Research Article

A New Integral Inequality and Delay-Decomposition with Uncertain Parameter Approach to the Stability Analysis of Time-Delay Systems

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This paper is concerned with the problem of delay-dependent stability of time-delay systems. Firstly, it introduces a new useful integral inequality which has been proved to be less conservative than the previous inequalities. Next, the inequality combines delay-decomposition approach with uncertain parameters applied to time-delay systems, based on the new Lyapunov-Krasovskii functionals and new stability criteria for system with time-delay have been derived and expressed in terms of LMIs. Finally, a numerical example is provided to show the effectiveness and the less conservative feature of the proposed method compared with some recent results.

1. Introduction

In recent years, the investigation about the stability of time-delay systems has already become a nuclear problem with the emergence of both suitable theoretical tools and more complex practical issues in the engineering field and information technology such as ecology, economics, or biology (see [1–6]). For the linear delay system, the stability conditions of systems are derived by using a lot of techniques. Among them, there are two most popular different approaches that allowed us to gain more efficient criteria. The first one is a classical methodology based on the survey of the roots of the related characteristic equation (see [7–11]); it is very effective in practice (see [12–14]) and perfect describer of the stability of systems. To some extent, however, this process exposes some limitations of itself that it cannot be spread straightway to other cases: the robust case and the system with time varying delay. Another one is the use of Lyapunov-Krasovskii functionals. As we all know, their general structures are simple, but the numeric is difficult to operate (see [15–19]). Furthermore, if some additional hypotheses are formulated on the Lyapunov functional, we can find that some conservative results are expressed in terms of LMIs.

Generally, we resolve these problems by either choosing extended state based on Lyapunov-Krasovskii functional (see [19–23]) and/or discretized Lyapunov functional (see [6–8]) and/or improving the existing integral inequality (see [21, 22, 24–26]). But, beyond that, another important technique reduces the conservatism, which is to bound some cross terms that have arisen when computing the derivative of Lyapunov functional. Looking through the literature for this purpose (see [23, 25, 27, 28]), the similar characteristic of all these approaches is a resort to the slack variables (see [21, 22, 24]) and Jensen's inequality (see [29–32]).

Fortunately, to the best of the authors' knowledge, we proposed a new more accurate inequality, which is proved to be less conservative than the existing inequalities based on Jensen's theorem, and constructed a more simple Lyapunov-Krasovskii functional by combining delay-decomposition approach with uncertain parameter in this paper. It is worth noting that this new functional depends not only on $x(t)$ and $x(t-h)$ but also on $x(t-\alpha h)$, $\int_{t-\alpha h}^t x(s)ds$, and $\int_{t-h}^{t-\alpha h} x(s)ds$, such that we can get more better result. Then, the last signal is directly integrated into a new appropriate Lyapunov-Krasovskii functional to highlight the feature of the new

inequality. Finally, new stability criteria for system with time-delay will be derived and will be expressed straightaway in terms of LMIs.

Notations. Throughout the paper, R^n denotes the n -dimensional Euclidean space with vector norm $|\cdot|$, and $R^{n \times m}$ is the set of all $n \times m$ real matrices. The notation $P > 0$ (or $P < 0$), for any symmetric matrix $P \in R^{n \times n}$, means that P is positive (or negative) definite. The notation $\begin{pmatrix} A & B \\ * & C \end{pmatrix}$, for any square matrices A and B in $R^{n \times m}$, stands for $\begin{pmatrix} A & B \\ B^T & C \end{pmatrix}$. Moreover, the notation I stands for the identity matrix. The expression e_i represents $(\underbrace{0, \dots, 0}_{i-1}, \underbrace{1, 0, \dots, 0}_{n-i})$.

2. Preliminaries

In this section, to show the priority of our approach, we consider the following linear time-delay systems:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h), \quad \forall t \geq 0, \\ x(t) &= \phi(t), \quad \forall t \in [-h, 0], \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state vector, ϕ is the initial condition, and $A, A_d \in R^{n \times n}$ are constant matrices. The delay h is a scalar and satisfies the constraint: $h \geq 0$. Before presenting our results, the following lemmas are provided, which are very important to derive the later delay-dependent stability conditions.

