21)$$

$$\tau_{23} = \tau_{23}^r. \quad (22)$$

Equation (22) may be written in terms of displacement as

$$\mu_r \frac{\partial u_2}{\partial z} + (\mu_L - \mu_r) a_3 \left( a_1 \frac{\partial u_2}{\partial x} + a_3 \frac{\partial u_2}{\partial z} \right) = \mu_r \frac{\partial u_2^r}{\partial z} + (\mu_L^r - \mu_r) a_3 \left( a_1 \frac{\partial u_2^r}{\partial x} + a_3 \frac{\partial u_2^r}{\partial z} \right). \quad (23)$$

Using (18)–(20) into boundary conditions (21) and (23), we have

$$\frac{A}{A_0} - \frac{B}{A_0} = -1, \quad (24)$$

$$r_1 \frac{A}{A_0} + r_2 \frac{B}{A_0} = r_0.$$  

where

$$r_0 = \mu_T k_0 \cos \theta_0 + (\mu_L - \mu_T) a_3 (a_3 k_0 \cos \theta_0 + a_1 \sin \theta_0),$$

$$r_1 = \mu_T k_2 \cos \theta_1 + (\mu_L - \mu_T) a_3 (a_3 k_2 \cos \theta_1 - a_1 k_0 \sin \theta_0),$$

$$r_2 = \left\{ \mu_T^r k_2^r \cos \theta_2 + (\mu_L^r - \mu_T^r) a_3 \left( a_3 k_2^r \cos \theta_2 + a_1 k_0 \sin \theta_0 \right) \right\}.$$  

These equations will give the amplitude ratios corresponding to the reflected and refracted $SH$-waves.

6. Amplitude and Energy Ratio

The amplitude ratios of the reflected and refracted $SH$-waves are defined as the ratio of the amplitudes corresponding to the reflected and refracted waves to that of the incident wave. Solving (24), we have

$$\frac{A}{A_0} = \frac{r_0 - r_2}{r_1 + r_2}, \quad (26)$$

$$\frac{B}{A_0} = \frac{r_0 + r_1}{r_1 + r_2},$$

where $A/A_0$ is the amplitude ratios corresponding to the reflected $SH$-wave and $B/A_0$ is that of the refracted $SH$-wave. We have observed that these ratios depend on the elastic constants, fibre orientation, inhomogeneity parameter, and the angle of incidence.

Let us consider the energy partition of the reflected and refracted $SH$-waves due to the plane interface, $z = 0$. The energy transmission per unit area may be given as (see [30])

$$\psi^* = \tau_{23} \cdot u_2 + \tau_{23}^r \cdot u_2^r. \quad (27)$$

The expression of the energy due to incident wave is given by

$$E_{inc} = J_0 \omega A_0^2 \exp \left\{ 2i \{ \omega t - k_0 (x \sin \theta_0 + z \cos \theta_0) \} \right\}, \quad (28)$$

where $J_0 = k_0 \{ \mu_L \cos \theta_0 + (\mu_L - \mu_T) a_3 (a_1 \cos \theta_0 + a_1 \sin \theta_0) \}$.

The energy ratio of the reflected and refracted $SH$-waves may be defined as the ratios of the energy corresponding to the reflected and refracted $SH$-waves to that of the incident wave. The modulus of energy ratios of the reflected and refracted $SH$-waves is given as

\begin{align*}
E_1 &= \left| \frac{f_1}{f_0} \right| \left| \frac{A}{A_0} \right|^2, \\
E_2 &= \left| \frac{f_2}{f_0} \right| \left| \frac{B}{A_0} \right|^2.
\end{align*}
where $E_1$ is the energy ratio of the reflected $SH$-wave and $E_2$ is that of refracted $SH$-wave and the expressions of $J_1$ and $J_2$ are given by

$$J_1 = -k_2 \left[ \mu_T \cos \theta_1 
+ (\mu_L - \mu_T) a_3 (a_3 \cos \theta_1 - a_1 \sin \theta_1) \right],$$

$$J_2 = k_2' \left[ \mu_T^{(0)} \cos \theta_2 
+ (\mu_L^{(0)} - \mu_T^{(0)}) a_3 (a_3 \cos \theta_2 + a_1 \sin \theta_2) \right] (1 - \varepsilon).$$

We come to know that the energy ratios are functions of the amplitude ratios, elastic constants, fibre orientation, inhomogeneity parameter, and the angle of incidence of the incident $SH$-wave.

