Research Article

Involutions in the Automorphism Groups of Small Sporadic Simple Groups

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For each of fifteen of the sporadic finite simple groups we determine the suborbits of its automorphism group in its conjugation action upon its involutions. Representatives are obtained as words in standard generators.

1. Introduction

Groups permeate many areas of mathematics. Sometimes they have cameo roles; other times they are centre stage. Frequently it is involutions, elements of order two, that are in the spotlight. For instance, in the topological arena we have involutory maps on the 3-sphere in connection with the Smith Conjecture (see, e.g., [1–3]), while in Banach spaces we encounter such things as involutive gradings and fixed points of involutions (see [4, 5]). In areas of algebra, such as associative rings and algebraic groups, there are many sightings of involutions (see [6–8]). Involutions can often have a considerable influence on the structure of the group to which they belong. Even their absence can be telling—witness the Feit Thompson theorem [9]. For a finite group of even order, Brauer and Fowler [10] establish many results concerning involutions and other properties of the group. For example, they bound the index of a proper normal subgroup in terms of the number of involutions the group possesses. In a similar vein, for a finite group with at least two conjugacy classes of involutions the Thompson order formula ([11], Theorem 35.1) gives its order using data closely associated with the involutions. In the case when we have a finite nonabelian simple group, more often than not, its involutions play a dominant role (see, e.g., [12]).

This paper studies the involutions in $\text{Aut}(K)$ where K is a small sporadic finite simple group. By small we mean that K is isomorphic to one of the following groups:

$M_{11}, M_{12}, J_1, M_{22}, J_2, M_{23}, HS, J_3, M_{24}, McL, He, Ru, Suz, Co_3, Co_2.$

The diminutive appellation aligns, more or less, with said group having a nontrivial permutation representation of degree at most 6156. Several of the larger sporadic groups have been studied individually in [13–16]. So for the remainder of this paper K is assumed to be a small sporadic simple group and G is a subgroup of $\text{Aut}(K)$ containing K . Also t will denote an involution of G . Put $X = t^G$, the G -conjugacy class of t . Our aim is to study the suborbits of G in its conjugation action on X , or, in other words, to determine the action of $C_G(t)$ on $X \setminus \{t\}$. This we do employing the services of the computational algebra packages Gap [17] and Magma [18] partnered by the electronic Atlas [19]. It goes without saying therefore that we use the Atlas notation and conventions as given in [20].

2. Calculating Orbit Representatives

As our starting point we take the smallest nontrivial permutation representation of G as described in [19] with K being generated by standard generators denoted here, as in [19], by a and b . In the case when $G \neq K$, the standard generators for G are, again as in [19], denoted by c and d . Having chosen a suitable element in X to play the role of t , we then forage for elements $g_1, g_2, \dots, g_k \in G$ so that $\{t, t^{g_1}, \dots, t^{g_k}\}$ is a complete set of representatives for the $C_G(t)$ -orbits of X . In doing this we make frequent use of the standard command `IsConjugate(H, x, y)`—this works effectively here since the degree of the permutation representation of G is no more than 6156. Also observe that

```

F<a,b>:=FreeGroup(2);
phi:=hom< F->G | <a,G.1>,<b,G.2> >;
r,h:=IsConjugate(G,t,x);
sh:=h@phi;
lengthmin:=#sh;
hmin:=h;
Ct:=Centralizer(G,t);
for c in Ct do
  sch:=(c*h)@phi;
  if #sch lt lengthmin then
    lengthmin:=#(sch);
    hmin:=c*h; shmin:=sch;
  end if;
end for;

```

ALGORITHM 1

we may write our conjugating element g_i as a word in a and b . Thus, our aim is to find g_i which have relatively “small” length relative to the generating set $\{a, b\}$ for K . However we may not always achieve the minimum possible length. In more detail we proceed as follows: define $D_0(a, b) = \{1\}$ and for $j \in \mathbb{N}$

$$D_j(a, b) = \{xa, xb \mid x \in D_{j-1}(a, b)\}. \quad (1)$$

For $n \in \mathbb{N}$ set

$$D_{[n]}(a, b) = \bigcup_{j \leq n} D_j(a, b). \quad (2)$$

The main purpose of $D_{[n]}(a, b)$ is to produce a colony of short words in a and b , a number of which may well yield the same element of G . To speed matters up we prune out these duplicates. We hunt through $D_{[n]}(a, b)$ for n typically at most 20, so as to ensnare suitable conjugating elements g_i . If this does not yield enough representatives for the $C_G(t)$ -orbits, then we recalculate $D_{[n]}(a, b)$ with a and b replaced by short words in a and b which are also generators for G . For example, we might try replacing a by ab and keeping b the same, or try a more complicated substitution such as replacing b with $(ab)^5a$. Such a substitution was used to produce a g_i in the case when $G \cong M_{24}$ and tx in the class 3A, starting from the word bab^2abab^2ab . Some of the words for the g_i may be further simplified and this was done by hand.

On a number of occasions this approach fails to deliver conjugating elements for some (often of small size) $C_G(t)$ -orbits. To deal with such elusive $C_G(t)$ -orbits, say $\mathcal{O}$, we begin by finding an x in $\mathcal{O}$, usually by a random search. Then we obtain an element h in G such that $t^h = x$. Now we run through some or all of the elements c in $C_G(t)$, seeking a c for which ch has small “length” (i.e., the total number of symbols a, a^{-1}, b, b^{-1} in the given expression for ch). Sometimes we may also vary h . In somewhat more detail the Magma code employed is as shown in Algorithm 1.

We remark that this procedure is not usually as efficient as using the set $D_{[n]}(a, b)$ and moreover usually yields more complicated expressions for the conjugating elements.

We say a few words about how we guarantee that we have a representative of every $C_G(t)$ orbit. By using

the `IsConjugate(Ct,x,y)` command, where t is a fixed involution of the G -conjugacy class and Ct denotes $C_G(t)$, we ensure that no two representatives are in the same $C_G(t)$ -orbit. We terminate our calculations for the G -conjugacy class of t when the sum of the $C_G(t)$ -orbit sizes equals the size of the conjugacy class.

One may envision the information presented here to be useful in the following circumstances. Suppose H is a finite group with $x, y \in H$. Further suppose that we have identified the subgroup $L = \langle x, y \rangle$ as being isomorphic to G via recognizing x and y as standard generators. (See [21] for a discussion of standard generators.) Then we may translate our information from G to L so that we see how L acts upon its involution conjugacy classes within the group H . Moreover, if H is large from a computational standpoint (e.g., $|H|$ is large or H is a matrix group of large degree), then having the g_i as short words may be beneficial. To facilitate applications such as these, computer files containing the g_i as words in a and b are available on request from the second author.

The following section gives the permutation ranks of G on X and the elements g_i together with some additional information.