Lemma 1 (see [12]). *For given symmetric positive definite matrices $R > 0$ and any differentiable function w in $[t-h, t] \rightarrow R^n$, the following inequality holds:*

$$\begin{aligned} & - \int_{t-h}^t w^T(s) R w(s) ds \\ & \leq -\frac{1}{h} \left(\int_{t-h}^t w(s) ds \right)^T R \left(\int_{t-h}^t w(s) ds \right). \end{aligned} \quad (2)$$

Lemma 2 (see [1]). *For given symmetric positive definite matrices $R > 0$ and any differentiable function w in $[t-h, t] \rightarrow R^n$, the following inequality holds:*

$$\int_{t-h}^t \int_{\theta}^t \dot{w}^T(s) R \dot{w}(s) ds d\theta \geq \frac{2}{h^2} \Omega_0^T R \Omega_0, \quad (3)$$

where

$$\Omega_0 = h w(t) - \int_{t-h}^t w(s) ds. \quad (4)$$

Lemma 3. *For given symmetric positive definite matrices $R > 0$ and any differentiable function w in $[a, b] \rightarrow R^n$, the following inequality holds:*

$$\int_a^b \dot{w}(u)^T R \dot{w}(u) du \geq \frac{3}{4(b-a)} \Omega^T K \Omega, \quad (5)$$

where

$$\begin{aligned} \Omega &= \left(w^T(b), w^T(a), \frac{1}{(b-a)} \int_a^b w^T(u) du \right)^T, \\ K &= \begin{pmatrix} 5R & 2R & -7R \\ * & 4R & -6R \\ * & * & 13R \end{pmatrix}. \end{aligned} \quad (6)$$

Proof. For any differentiable function $w(u) \in W[a, b]$, consider a signal $z(u)$ given by

$$\begin{aligned} z(u) &= \dot{w}(u) - \frac{1}{(b-a)} \Omega_1 + \frac{3g(u)}{(b-a)^2} \Omega_2 \\ &\quad - \frac{f(u)}{2(b-a)^2} \Omega_3, \end{aligned} \quad (7)$$

where

$$\begin{aligned} \Omega_1 &= w(b) - w(a), \\ \Omega_2 &= w(b) + w(a) - \frac{2}{b-a} \int_a^b w(u) du, \\ \Omega_3 &= 2w(b) + w(a) - \frac{3}{b-a} \int_a^b w(u) du, \\ g(u) &= b + a - 2u, \\ f(u) &= 3u - 2a - b. \end{aligned} \quad (8)$$

Simple calculus ensures that

$$\begin{aligned} \int_a^b g(u) du &= 0, \\ \int_a^b g(u)^2 du &= \frac{1}{3} (b-a)^3, \\ \int_a^b g(u) \dot{w}(u) du &= -(b-a) \Omega_2, \\ \int_a^b \dot{w}(u) du &= w(b) - w(a) = \Omega_1, \\ \int_a^b f(u) du &= \frac{1}{2} (b-a)^2, \\ \int_a^b f(u)^2 du &= (b-a)^3, \\ \int_a^b f(u) \dot{w}(u) du &= (b-a) \Omega_3, \\ \int_a^b f(u) g(u) du &= -\frac{1}{2} (b-a)^3. \end{aligned} \quad (9)$$

The computation of $\int_a^b \int_\theta^t z^T(u) R z(u) du d\theta$ leads to

$$\begin{aligned}
 & \int_a^b \int_\theta^t z^T(u) R z(u) du d\theta \\
 &= \int_a^b \dot{w}(u)^T R \dot{w}(u) du - \frac{1}{b-a} \Omega_1^T R \Omega_1 \\
 &\quad - \frac{3}{b-a} \Omega_2^T R \Omega_2 - \frac{1}{2(b-a)} \Omega_3^T R \Omega_3 \\
 &\quad - \frac{1}{4(b-a)} \Omega_1^T R \Omega_5 - \frac{1}{4(b-a)} \Omega_3^T R \Omega_1 \\
 &\quad - \frac{3}{4(b-a)} \Omega_2^T R \Omega_3 - \frac{3}{4(b-a)} \Omega_3^T R \Omega_2 \\
 &= \int_a^b \dot{w}(u)^T R \dot{w}(u) du \\
 &\quad - \frac{1}{4(b-a)} \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}^T \begin{pmatrix} 4R & 0 & -R \\ * & 12R & -3R \\ * & * & 3R \end{pmatrix} \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}. \tag{10}
 \end{aligned}$$