### 7. Particular Cases

#### 7.1. Case I.
When the inhomogeneous fibre-reinforced elastic half-space, $H'$, reduces to homogeneous fibre-reinforced elastic medium, the problem reduces to the reflection and refraction of $SH$-waves at the plane interface between the two dissimilar homogeneous fibre-reinforced elastic half-spaces. Under this condition, $\varepsilon = 0$ and (14) reduces to

$$r_0^{(1)} r_2^{(1)} = P(0) P_1^2 + Q(0) P_1^2 + 2R(0) P_1 P_3.$$ (31)

The amplitude and energy ratios corresponding to the reflected and refracted $SH$-waves are given by (26) and (29) with the following modified values:

$$r_0 = \mu_T k_0 \cos \theta_0 + (\mu_L - \mu_T) a_3 \left( a_3 k_0 \cos \theta_0 
+ a_1 k_0 \sin \theta_0 \right),$$

$$r_1 = \mu_T k_2 \cos \theta_1 + (\mu_L - \mu_T) a_3 \left( a_3 k_2 \cos \theta_1 
- a_1 k_2 \sin \theta_1 \right),$$

$$r_2 = \mu_T^{(0)} k_2' \cos \theta_2 + (\mu_L^{(0)} - \mu_T^{(0)}) a_3 \left( a_3 k_2' \cos \theta_2 
+ a_1 k_2' \sin \theta_2 \right),$$

$$J_1 = -k_2 \left[ \mu_T \cos \theta_1 
+ (\mu_L - \mu_T) a_3 (a_3 \cos \theta_1 - a_1 \sin \theta_1) \right],$$

$$J_2 = k_2' \left[ \mu_T^{(0)} \cos \theta_2 
+ (\mu_L^{(0)} - \mu_T^{(0)}) a_3 (a_3 \cos \theta_2 + a_1 \sin \theta_2) \right].$$ (32)

These results are similar to those of Singh and Tomar [12].

#### 7.2. Case II.
When the two half-spaces $H$ and $H'$ reduce to homogeneous isotropic elastic media, the problem reduces to the reflection and refraction of $SH$-waves between two dissimilar isotropic elastic half-spaces. Under this condition,

$$P^{(0)} = Q^{(0)} = \mu_T^{(0)} = \mu_L^{(0)} = \mu_0,$$

$$P = Q = \mu_T = \mu_L = \mu,$$

$$R^{(0)} = R = 0,$$ (33)

$$c_2^2 = \frac{\mu}{\rho},$$

$$c_2^2 = \frac{\mu_0}{\rho}.$$ (34)

The amplitude and energy ratios corresponding to the reflected and refracted $SH$-waves are given by (26) and (29) with the following modified values:

$$r_0 = \mu k_0 \cos \theta_0,$$

$$r_1 = \mu k_2 \cos \theta_1,$$

$$r_2 = \mu_0 k_2' \cos \theta_2,$$ (35)

$$J_1 = -k_2 \left[ \mu \cos \theta_1 
+ (\mu_L - \mu_T) a_3 (a_3 \cos \theta_1 - a_1 \sin \theta_1) \right],$$

$$J_2 = k_2' \left[ \mu_T^{(0)} \cos \theta_2 
+ (\mu_L^{(0)} - \mu_T^{(0)}) a_3 (a_3 \cos \theta_2 + a_1 \sin \theta_2) \right].$$ (36)

These results exactly match with that of Achenbach [30].