3. Orbit Representatives

So t is a suitably chosen (and then fixed) involution of the conjugacy class X of G . For C a G -conjugacy class we define

$$X_C = \{x \in X \mid tx \in C\}. \quad (3)$$

Since X_C is $C_G(t)$ -invariant, X_C will be a union of $C_G(t)$ -orbits for each G -conjugacy class C for which $X_C \neq \emptyset$. As is well known, $|X_C|$ may be calculated for any G -conjugacy class using the complex character table (see, e.g., [22]), and this is easily carried out in Gap. As we proceed by breaking each X_C (for $|X_C| \neq 0$) into $C_G(t)$ -orbits this is useful information.

Our first table gives an overview of the permutation ranks of G in its action on X . The succeeding tables consider in turn the possibilities for K , with the first column identifying X_C and the second the size of the $C_G(t)$ -orbit $\mathcal{O}$ (contained in X_C) and the third supplies a group element g for which $t^g \in \mathcal{O}$.

We emphasize that the following tables give the $C_K(t)$ -orbits of X —in the case when $K < G = \text{Aut}(K)$ the tables are annotated so as to also yield the $C_G(t)$ -orbits of X . Before explaining how this is done, we remark that in all instances here when $K < G = \text{Aut}(K)$ we have $C_K(t) < C_G(t)$. Thus X is a G -conjugacy class and $G = KC_G(t)$. Now suppose that C_1 and C_2 are two K -conjugacy classes which fuse in G (so $C_1 \cup C_2$ is a G -conjugacy class). Then $C_1^g = C_2$ for some $g \in C_G(t)$ and consequently $X_{C_1}^g = X_{C_2}$. Hence, if $\mathcal{O}$ is a $C_K(t)$ -orbit contained in X_{C_1} , $\mathcal{O} \cup \mathcal{O}^g$ will be a $C_G(t)$ -orbit of X . In this circumstance a broken horizontal line indicates that C_1 and C_2 are fused in $G = \text{Aut}(K)$ and if $\mathcal{O}$ is obtained by using the i th listed conjugating element in X_{C_1} then $\mathcal{O}^g$ is obtained using the i th listed conjugating element in X_{C_2} . When a K -conjugacy class C is also a G -conjugacy class, it may be the case that a $C_G(t)$ -orbit contained in X_C is the union of two $C_K(t)$ -orbits in X_C . A vertical line connecting

TABLE 1

G	$2A$	$2B$	$2C$	$2D$
M_{11}	8			
M_{12}	10	11	13	
J_1	22			
M_{22}	13	5	10	
J_2	6	31	14	
M_{23}	9			
HS	9	27	5	32
J_3	27	17		
M_{24}	14	26		
McL	6	28		
He	12	34	41	
Ru	22	42		
Suz	9	66	15	41
Co_3	7	61		
Co_2	5	13	178	

TABLE 2

X_C	$ \mathcal{O} $	g
$2A$	12	bab^3ab
$3A$	24	b^2
	8	b^3ab
$4A$	24	bab
$5A$	48	$babab$
$6A$	24	b
	24	b^3

TABLE 3

X_C	$ \mathcal{O} $	g
$2A$	10	b^2abab^2abab
	10	$b^2ab^2abab^2ab$
$2B$	15	a
$3B$	60	ab
$4A$	30	ab^2aba
$4B$	30	$bababab^2a$
$5A$	120	ba
$6A$	60	b
	60	b^2

two $C_K(t)$ -orbits in X_C signifies that their union is a $C_G(t)$ -orbit.

3.1. *Permutation Ranks.* See Table 1.

3.2. $K \cong M_{11}$

3.2.1. $X = 2A, t = a$. See Table 2.

3.3. $K \cong M_{12}$

3.3.1. $X = 2A, t = ((ab)^2ab^2)^3$. See Table 3.

TABLE 4

X_C	$ \mathcal{O} $	g
$2B$	24	bab^2ab
	6	$b^2ababab^2ababab^2abab$
$3A$	32	b^2ab
	32	bab^2
$3B$	16	bab^2ab^2abab
$4A$	48	$babab^2$
$4B$	48	b^2abab
$5A$	96	bab
$6B$	96	b
	96	b^2

TABLE 5

X_C	$ \mathcal{O} $	g
$2A$	15	$bab^2ababab$
	1	$bababab^2ab^2abab^2ab$
$2B$	15	$babab^2$
$3A$	20	$b^2ababab$
	20	$abab^2aba$
$3B$	60	bab^2
$5A$	60	$abab^2a$
	60	ab
$6A$	60	$baba$
$6B$	60	b^2ab
	60	a
$10A$	60	bab^2aba
	60	$babab$
$11A$	120	b
$11B$	120	ba

3.3.2. $X = 2B, t = a$. See Table 4.

3.3.3. $X = 2C, t = c$. See Table 5.

3.4. $K \cong J_1$

3.4.1. $X = 2A, t = a$. See Table 6.

3.5. $K \cong M_{22}$

3.5.1. $X = 2A, t = a$. See Table 7.

3.5.2. $X = 2B, t = c$. See Table 8.

3.5.3. $X = 2C, t = (cdcd^2)^5$. See Table 9.

3.6. $K \cong J_2$

3.6.1. $X = 2A, t = (bab^2(ab)^2b(ab)^3)^5$. See Table 10.

TABLE 6

X_C	$ \mathcal{O} $	g
2A	15	$bab^2abab^2abab^2abab$
	15	$bab^2ab^2abab^2ababab^2abab$
3A	60	$bab^2ababab^2ab$
	12	$b^2ab^2abab^2abab^2abab$
5A	60	$babab^2ab^2abab$
	20	$b^2ababab^2ab^2ab$
5B	60	$babab^2abab^2abab$
	20	b^2abab^2abab
6A	60	bab^2abab
	60	$babab^2ab$
7A	120	bab^2ab^2abab
10A	60	$b^2ababab$
	60	$bababab^2$
10B	60	b^2abab
	60	$babab^2$
11A	120	$babab^2abab$
15A	120	b^2ab^2abab
15B	120	bab
19A	120	b
19B	120	b^2ab
19C	120	b^2abab^2ab

TABLE 7

X_C	$ \mathcal{O} $	g
2A	24	bab^3ab
	12	$(ba)^2b^2abab$
	8	$b^2abab^2(ab)^3$
	6	$b^3(ab)^2(ab^3)^2ab$
3A	192	bab
4A	48	bab^2ab
	48	b^3abab
	48	$babab^3$
4B	96	b^2abab
	96	$babab^2$
5A	384	b^2
6A	192	b

TABLE 8

X_C	$ \mathcal{O} $	g
2A	42	b
	7	$(b^2a)^2ba$
3A	112	a
4A	168	$ababa$

TABLE 9

X_C	$ \mathcal{O} $	g
2A	20	b^2
	5	bab^2ab^3
3A	80	b
4A	80	$bababa$
4B	40	bab^2a
	40	bab^2ab
5A	320	ab
6A	80	a
	80	ab^2a
11A	320	b^2a
11B	320	ba

TABLE 10

X_C	$ \mathcal{O} $	g
2A	10	ba
3B	160	b
4A	80	a
5A	32	ab^2aba
5B	32	bab^2

3.6.2. $X = 2B, t = a$. See Table 11.