Consider now the right-hand side of expression (10); let

$$\Omega^T = \left(w^T(b), w^T(a), \frac{1}{(b-a)} \int_a^b w^T(u) du \right), \tag{11}$$

and the following formulation will be obtained:

$$\begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \begin{pmatrix} I & -I & 0 \\ I & I & -2I \\ 2I & I & -3I \end{pmatrix} \Omega. \tag{12}$$

It can be seen that the left-hand side of expression (10) is positive definite since $R > 0$. Combining expression (10) and (12), we can complete the proof. $\square$

Remark 4. It is worth mentioning that the choice of function $z(u)$ is very necessary to Lemma 3. On one hand, functions $g(u)$ and $f(u)$ should be constructed so as the cross term $\int_a^b g(u) du \Omega_1^T R \Omega_2 = 0$ and $\int_a^b f(u) du \Omega_2^T R \Omega_3 \neq 0$ in computing $\int_a^b z^T(u) R z(u) du$. On the other hand, to unify the coefficient $1/(b-a)$ of the expression, $z(u)$ should contain the coefficient $1/(b-a)^2$.

Remark 5. Meanwhile, the choice of functions $g(u)$ and $f(u)$ is very important and the coefficient of parameters u , a , and b in functions $g(u)$ and $f(u)$ must be ensured so that the sum of the three coefficients is zero. Otherwise, we cannot guarantee that expression (9) is true.

Remark 6. It is interesting that inequality (5) will transform into inequality (2) when $\Omega_2 = 0$ and $\Omega_3 = 0$; this mean that inequality (5) encompass Jensen's inequality. Thus, we can say that inequality (5) improved Jensen's inequality.

3. Main Results

This section will state the stability analysis for time-delay systems. In the following context, we aim at assessing the stability of system (1) and illustrating the application and analysis of new integral inequalities combined with a simple Lyapunov-Krasovskii functional which is constructed by utilizing delay-decomposition approach with uncertain parameter. Based on the previous inequalities, the following theorem is provided.

Theorem 7. For a given constant delay h , system (1) is asymptotically stable if there exist $n \times n$ -matrices $P_{11} = P_{11}^T > 0$, $P_{ij} = P_{ji}^T$, $Q_i = Q_i^T > 0$, ($i, j = 1, 2$), and $S = S^T > 0$ such that the following conditions hold:

$$\begin{aligned}
 & \Phi \\
 &= \begin{pmatrix} P_{11} + 2S & P_{12} - \frac{2}{h}S & P_{12} - \frac{2}{h}S \\ * & P_{22} + \frac{1}{\alpha h}Q_1 + \frac{2}{h^2}S & P_{22} + \frac{2}{h^2}S \\ * & * & P_{22} + \frac{1}{(1-\alpha)h}Q_2 + \frac{2}{h^2}S \end{pmatrix} \\
 &> 0, \tag{13}
 \end{aligned}$$

$$\Psi = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & M_{23} & M_{24} & M_{25} \\ * & * & M_{33} & M_{34} & M_{35} \\ * & * & * & M_{44} & 0 \\ * & * & * & * & M_{55} \end{pmatrix} < 0,$$