### 8. Numerical Results and Discussion

For the numerical computations, we take the following relevant parameters for fibre-reinforced elastic half-space, $H$, and the nonhomogeneous fibre-reinforced elastic half-space, $H'$ [13].

For the homogeneous half-space $H$,

$$\mu_L = 7.07 \times 10^9 \text{ N/m}^2,$$

$$\mu_T = 3.5 \times 10^9 \text{ N/m}^2,$$ (37)

$$\rho = 1600 \text{ Kg/m}^3.$$ (38)

For the nonhomogeneous half-space $H'$,

$$\mu_L^{(0)} = 5.66 \times 10^9 \text{ N/m}^2,$$

$$\mu_T^{(0)} = 2.46 \times 10^9 \text{ N/m}^2,$$ (39)

$$\rho_0 = 7800 \text{ Kg/m}^3,$$

$$a_1 = 0.00316227,$$

$$\varepsilon = 0.0, 0.1, 0.2.$$ (40)

The variation of the modulus of amplitude ratios corresponding to the reflected and refracted $SH$-waves with the angle of incidence, $\theta_0$, is depicted in Figures 2 and 3 with and without fibre-reinforcement at different values of inhomogeneity parameter, $\varepsilon$. In Figures 2(a) and 2(b), the amplitude
ratio, $A/A_0$, corresponding to the reflected SH-wave starts from certain values and increases with the increase of $\theta_0$ which attains the maximum value at the grazing angle of incidence. The values of $A/A_0$ decrease with the increase of inhomogeneity parameter.

All Curves I, II, and III in Figures 3(a) and 3(b) show that $B/A_0$ corresponding to the refracted SH-wave decreases with the increase of $\theta_0$ and attains the minimum value at the grazing angle of incidence. The values of $B/A_0$ increase with the increase of inhomogeneity parameter. Figures 4 and 5 show the variation of the modulus of energy ratios corresponding to the reflected and refracted SH-waves with the angle of incidence with and without fibre-reinforcement at different values of inhomogeneity parameter, $\epsilon$. In Figures 4(a) and 4(b), $E_1$ corresponding to the reflected SH-wave starts from certain values and increases up to $\theta_0 = 88^\circ$ and decreases to $\theta_0 = 89^\circ$ which increases thereafter with the increase of $\theta_0$.

The modulus of energy ratio, $E_2$, corresponding to the refracted SH-wave in Figures 5(a) and 5(b) starts from certain
values and decreases with the increase of $\theta_0$ and attains the minimum value at the grazing angle of incidence. We have observed that the sum of energy ratios is close to unity.

9. Conclusion

The problem of reflection and refraction of $SH$-waves at the plane interface between the homogeneous fibre-reinforced half-space and the nonhomogeneous fibre-reinforced half-space has been investigated. The amplitude and energy ratios corresponding to the reflected and refracted $SH$-waves have been obtained and computed numerically. We may conclude the following points:

(i) The amplitude and energy ratios corresponding to the reflected and refracted $SH$-waves are functions of elastic constants, fibre orientation, inhomogeneity parameter, and angle of incidence.

(ii) The amplitude ratio, $A/A_0$, and energy ratio, $E_1$, attain the maximum value at grazing angle of incidence.

(iii) The values of $A/A_0$ and $E_1$ decrease with the increase of $\varepsilon$. 

Figure 4: Variation of the modulus of energy ratio, $E_1$ with $\theta_0$.

Figure 5: Variation of the modulus of energy ratio, $E_2$ with $\theta_0$. 

(iv) The values of $A/A_0$ and $E_1$ with fibre-reinforcement are greater than those values without fibre-reinforcement.

(v) The values of $B/A_0$ and $E_2$ increase with the increase of $\varepsilon$.

(vi) The sum of energy ratios is close to unity.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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