3.6.3. $X = 2C, t = c$. See Table 12.

3.7. $K \cong M_{23}$

3.7.1. $X = 2A, t = a$. See Table 13.

3.8. $K \cong HS$

3.8.1. $X = 2A, t = a$. See Table 14.

3.8.2. $X = 2B, t = ((ab)^2a)^5$. See Tables 15 and 16.

3.8.3. $X = 2C, t = c$. See Table 17.

3.8.4. $X = 2D, t = ((dc)^2d^4cd)^5$. See Tables 18 and 19.

3.9. $K \cong J_3$

3.9.1. $X = 2A, t = a$. See Table 20.

3.9.2. $X = 2B, t = c$. See Table 21.

3.10. $K \cong M_{24}$

3.10.1. $X = 2A, t = ((ab)^3ab^2)^4$. See Tables 22 and 23.

TABLE 11

X_C	$ \mathcal{O} $	g
2A	15	$b(ba)^2b(ba)^2b(ba)^2$
2B	15	$((ba)^2b)^3(ba)^3$
	15	$b(ba)^2(b(ba)^3b^2a)^2$
	1	$(ba)^2b^2a(b(ba)^2)^2b^2ab(ba)^3$
	1	$b^2ab(ba)^3b(ba)^2b(ba)^3(b^2a)^2$
3A	24	$b(ba)^2b(ba)^2$
3B	60	$b(ba)^3b(ba)^2b(ba)^3b(ba)$
4A	60	$(ba)b(ba)^2$
5A	60	$(ba)^2b(ba)^2b(ba)^3b(ba)b(ba)^2$
5B	60	$(ba)b(ba)b(ba)^3b(ba)^2b(ba)^3$
5C	120	$b(ba)^3b(ba)^3$
	24	$b(ba)b(ba)^2b(ba)^3$
5D	120	$(ba)b(ba)^2b(ba)^3$
	24	$(ba)^2b(ba)^2b(ba)b(ba)$
6A	120	$(ba)^3$
6B	60	$(ba)^2b(ba)^3$
	60	$(ba)^3b(ba)^4$
7A	240	$b(ba)^4$
10A	60	$b(ba)b(ba)^2b(ba)^2$
	60	$b(ba)^2b(ba)^3$
10B	60	$(ba)^2b(ba)^2b(ba)$
	60	$(ba)b(ba)^2b(ba)b(ba)$
10C	120	$(ba)b(ba)b(ba)$
	120	$(ba)^2b(ba)$
10D	120	$b(ba)^3$
	120	$b(ba)b(ba)^2$
12A	120	$b(ba)$
	120	(ba)
15A	240	$(ba)b(ba)b(ba)^2b(ba)^2$
15B	240	$(ba)b(ba)^2b(ba)b(ba)^2$

TABLE 12

X_C	$ \mathcal{O} $	g
2A	21	$(bab^2a)^2(ba)^3$
2B	28	bab^2
3A	14	$(ab)^3(ab^2)^2ab$
	14	$(b^2aba)^2bab$
3B	168	$baba$
4A	84	$abab^2ab$
	42	b^2abab
6A	84	b^2
	84	b
	42	$abababa$
	42	$abab^2abab$
6B	168	ab
7A	336	bab
8A	168	ab^2a
	168	aba
12A	168	ba
	168	a

TABLE 13

X_C	$ \mathcal{O} $	g
2A	84	bab^3ab
	14	b^3ab^2ab
3A	448	b^2
4A	336	$babab$
	336	$bababab$
	336	$bababab^2$
5A	896	bab
6A	1344	b

TABLE 14

X_C	$ \mathcal{O} $	g
2A	80	$b^2ab^3ab^2$
	30	$b^2abab^2abab^2$
3A	640	bab
4B	480	bab^3ab
4C	960	b^2ab
5B	128	bab^2ab
5C	1536	b
6B	1920	b^2

TABLE 15

X_C	$ \mathcal{O} $	g
2A	45	bab^3ab
	30	b^4ab^4abab
2B	36	$b^2ab^3ab^2$
	36	$bababab^3a$
3A	360	b^3ab^2abab
	24	$ab^2ab^2abab^2ab^2a$
4B	180	a
4C	360	$babab^4ab^2a$
	180	$bababab^2$
	180	$b^2ababab$
5A	144	bab^4ab^3ab
	144	bab^3ab^4ab
5B	720	$babab^2a$

TABLE 16

X_C	$ \mathcal{O} $	g
	720	bab
6B	360	bab^2a
	360	ab^2ab
7A	2880	b
8B	720	b^4ab^2a
	720	ab^3ab
8C	720	bab^3a
	720	ab^2ab^4
10A	720	b^2
	720	b^3
10B	720	aba
	720	ba
15A	2880	b^2a

TABLE 17

X_C	$ \mathcal{O} $	g
2A	105	a
2B	28	ba
3A	336	b^2
4B	630	b

TABLE 18

X_C	$ \mathcal{O} $	g
2A	60	ab^3abab^2a
	5	ab^3ab^2a
2B	60	b^2a
3A	240	a
	160	ab^3abab^2aba
4A	120	bab^3aba
4B	240	ab
	120	bab^2abab
	30	ab^2a
4C	120	ab^2ab^2a
	120	ab^3aba
5A	192	$abab^3$
	192	bab^2ab^3
5B	320	b^4ab
5C	1920	b^2ab^2
6A	480	ab^2ab
	480	bab^2a

TABLE 19

X_C	$ \mathcal{O} $	g
6B	480	b^4abab
	480	b^2ab^2a
7A	1920	b
8A	480	$babab^2$
	480	bab^2ab^2ab
10A	960	$b^2ab^2ab^2a$
	960	bab
10B	960	ab^2
11A	1920	b^2aba
11B	1920	$babab$
12A	960	ba
	960	b^3
15A	1920	bab^3
20A	960	$baba$
	960	b^2ab
20B	960	b^2ab^3a
	960	b^4aba

TABLE 20

X_C	$ \mathcal{O} $	g
2A	120	$b(ba)^3b(ba)b(ba)^5$
	10	$(ba)b(ba)^4b(ba)^2$
3A	192	$b(ba)^6$
3B	640	$(ba)b(ba)^2$
4A	480	$b(ba)^5b(ba)$
	80	$(ba)^3b(ba)^3b(ba)^4$
5A	960	$(ba)^3b(ba)^3b(ba)^2$
	320	$(ba)^2b(ba)^2$
5B	960	$(ba)b(ba)^3b(ba)^4$
	320	$(ba)b(ba)^3$
6A	960	$(ba)b(ba)^6$
8A	960	$(ba)^4b(ba)^2$
	960	$(ba)b(ba)^5$
9A	1920	$b(ba)^2b(ba)^2$
9B	1920	(ba)
9C	1920	$b(ba)^2$
10A	960	$(ba)^4b(ba)$
	960	$b(ba)^4b(ba)b(ba)$
10B	960	$b(ba)b(ba)b(ba)b(ba)^2$
	960	$b(ba)^5$
12A	960	$(ba)^2b(ba)^4$
	960	$(ba)^6$
15A	1920	$(ba)^2b(ba)$
15B	1920	$b(ba)^3$
17A	1920	$(ba)^2$
17B	1920	$(ba)^2b(ba)^3$