where

$$M_{11} = A^T P_{11} + P_{11} A + P_{12}^T + P_{12} + Q_1 + h A^T S A$$

$$- \frac{15}{4\alpha h} S,$$

$$M_{12} = -\frac{3}{2\alpha h} S,$$

$$M_{22} = -Q_1 + Q_2 - \frac{3}{\alpha h} S - \frac{15}{4(1-\alpha)h} S,$$

$$M_{13} = P_{11} B - P_{12} + h A^T S B,$$

$$M_{23} = -\frac{3}{2(1-\alpha)h} S,$$

$$M_{33} = -Q_2 + h B^T S B - \frac{3}{(1-\alpha)h} S,$$

$$M_{14} = \alpha h \left(A^T P_{12} + P_{22} \right) + \frac{21}{4\alpha h} S,$$

$$M_{24} = \frac{9}{2\alpha h} S,$$

$$M_{34} = \alpha h \left(B^T P_{12} - P_{22} \right),$$

$$M_{44} = -\frac{39}{4\alpha h} S,$$

$$\begin{aligned}
M_{15} &= (1 - \alpha) h (A^T P_{12} + P_{22}), \\
M_{25} &= \frac{21}{4(1 - \alpha) h} S, \\
M_{35} &= (1 - \alpha) h (B^T P_{12} - P_{22}) + \frac{9}{2(1 - \alpha) h} S, \\
M_{55} &= -\frac{39}{4(1 - \alpha) h} S.
\end{aligned} \tag{14}$$

Proof. Consider the following Lyapunov-Krasovskii functional given by

$$V(t, x_t) = V_1(t, x_t) + V_2(t, x_t) + V_3(t, x_t), \tag{15}$$

where

$$\begin{aligned}
V_1(t, x_t) &= \begin{pmatrix} x(t) \\ \int_{t-h}^t x(s) ds \end{pmatrix}^T \\
&\quad \cdot \begin{pmatrix} P_{11} & P_{12} \\ * & P_{22} \end{pmatrix} \begin{pmatrix} x(t) \\ \int_{t-\alpha h}^t x(s) ds + \int_{t-h}^{t-\alpha h} x(s) ds \end{pmatrix}, \\
V_2(t, x_t) &= \int_{t-\alpha h}^t x^T(s) Q_1 x(s) ds \\
&\quad + \int_{t-h}^{t-\alpha h} x^T(s) Q_2 x(s) ds, \\
V_3(t, x_t) &= \int_{t-h}^t \int_{\theta}^t \dot{x}^T(s) S \dot{x}(s) ds d\theta.
\end{aligned} \tag{16}$$

Let

$$\zeta^T = \left(x^T(t), \int_{t-\alpha h}^t x^T(s) ds, \int_{t-h}^{t-\alpha h} x^T(s) ds \right), \tag{17}$$

and then employing Lemmas 1 and 2, respectively, to the above expression, we can obtain

$$\begin{aligned}
V_1(t, x_t) &= \zeta^T \begin{pmatrix} e_1 \\ e_2 + e_3 \end{pmatrix}^T \begin{pmatrix} P_{11} & P_{12} \\ * & P_{22} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 + e_3 \end{pmatrix} \zeta, \\
V_2(t, \dot{x}_t) &\geq \frac{1}{\alpha h} \left(\int_{t-\alpha h}^t x(s) ds \right)^T Q_1 \left(\int_{t-\alpha h}^t x(s) ds \right) \\
&\quad + \frac{1}{(1 - \alpha) h} \left(\int_{t-h}^{t-\alpha h} x(s) ds \right)^T Q_2 \left(\int_{t-h}^{t-\alpha h} x(s) ds \right) \\
&= \zeta^T \left[\frac{1}{\alpha h} e_2^T Q_1 e_2 + \frac{1}{(1 - \alpha) h} e_3^T Q_2 e_3 \right] \zeta, \\
V_3(t, \dot{x}_t) &\geq \frac{2}{h^2} \Omega_4^T S \Omega_4,
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\Omega_4 &= hx(t) - \int_{t-h}^t x(s) ds \\
&= hx(t) - \int_{t-\alpha h}^t x(s) ds - \int_{t-h}^{t-\alpha h} x(s) ds \\
&= [he_1 - e_2 - e_3] \zeta.
\end{aligned} \tag{19}$$

By (18), we obtain the following lower bound of $V(t, x_t)$: $V(t, x_t) \geq \zeta^T \Phi \zeta$. It is easy to see that the positive definiteness of the matrices Φ implies the positive definiteness of the functional $V(t, x_t)$.