TABLE 21

X_C	$ \mathcal{O} $	g
2A	153	$ab^2ababab$
3A	102	b^2
	102	b
3B	272	$abab^2a$
4A	612	$bababab^2a$
	306	bab^2ababa
6A	612	bab^2aba
	612	$babab^2a$
	306	ab^2ab^2ababa
	306	$baba$
8A	1224	$ababab$
	1224	b^2ababa
9A	816	b^2abab^2ab
9B	816	$babab^2ab$
9C	816	bab^2
12A	1224	ab
	1224	ab^2aba
17A	2448	ba
17B	2448	a
19A	2448	ab^2ab
19B	2448	b^2aba

TABLE 22

X_C	$ \mathcal{O} $	g
	168	$b^2 ababab^2 ab$
2A	14	aba
	14	$abab^2 ab$
2B	84	$b^2 ab$
3A	896	b
3B	128	$b^2 ab^2 abab^2 abab$

TABLE 23

X_C	$ \mathcal{O} $	g
4A	112	$bababab$
	112	$b^2 ab^2 ab^2 ab^2$
4B	1344	$ab^2 aba$
	672	$ab^2 abab^2 ab$
	672	$b^2 abab^2 aba$
5A	1792	$baba$
6A	5376	a

TABLE 24

X_C	$ \mathcal{O} $	g
2A	60	$b^2 abab^2 ab^2 abab^2 ababab^2 ab$
	15	$bab^2 ab^2 ababab^2$
2B	120	$bab^2 abab^2 abab^2$
	80	$b^2 abab^2 abab^2 ab$
	2	$(ba)^8 (b^2 a)^2 bab^2 (ab)^2 (ab^2)^2 (ab)^3 (ab^2 ab)^2 b$
3A	320	$b^2 ab^2 abab^2 ababab$
3B	960	$b^2 abab^2 ab$
4A	120	$bab^2 ab^2$
	120	$babab^2$
4B	480	$b^2 ababab^2 ab$
	240	$bab^2 ababab^2 abab^2 ab$
	240	$b^2 abababababab$
4C	960	$bab^2 ab$
	960	$b^2 abab^2$
	160	$bababab^2 abababab^2 ab^2$
	160	$b^2 abababab^2 ab^2 ababab$
5A	1920	bab
6A	1920	$bababab^2$
6B	1920	bab^2
	1920	$b^2 ab$
10A	1920	$b^2 abab^2 abab$
	1920	$b^2 abababab^2$
11A	7680	$babab$
12B	3840	b
	3840	b^2

3.10.2. $X = 2B$, $t = a$. See Table 24.

3.11. $K \cong McL$

3.11.1. $X = 2A$, $t = a$. See Table 25.

3.11.2. $X = 2B$, $t = c$. See Table 26.

TABLE 25

X_C	$ \mathcal{O} $	g
2A	210	$b^2 ab^3 ab^2$
3B	2240	$babab^2$
4A	5040	$b^2 ab$
5B	8064	b
6B	6720	b^2

TABLE 26

X_C	$ \mathcal{O} $	g
2A	165	$ab^2 abab^2 ab$
3A	220	$b^2 ab^3 ab^2 abab^2 a$
3B	660	$ab^2 ab^2$
	220	$b^2 ababab^3$
4A	1980	$b^2 ab^3 ababa$
	990	a
5A	792	$b^2 ab^2 ab^3$
	792	$b^2 ababab^2$
5B	7920	b^2
6A	1980	ab
6B	1980	$bab^2 abab$
	1980	$ab^2 ab$
	660	$abab^3$
	660	$bab^3 ab^4 ab$
7A	3960	$babab^2 a$
	3960	bab^2
7B	3960	$bab^3 a$
	3960	$b^2 a$
8A	3960	$b^2 ab^2 a$
	3960	$ab^3 ab^3$
9A	2640	$b^2 aba$
9B	2640	$abab^2 ab$
10A	3960	$babab$
	3960	$ababa$
12A	3960	$babab^3$
	3960	bab^3
14A	3960	$ab^2 ab^3$
	3960	$bab^2 ab$
14B	3960	$bababa$
	3960	$b^2 ababa$
15A	3960	bab
	3960	b^4
15B	3960	$abab$
	3960	b
30A	3960	ba
	3960	$bab^2 a$
30B	3960	$b^4 ab^2 a$
	3960	ab^2

TABLE 27

X_C	$ \mathcal{O} $	g
2A	240	$bababab$
	2	$b^5(ab^6)^2(ab)^3b^5a(b^2ab^3)^2$
2B	315	$b^2abab^2ab^2ab$
3A	1344	b^2ab
3B	960	bab^4ab
4A	480	b^3abab^2ab
	480	b^2abab^3ab
4B	2520	b^2abab
	2520	b^4ab
5A	2688	b^3
6A	13440	b

TABLE 28

X_C	$ \mathcal{O} $	g
2A	84	$(ab)^2b^5(ab)^2$
2B	168	$(ab)^2$
	168	$(ab)b^4(ab)b^2(ab)b^2(ab)b^4$
	14	$b^2(ab)^2b^4(ab)b^4(ab)b^2(ab)b^2$
	14	$b^2(ab)b^2(ab)^2b^4(ab)b^2(ab)b^2$
3A	896	$b^4(ab)b^2$
3B	2688	b^2
4B	1344	$b^2(ab)b^2(ab)b^2(ab)b^2$
	1344	$b^2(ab)b^4(ab)^2b^4$
	112	$(ab)b^2(ab)^4b^4(ab)^2$
	112	$(ab)^3b^4(ab)^4b^2$
4C	1344	$(ab)b^4(ab)b^4$
	1344	$b^4(ab)b^2(ab)b^2(ab)^3b^2$
	672	$b^2(ab)^2b^2(ab)^3$
	672	$b^2(ab)^4b^2(ab)^2$
	672	$b^2(ab)b^2(ab)^4b^2(ab)^2$
	672	$(ab)^4b(ab)^2$
5A	5376	$(ab)^3$
6A	5376	$(ab)b^2(ab)b^2(ab)^4$
6B	5376	$b^2(ab)^2b^2(ab)$
	5376	$(ab)b^2(ab)$
7C	3072	$b^2(ab)b^2(ab)b^2(ab)^2$
8A	5376	$(ab)^2b^2(ab)^2b^2$
	5376	$(ab)b^2(ab)^2b^2(ab)$
10A	5376	$b^2(ab)^2$
	5376	$b^2(ab)b^2(ab)b^2$
12B	10752	$b^2(ab)b^2(ab)^2$
	10752	$b^4(ab)^2$
15A	21504	$b^2(ab)^3$
17A	21504	$(ab)b^2(ab)^2$
17B	21504	(ab)
21A	21504	$(ab)b^2(ab)b^2$
21B	21504	$b^2(ab)$