Let

$$\begin{aligned}
\xi^T &= \left(x^T(t), x^T(t - \alpha h), x^T(t - h), \frac{1}{\alpha h} \right. \\
&\quad \cdot \left. \int_{t-\alpha h}^t x^T(s) ds, \frac{1}{(1 - \alpha) h} \int_{t-h}^{t-\alpha h} x^T(s) ds \right).
\end{aligned} \tag{20}$$

Calculating the derivative of the functional along the trajectories of system (1) and applying Lemma 3,

$$\begin{aligned}
\dot{V}_1(t, x_t) &= 2 \begin{pmatrix} \dot{x}(t) \\ x(t) - x(t - h) \end{pmatrix}^T \\
&\quad \cdot P \begin{pmatrix} x(t) \\ \int_{t-\alpha h}^t x(s) ds + \int_{t-h}^{t-\alpha h} x(s) ds \end{pmatrix} \\
&= 2\xi^T \left[\begin{pmatrix} Ae_1 + Be_3 \\ e_1 - e_3 \end{pmatrix}^T P \begin{pmatrix} e_1 \\ \alpha he_4 + (1 - \alpha) he_5 \end{pmatrix} \right] \xi, \\
\dot{V}_2(t, x_t) &= x^T(t) Q_1 x(t) - x^T(t - \alpha h) (Q_1 - Q_2) x(t - \alpha h) \\
&\quad - x^T(t - h) Q_2 x(t - h) = \xi^T \left[e_1^T Q_1 e_1 - e_2^T (Q_1 - Q_2) e_2 - e_3^T Q_2 e_3 \right] \xi,
\end{aligned} \tag{21}$$

$$\begin{aligned}
\dot{V}_3(t, x_t) &= h \dot{x}^T(t) S \dot{x}(t) - \int_{t-\alpha h}^t \dot{x}^T(s) S \dot{x}(s) ds \\
&\quad - \int_{t-h}^{t-\alpha h} \dot{x}^T(s) S \dot{x}(s) ds \leq h (Ax(t) + Bx(t - h))^T S (Ax(t) + Bx(t - h)) \\
&\quad - \frac{1}{\alpha h} \Omega_5^T K \Omega_5 - \frac{1}{(1 - \alpha) h} \Omega_6^T K \Omega_6 \\
&= \xi^T \left[h (Ae_1 + Be_3)^T S (Ae_1 + Be_3) - \frac{1}{\alpha h} \Omega_1^T K \Omega_1 - \frac{1}{(1 - \alpha) h} \Omega_2^T K \Omega_2 \right] \xi,
\end{aligned}$$

where

$$\begin{aligned}
 \Omega_5 &= \left(x^T(t), x^T(t - \alpha h), \frac{1}{\alpha h} \int_{t-\alpha h}^t x^T(s) ds \right) \\
 &= (e_1, e_2, e_4) \xi, \\
 \Omega_6 &= \left(x^T(t - \alpha h), x^T(t - h), \frac{1}{(1 - \alpha)h} \right. \\
 &\quad \cdot \left. \int_{t-h}^{t-\alpha h} x^T(s) ds \right) = (e_2, e_3, e_5) \xi, \\
 P &= \begin{pmatrix} P_{11} & P_{12} \\ * & P_{22} \end{pmatrix}, \\
 K_1 &= \frac{3}{4} \begin{pmatrix} 5S & 2S & -7S \\ * & 4S & -6S \\ * & * & 13S \end{pmatrix}.
 \end{aligned} \tag{22}$$

Combined with (21), we can get $\dot{V}(t, x_t) \leq \xi^T \Psi \xi$. Therefore, system (1) is asymptotic from the Lyapunov-Krasovskii stability theorem [12] if $\Psi < 0$ holds. This completes the proof. $\square$

Remark 8. It is worth noting that the construction of our Lyapunov-Krasovskii functionals is very general and simple, especially the functionals $V_1(t, x_t)$ and $V_2(t, x_t)$, which play an important role in reducing the conservativeness of stability criterion for neutral time-delay systems, since the functional includes more cross quadratic terms.