TABLE 29

X_C	$ \mathcal{O} $	g
2A	105	$(ab)b(ab)^3b(ab)^3$
	63	$b^6ab^3ab^3ab^2ab^6ab^2ab^6ab^4a$
2B	315	$(ab^2)^2(ba)^2b^2a$
3A	630	$(ab)b(ab)b(ab)b$
	126	$b(ab)^2b^6$
3B	2520	$(ab)b^6$
4A	1260	$(ab)^2b^5(ab)$
	1260	$(ab)b(ab)b^3$
4B	945	$(ab)^2b^2(ab)b$
	945	$b(ab)b(ab)b^2(ab)b$
	315	$b(ab)^5b$
	315	$b(ab)^3b^2(ab)b$
4C	1890	$b^3(ab)^2$
	1890	$(ab)b^2(ab)^2b$
5A	2520	$(ab)^3b(ab)$
	3780	$(ab)^3b^2(ab)^2$
6A	1890	$b(ab)b(ab)$
	1890	$b(ab)b(ab)b(ab)^3$
	1260	$b(ab)b^2(ab)b(ab)^2$
6B	7560	(ab)
7A	1890	$(ab)b(ab)^4b^2$
	90	$b(ab)b^3(ab)^2b$
7B	1890	$b^5(ab)b(ab)$
	90	$b^5(ab)b^5(ab)b^2(ab)b^5$
7C	2160	$b(ab)b(ab)^4$
7D	7560	$(ab)b(ab)^4$
	1080	$(ab)b^3(ab)$
7E	7560	$(ab)^3b(ab)^2$
	1080	$(ab)b^4(ab)^2b(ab)$
8A	3780	$(ab)^2b(ab)^2$
	3780	$(ab)^4$
	3780	$(ab)^3b(ab)b(ab)b$
	3780	$(ab)b(ab)b(ab)^2$
10A	7560	$(ab)^3$
12A	7560	$b(ab)b$
	7560	$b(ab)b^2$
12B	7560	$b(ab)^2b$
	7560	b
14A	3780	$(ab)b^3(ab)^2$
	3780	$(ab)^2b^2(ab)$
	1890	$(ab)b^2(ab)$
	1890	$b(ab)^3b$
14B	3780	$(ab)b(ab)^2b(ab)$
	3780	$(ab)b^4$
	1890	$(ab)^2b^2(ab)b(ab)^2$
	1890	$b(ab)b(ab)^2$

TABLE 29: Continued.

X_C	$ \mathcal{O} $	g
14C	7560	$b(ab)$
	7560	$b(ab)^3$
14D	7560	$b^3(ab)$
	7560	$b(ab)^2$
15A	15120	$(ab)b^2$
21A	15120	$(ab)^4b(ab)$
21B	15120	$(ab)b$
21C	15120	$(ab)^3b$
21D	15120	b^2
28A	7560	$b^2(ab)b^2$
	7560	$(ab)^2$
28B	7560	$b^4(ab)^3$
	7560	$(ab)b(ab)^2$

TABLE 30

X_C	$ \mathcal{O} $	g
2A	640	ab^3ab^2aba
	480	$ab^2ab^2abababab^2ab^2a$
	240	$ab^2ab^3ab^3ab^3ab^2abab^2ab^3abababab^3ab^3$
	30	$ab^2ababab^2ab^2ab^3ab^3ababab^2ab^3ab^2ab^2ab^3ab^2ababab^3a$
3A	10240	$ab^3ababababab^2a$
4A	5120	$abababa$
4C	7680	$abab^2ab^2ab^2aba$
	3840	$ab^2ab^3abab^2aba$
	3840	$abab^2abab^3ab^2a$
4D	15360	aba
	7680	$ab^2ab^2abab^2abab^2a$
5A	12288	$ab^3abab^2abababa$
5B	4096	$ababa$
6A	61440	ab^2abab^2aba
	30720	$abababababa$
7A	61440	a
8C	61440	$abab^2ab^2a$
	61440	ab^2ab^2aba
10A	61440	$ab^2ab^2abababa$
12A	122880	ab^2aba
13A	61440	ab^3aba

3.12. $K \cong He$ 3.12.1. $X = 2A$, $t = a$. See Table 27.3.12.2. $X = 2B$, $t = (ab^2)^6$. See Table 28.3.12.3. $X = 2C$, $t = d^3$. See Table 29.3.13. $K \cong Ru$ 3.13.1. $X = 2A$, $t = b^2$. See Table 30.3.13.2. $X = 2B$, $t = a$. See Table 31.3.14. $K \cong Suz$ 3.14.1. $X = 2A$, $t = ((ab)^2ab^2)^6$. See Table 32.

TABLE 31

X_C	$ \mathcal{O} $	g
2A	455	$bab^3ab^2ab^3ab$
	455	$bab^3ab^2ababababab$
2B	455	$bababababab^2ab^3ab$
	1	$b^3ab^3ab^3ab^2ababab^2ab^2ab^3abab^2ab^2ab^2$
	1	$bab^2abab^3abab^2ab^2abab^2abab^3ab^3ababab^3abab^2abab^2ab^3abab^3$
3A	5824	$b^2ababababab$
4A	3640	$(ab)^{10}a(ab)^{15}a((ab)^5a)^2$
4C	3640	$b^3ababab^2ab^2$
4D	3640	$b^3abab^2ab^3ab$
5A	5824	$b^3abab^2abab^2$
	5824	$b^2ab^3ab^2ab$
5B	29120	b^2abab
6A	29120	$b^2ababab^2ab$
	29120	b^2ab^2
	29120	b
7A	29120	b^2
	29120	b
10A	29120	$babab^2abab$
	29120	$babab$
10B	29120	$babab^3ab^3ab$
	29120	bab^3ab^3abab
12A	58240	$b^3ababab^2ab$
13A	29120	bab
	29120	$bababab$
	29120	b^3ab^2ab
14A	29120	$b^3ab^2ab^2abab$
	29120	$bab^2ab^2ababab$
14B	29120	$b^2ab^2ababab$
	29120	$bababab^2ab^2$
14C	29120	$bab^2abababab^2$
	29120	$b^2abababab^2ab$
15A	116480	$b^3ab^2ababab$
20A	58240	b^3abab^2
	58240	b^2ab^3ab
26A	29120	bab^2
	29120	b^2ab
26B	29120	bab^2ab^2
	29120	b^2ab^2ab
26C	29120	$b^3ababab$
	29120	$babab^2ab^2$
29A	116480	bab^2ab
29B	116480	b^3abab

TABLE 32

X_C	$ \mathcal{O} $	g
2A	360	a
	54	$((ab)^2a)^4a((ab)^2a)^3$
3B	5120	$b^2ab^2abababababa$
3C	9216	$baba$
4A	1728	$b^2ab^2ab^2ababab$
4C	17280	b^2ab
5B	55296	b^2ababa
6D	46080	b

TABLE 33

X_C	$ \mathcal{O} $	g
2A	315	$bab^2 ababab^2 ab^2 ab^2 ab^2 ababab^2 ab$
	630	$b^2 abababab^2 ababab^2 ab^2 ababab^2 ab^2 abab^2 ab^2 ab^2 ab$
2B	560	$bab^2 abab^2 abab^2 abab$
	2	$(bab^2 a)^2 (b^2 a)^2 (ba(b^2 a)^2)^2 (b^2 aba)^2 ((b^2 a)^2 b^2 aba)^2 bab^2 (ab)^2$
3A	224	$(bab^2 a)^3 b(ba)^3 (b^2 a)^2 (ba)^5 b(ba)^4 (b^2 a)^2 b$
3B	2240	$b^2 ab^2 ababab^2 abab$
3C	20160	$bab^2 abab^2 ab$
4A	2520	$bababab^2 ab^2 ab^2 ab^2 ababab$
	1260	$b^2 ababab^2 ababab^2 abab$
	2520	$b^2 ab^2 ababab^2 abab^2 ab^2 abab$
4B	2520	$bab^2 ab^2 ababab^2 abababab$
	1260	$b^2 ababab^2 ab^2 ab$
	1260	$b^2 ab^2 abab^2 abab$
	5040	$bab^2 abab^2 ababab^2 abab^2 ab$
4C	2520	$bababab^2 ab^2 ababab$
	2520	$b^2 ab^2 ababab^2 abab^2 ab^2 abab$
	10080	$bab^2 ab^2 ab^2 abab^2 ab^2 abab$
4D	10080	$babab^2 ab^2 abab^2 ab^2 ab^2 ab$
	1120	$(ab)^4 ((ab)^4 a)^4 a(ab)^8 ((ab)^4 a)^2$
	1120	$(a(b^2 a)^4)^2 (b^2 a)^8 a(a(b^2 a)^4)^4 (b^2 a)^4$
5A	20160	$babab$
5B	40320	$b^2 ababab^2 ab$
6A	10080	$babababab^2 abababab$
6D	20160	$bab^2 ab^2 ab$
6E	40320	$b^2 ababab^2 ababab$
	40320	$b^2 abab^2 ab^2 ab^2 ababab$
7A	80640	$b^2 abababab$
8A	20160	$bab^2 abab^2 ab^2 ab^2 ab$
	20160	$bab^2 ab^2 ab^2 abab^2 ab$
	20160	$bab^2 abababab^2 ab^2 abab^2 ab$
	20160	$babab^2 ab^2 ab$
8B	20160	$bab^2 ab^2 abab$
	20160	$bab^2 abab^2 ab^2 abababab^2 ab$
	10080	$b^2 ab^2 ab$
	10080	$b^2 abab$
	20160	$b^2 abab^2 ababab$
8C	20160	$b^2 ababab^2 abababab^2 ab$
	20160	$b^2 ab^2 ab^2 abab^2 ab$
	20160	$bab^2 ab^2 abab^2 ab^2 ababab$
	40320	$b^2 abab^2 abab^2 abab$
10A	40320	$bababab^2 ababab$
	20160	$b^2 ab^2 ababab^2 ababab$
	20160	$b^2 ab^2 ab^2 abab^2 ab^2 abab$
10B	40320	$babababab^2 ab$
	40320	$bab^2 abababab$
11A	161280	$b^2 abab^2 abab$
12A	40320	$b^2 ab^2 abab^2 abab^2 abab$
	20160	$b^2 abab^2 ab^2 ababab^2 ab$
12B	40320	$b^2 ababababab$
	40320	$b^2 ab^2 ab^2 ab^2 ab^2 ab$
12C	40320	$bab^2 abab^2 ab^2 ababab$
12D	80640	$bab^2 ababab$
	80640	$bababab^2 ab$
13A	161280	bab
13B	161280	$b^2 ab^2 abab$
14A	80640	$b^2 abab^2 ab^2 abab$
15A	161280	b
15B	161280	$b^2 ab$
15C	161280	$b^2 ab^2 abab^2 ababab$

TABLE 33: Continued.

X_C	$ \mathcal{O} $	g
20A	80640	$bab^2 ab^2 ab^2 abab$
	80640	$babab^2 ab^2 ab^2 ab$
21A	161280	$bab^2 ab^2 ab^2 ababab$
21B	161280	$bab^2 ab^2 ab^2 ab$
24A	80640	$babab^2 ab$
	80640	$bab^2 abab$

TABLE 34

X_C	$ \mathcal{O} $	g
2A	315	$((ab)^3 a)^2 a((ab)^3 a)^4 a(ab)^3 ((ab)^3 a)^2 a$
2B	1800	$ababab^2$
3A	200	$b^2 ab^2 abab^2 abab$
3C	16800	$ab^2 ab^2 ababa$
4A	3150	$bab^2 abab^2 a$
	630	$bab^2 ababababab^2 a$
4C	12600	$bab^2 aba$
5B	20160	aba
6A	12600	$ababab^2 ab$
6E	100800	ba
7A	100800	b
8A	25200	$ababab$
	25200	$bababab$
12A	50400	a

TABLE 35

X_C	$ \mathcal{O} $	g
2A	495	$(ab)^5 a(ab)^2 (a(ab)^5)^4 (ab)^5$
2B	792	$b^2 abab^2 ababab^2 abab$
3A	264	$a(ab)^9 ((ab)^9 a)^4 a(ab)^9 ((ab)^9 a)^2$
3B	5280	$a(ab)^6 ((ab)^3 a)^4$
	1760	$(a((ab)^3 a)^2)^4 a$
3C	2640	$b^2 ababab^2 ababab^2 a$
	2970	$babab^2 abab^2 aba$
4A	495	$b^2 abababab$
	495	$bababab^2 ababa$
4B	11880	$bab^2 ab^2 abab^2 aba$
4C	11880	$b^2 ab$
5A	19008	$baba$
5B	47520	$ab^2 ababa$
	11880	aba
6A	3960	$ababab^2 a$
	3960	$abab^2 ab^2 a$
	15840	$ab^2 abab$
6B	15840	$ab^2 ababababab$
	15840	$b^2 ab^2 aba$
6C	15840	$b^2 ababababab^2 ab^2$
	47520	$ab^2 abababa$
6D	15840	$ab^2 ababab^2 aba$

TABLE 35: Continued.