Remark 9. To illustrate the effectiveness and the less conservativeness of our new inequality, constructing the functional $V_3(t, x_t)$ is important and necessary, because it can induce single integral term, and then we can use the delay-decomposition approach and our new inequality.

Remark 10. The delay-decomposition approach with uncertain parameter has been applied to construct the Lyapunov-Krasovskii functionals such that the discussing interval of the delay of time-delay systems is extended to two segments, and according to different parameter value we can get more accurate and better results of delay h .

4. Example

In this section, our main purpose is to show how the inequality is given in Section 2 and the delay-decomposition approach with uncertain parameter reduces the conservatism in the stability condition. Furthermore, the simulated picture is provided to visually illustrate the effectiveness of our results.

Example 1. Consider the following example of the time-delay systems (1) with the matrices

$$\begin{aligned}
 A &= \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}, \\
 B &= \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}.
 \end{aligned} \tag{23}$$

TABLE 1: The maximal allowed delays of example (23) for different values of delay h .

Theorems	$h_{\max}$	Number of variables
[19]	6.11	$1.5n^2 + 9n + 9$
[21] $D_d = 3$	5.96	$6.5n^2 + 2.5n$
[21] $D_d = 4$	6.05	$10n^2 + 3n$
[24] $D_d = 1$	6.053	$7.5n^2 + 3.5n$
[24] $D_d = 2$	6.165	$10.5n^2 + 4.5n$
[24] $D_d = 3$	6.171	$14.5n^2 + 4.5n$
[24] $D_d = 4$	6.171	$20.5n^2 + 5.5n$
[15] $D_p = 2$	5.90	$12.5n^2 + 4.5n$
[15] $D_p = 3$	6.10	$21n^2 + 6n$
[32] ($N = 1$)	6.059	$3n^2 + 2n$
[32] ($N = 2$)	6.168	$5.5n^2 + 2.5n$
Theorem 7 ($\alpha = 0.6563$)	6.1721	$3n^2 + 3n$
Theorem 7 ($\alpha = 0.6564$)	6.1719	$3n^2 + 3n$
Theorem 7 ($\alpha = 0.6565$)	6.1716	$3n^2 + 3n$

FIGURE 1: Example (23) for delay $h = h_{\max} = 6.1721$.

This system is a well-known delay-dependent stable time-delay system. That is, the delay free system is stable and the maximum allowable delay $h_{\max} = 6.1721$ can be easily computed by delay sweeping techniques. To intuitively demonstrate the effectiveness and veracity of our approach, we will compare the results with the literature in Table 1 and the related simulated picture is reported in Figure 1.

In Table 1, the notation D_p in [15] means that the degree of freedom comes from the degree of the polynomial matrices, and the notation D_d in [21, 24] means that the degree of the polynomial is always 1, but the degree of freedom comes from the degree of discretization. Apparently, Table 1 shows that our result is competitive with the most accurate stability conditions from the literature. For the case of constant and known delay, it delivers a better result of delay h than the one provided by other theorems, such as [15, 19, 21, 24, 32], and it has a lower number of variables although the result of h in our paper is a little smaller than the other results.

Meanwhile, it is interesting that the simulated figure describes a distinct phenomenon that the system tends to be stable within a limited time; please look at Figure 1. In other words, our result is very suitable for time-delay systems producing some less conservative conditions than those from these literatures.

5. Conclusion

In this paper, we provide a new useful integral inequality, which has been proved to be suitable for the stability analysis of time-delay systems. Then, we construct a simple Lyapunov-Krasovskii functional and derive new stability criteria for time-delay systems, by employing this new integral inequality and combining with delay-decomposition approach with uncertain parameter. The new result we gained has been expressed in terms of LMIs and shows a larger improvement than the existing results through a numerical example. More generally, this new inequality could be coupled to more elaborated Lyapunov functional to resolve more problems about time-delay systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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