X_C	$ \mathcal{O} $	g
6E	15840	$abababab^2abab^2ab^2ab$
7A	47520	bab^2aba
8B	47520	$b^2ababababa$
	47520	$ababab$
9A	95040	$bababab$
	31680	$babab^2ab^2abab^2a$
9B	95040	b^2ababa
	31680	$ab^2abababababa$
10A	95040	ab
10B	47520	$babab^2ab^2a$
	47520	bab^2ab^2aba
11A	190080	a
12A	47520	b^2aba
12B	47520	$abab^2ababab^2$
	47520	$abababababa$
	15840	b^2ab^2abab
	15840	$bababa$
12C	47520	ab^2aba
	47520	$abababababab^2$
12E	47520	b^2ab^2a
	47520	$abab$
	47520	$babababab$
	47520	$b^2abababa$
14A	47520	ab^2abab^2abab
	47520	b
15C	190080	$ababa$
18A	95040	bab^2ababa
	95040	ab^2ab^2abab
18B	95040	b^2ab^2ababa
	95040	$bababab^2a$
20A	95040	$babab^2aba$
	95040	ba

TABLE 36

X_C	$ \mathcal{O} $	g
2A	630	$ab^2a^2b^2a$
3B	8960	ab^2a^2
3C	1920	a^2ba^2

TABLE 37

X_C	$ \mathcal{O} $	g
4B	30240	ab^2a
5B	48384	aba^2
6C	80640	a

TABLE 38

X_C	$ \mathcal{O} $	g
2A	495	$baba^2ba^2bab^2ab$
2B	396	$ab^3ab^2a^2ba$
	396	$a^2b^3ab^2a^2ba^2$
3A	1320	$ba^2ba^2b^2ab$
	440	ab^2abab^2abab
3B	2640	$a^2b^2abab^3a$
	1584	$ababa^2b^3a^2$
3C	7920	bab^2ab^2ab
4A	2970	$bababab^2aba$
4B	11880	a^2b
	5940	b^2
	2970	$babab^2ab$
5A	9504	b^2aba^2
	9504	$ba^2b^3a^2ba$
5B	47520	a^2b^2a
6A	11880	$ba^2b^2ab^2a$
	3960	$ab^2a^2baba^2ba$
6B	15840	a^2
	15840	a
6C	23760	$babab^3a$
	23760	ba^2b^2aba
	15840	bab^3aba
6E	23760	b^2a
	23760	a^2b^2
7A	95040	ba^2
	31680	bab^2aba
8A	23760	$babab^2a$
	23760	$b^2ababab$
8B	23760	$abab^2a$
	23760	b^2a^2ba

3.14.2. $X = 2B$, $t = a$. See Table 33.

3.14.3. $X = 2C$, $t = c$. See Table 34.

3.14.4. $X = 2D$, $t = (c(cd)^{12})^{11}$. See Table 35.

3.15. $K \cong Co_3$

3.15.1. $X = 2A$, $t = b^2$. See Tables 36 and 37.

TABLE 39

X_C	$ \mathcal{O} $	g
8C	23760	ab^2ab^2
	23760	$b^2a^2b^2a^2$
	23760	b^2aba
	23760	ba^2bab^3
9A	31680	bab^2ab^3
	31680	$abababab$
9B	63360	$ababab^2$
10A	47520	aba^2b^2
	47520	$babab^2$
10B	47520	aba
	47520	$a^2b^3a^2$
12A	23760	a^2ba^2
	23760	ab^3a
12B	47520	b
	47520	$ababa^2$
	47520	bab^2ab
	47520	bab^2ab
14A	95040	$baba$
	95040	ab^2aba
15A	95040	ab^2
	95040	$abab$
15B	190080	$abab^2$
18A	95040	ab^2ab
	95040	$babab$
21A	190080	ba^2ba^2
24A	95040	ab^2a
	95040	$a^2b^2a^2$
24B	95040	a^2ba
	95040	ba
30A	95040	aba^2
	95040	ab^3a^2

3.15.2. $X = 2B$, $t = ((a^2b)^3ab)^5$. See Tables 38 and 39.

3.16. $K \cong Co_2$

3.16.1. $X = 2A$, $t = a$. See Table 40.

3.16.2. $X = 2B$, $t = a(ababa)^2a(ababa)^7$. See Table 41.

3.16.3. $X = 2C$, $t = (b^{-1}(ab^2)^2ab^{-1}(ab^2)^4)^5$. See Table 42.

TABLE 40

X_C	$ \mathcal{O} $	g
2B	1260	bab^2ab
2C	1008	b^3abab^3
3B	14336	bab
4C	40320	b

TABLE 41

X_C	$ \mathcal{O} $	g
2A	15	$(b^4a)^2b^3ab^2ab^4(bab)^2a(bab)^7abab^3ab^2(ab)^2$
2B	1680	b^3abab^3
	210	b^4ab^4abab
2C	2520	$(ab)^5a(ab)^2a(ab)^3$
3B	35840	b^2ab
4A	1920	bab^3ab^2abab
4C	20160	bab^2ab
4E	161280	bab
4F	13440	$babababab$
	13440	$b^3abab^2ab^2$
5B	344064	b^2
6E	430080	b

TABLE 42

X_C	$ \mathcal{O} $	g
2A	240	$ab^4ab^2abab^4ab^3ab^4ab^3aba$
	45	$ab^2ab^4ab^3abab^3ab^4ab^4ab^4ab^2a$
2B	720	b^2a
	240	$b^4ab^4ab^3ab^4ab^4ab^2ab^3ab^2a$
	180	$ab^2ab^3ab^2abab^3ab^3ab^2$
	30	$abab^4a$
	1440	$abab^3ab^3ab^2abab^2a$
2C	1440	$ab^3ab^4ab^3ab^2ab^2ab^4a$
	1440	$b^2ababab^3ab^4ab^3$
	1152	$abab^3ab^3ab^3ab^4ab^2a$
	360	$ababab^3ab^2abab^3ab^3ab^2$
	10240	$ab^4ab^2abab^2abab^2a$
3A	15360	ab^3ab^2a
	7680	b^3ab^2
	1024	$ab^2ab^2ab^3ab^3a$
4A	3840	$b^4ab^4ab^3ab^4ab^4ab^4$
	1440	$abab^2ab^2ab^4ab^2$
	11520	$bab^4ab^4ab^3ab^3a$
	5760	$ab^2abab^2abab^3a$
	5760	$ab^2ab^4ab^3ab^4ab^3a$
4B	1440	$ab^4abab^2ab^4ab^2abab^3a$
	1440	$ab^2ab^4ab^3abab^3ab^4aba$
	720	$abab^4ab^2ab^2ab^2ab^3ab^3$
	720	$b^2ab^4ab^2ab^2ab^4ab^3ab^4a$

TABLE 42: Continued.

X_C	$ \mathcal{O} $	g
4C	11520	ab^2ab^3a
	11520	$ab^3abab^3ab^2ab^2$
	5760	$b^3ab^4ab^2a$
	5760	ab^3abab^2
	2880	$abab^4ab^2a$
	1440	$b^4ab^4ab^3ab^2ab^2ab^3a$
	1440	$ab^2ab^3ab^3ab^2abab$
	720	$abababab^2abab^3a$
4D	720	$ab^3ab^4ab^2a$
	23040	$bab^3abab^2ab^3ab^2$
	11520	$abab^3ab^3abab^3$
	5760	$ab^4abab^4ab^2a$
	2880	$ababab^4ab^2ab^4ab^4ab^3a$
4E	2880	$ab^2ababab^3abab^4ab^4a$
	23040	$bababab^2$
	23040	$ab^3ab^2ab^2$
	11520	$ab^3ab^2ab^4ab^2$
	11520	$b^3abab^3ab^2a$
	5760	b^3ab^4a
	5760	$abab^2$
4F	5760	$b^4ab^3ab^2abab^3a$
	23040	$abab^2ab^2ab^2$
	23040	$b^3ab^3ab^3ab^4a$
	11520	$b^4ab^4ab^2ab^4ab^3ab^2a$
	11520	$ab^3ab^4ab^3ab^2ab^2ab^2$
	5760	$b^2ab^4ab^2ab^2ab^2ab^3ab^2a$
	5760	$b^2ab^2ab^3ab^3ab^4ab^2ab^3a$
	5760	$ab^3ab^2ab^4ab^2ab^3ab^3ab^2a$
	5760	$ab^4ababab^3ab^2ab^2ab^3$
	5760	$ab^2ab^3abab^2ab^2ab^3ab^3$
	5760	$b^4ab^4ab^2ab^2ab^2ab^3ab^2a$
	2880	$b^3ab^2abab^2$
	2880	$b^3ab^4ab^3ab^2$
	1920	$abab^4ab^2ab^2ab^4ab^3$
	1920	$bab^3ab^3abab^2ab^4$
4G	46080	$b^3ab^3ab^2ab^3a$
	46080	$ab^2ab^3ab^2ab^2$
	23040	$abab^2ab^2ab^3ab^2a$
	23040	$ab^3ab^2ab^3ab^3ab^4a$
5A	73728	$ab^2ab^3abab^2a$
5B	184320	b^2ab^2a
	122880	$b^2ab^3ab^2$

TABLE 42: Continued.

X_C	$ \mathcal{O} $	g
6A	92160	$bab^3ab^4ab^3ab^2a$
6B	122880	$b^2ab^4ab^2a$
	46080	$b^2ab^3ab^3ab^2a$
	46080	$b^3abab^2ab^2$
6C	15360	$b^4ab^2ab^3ab^3a$
	15360	$ab^2ab^2ab^3ab$
	92160	$b^4abab^2ab^4a$
	23040	$b^4ab^3ab^4ab^2ab^2$
6D	15360	ab^2ab^2aba
	15360	$ab^4ab^3ab^3a$
	92160	$babab^2$
	92160	bab^4a
6E	46080	$b^3ab^2ab^3ab^2a$
	46080	bab^2ab^3a
	46080	b^2abab^4a
	46080	$abab^4ab^3$
	15360	$ab^2abab^4abab^2a$
6F	92160	ab^3aba
	92160	ab^4ab^2a
	46080	$b^3ab^2abab^2a$
	46080	$ab^4ab^4ab^3ab^3$
	46080	$ab^3ab^4ab^3ab^2$
	46080	b^2ab^2ababa
	737280	b^4ab^2
7A	92160	b^2aba
	92160	ab^4ab^3
8A	92160	$ab^4ab^2ab^2aba$
	92160	$ab^4ab^3ab^3aba$
	46080	$abab^3ab^2a$
	46080	$ab^3ab^2ab^4a$
	23040	$bab^3ab^3abab^3$
8B	23040	$b^2ab^4ab^2ab^2ab^4$
	92160	$b^3ab^3ab^3ab^4$
	92160	b^2
	92160	b^3
8C	92160	$bab^2ab^2ab^2$
	92160	$b^3ab^3ab^2$
	92160	$ab^2ab^3ab^3ab^4ab^2$
8D	92160	$b^4ab^3ab^3a$
	92160	bab^2abab^3a
	92160	$b^3ab^2ab^2$
	92160	

TABLE 42: Continued.

X_C	$ \mathcal{O} $	g
8E	92160	ab^3a
	92160	ab^2a
	46080	$ab^3abab^4ab^2$
	46080	$b^3abab^4ab^2a$
	46080	$ab^2ab^2abab^3$
	46080	b^2ababa
	46080	$b^2ab^4ab^3ab^3a$
	46080	$b^2ab^2ab^3ab^3a$
	46080	$ab^2ab^2ab^3ab^3$
	46080	$ab^4ab^4ab^3$
	23040	$ab^4ab^2ab^2abab^3$
	23040	$b^4ab^4ab^2ab^4ab^3$
8F	184320	ab^4abab^3a
	184320	ab^2ab^4aba
	92160	$abab^3abab^2ab^2$
	92160	$b^3ab^4ab^4ab^3ab^2a$
9A	737280	ab^4abab^2
	245760	$b^2ab^4ab^2$
10A	368640	$b^2ab^3ab^2a$
10B	368640	ab^2ab^2
	368640	b^3ab^3a
10C	368640	ab^4
	368640	$abab^3abab^3a$
11A	1474560	b^2ab^3a
12A	122880	bab^2abab^4a
	122880	$b^4abab^4ab^3$
12B	368640	$b^2abab^3ab^2$
	184320	$ab^4ab^2ab^4ab^2$
12C	368640	$ab^2ab^2ab^3ab^2$
	184320	$ab^4abab^2ab^2$
12D	184320	$b^4abab^2ab^2$
	184320	$ababab^3ab^3a$
	92160	$ab^4ab^2ab^3a$
	92160	ab^2ab^3aba
12E	368640	$b^3ab^2ab^4$
	368640	bab^3ab^2
12F	184320	b^2abab^2a
	184320	ab^3ab^4a
	184320	$abab^3ab^3$
	184320	$abab^2a$
	92160	ab^2aba
12G	92160	ab^4ab^3a
	368640	bab^4ab^2a

TABLE 42: Continued.

X_C	$ \mathcal{O} $	g
12H	184320	$b^4ab^3ab^3ab^2$
	184320	$ab^3ab^2ab^3a$
	184320	$ababab^4ab^2a$
	184320	$ab^4ab^3ab^4ab^2a$
	184320	$b^3ab^2ab^2ab$
	184320	$ab^2ab^3ab^2a$
14A	737280	ab^4a
15A	737280	b^4a
	737280	bab^2a
16A	368640	ab^3ab^2
	368640	ab^2abab^4
	368640	b^3ab^2a
	368640	bab^4ab^3a
18A	737280	b^3ab^3
20A	737280	ab^4ab^2
	737280	b^3aba
20B	737280	ab^4ab^4a
	737280	$ababa$
24A	737280	ab^2
	737280	b^3a
24B	737280	b^4
	737280	b
28A	737280	$abab^3ab^2$
	737280	$ab^4ab^2ab^2$
30A	737280	b^2ab^4
	737280	bab^3

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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