Research Article

System Performance of an Inertially Stabilized Gimbal Platform with Friction, Resonance, and Vibration Effects

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The research work evaluates the quality of the sensor to perform measurements and documents its effects on the performance of the system. It also evaluates if this performance changes due to the environments and other system parameters. These environments and parameters include vibration, system friction, structural resonance, and dynamic system input. The analysis is done by modeling a gimbal camera system that requires angular measurements from inertial sensors and gyros for stabilization. Overall, modeling includes models for four different types of gyros, the gimbal camera system, the drive motor, the motor rate control system, and the angle position control system. Models for friction, structural resonance, and vibration are analyzed, respectively. The system is simulated, for an ideal system, and then includes the more realistic environmental and system parameters. These simulations are run with each of the four types of gyros. The performance analysis depicts that for the ideal system; increasing gyro quality provides better system performance. However, when environmental and system parameters are introduced, this is no longer the case. There are even cases when lower quality sensors provide better performance than higher quality sensors.

1. Introduction

Inertially stabilized platforms (ISP) are used to aim and stabilize many different instruments. These can include infrared and optical cameras used for civilian or military applications including target tracking, telescope pointing, and image capture where the line of sight (LOS) of the instrument must be held steady and pointed to a desired location [1–6]. This becomes even more essential if the instrument is mounted to a moving base such as a land vehicle, airplane, helicopter, or a moving ship under environmental vibrations. Inertial measurement units (IMUs) consisting of accelerometers and rate gyros can be used to measure the acceleration and angular movement of both the base and stabilized platform in order to compensate for the external disturbances and stabilize the instrument concerned in order to hold the LOS [1–6]. The ability to stabilize a system and hold its LOS depends on many factors including, but not limited to, the

control algorithms used, the gain settings of these algorithms, and the quality of the sensor providing the measurements used to control the LOS. But determining what quality of sensor is needed for a given application is not always straight forward. While increased sensor quality should increase the quality of the system performance, this is not always the case. Depending on the dynamic environment of the system, the quality of the system performance can change for a given sensor. There may be instances that a lower quality sensor may provide equal performance or even outperform higher quality sensors.

Any ISP system includes integration of complex instrumentation; the model also needs to address the internal vibrations of the electromechanical system and external disturbances from the environment that can get challenging for simulations. The developed ISP model demonstrates the entire system dynamics to evaluate the performance of four different types of sensors under various testing conditions.

FIGURE 1: Block diagram of the system model.

The system model includes the gyro model, the gimbal model, the motor control, and the outer-loop angle control subsystems. The research is mainly focused on developing an analytical ISP model as a function of moment of inertia, motor drive, and gyro performance to measure the disturbances for stabilization and tracking purposes. The simulations from the gimbal, motor, and feedback control systems maintain the LOS for tracking purposes [7, 8].

In order to investigate this system performance based on sensor selection without performing hardware tests, an ISP system that requires inertial sensor measurements for stabilization is modeled and simulated. The modeling and simulation are conducted with four different qualities of inertial sensors to compare performance. The model including friction and structural resonance is subjected to different levels of environmental vibration to determine if the performance provided by each quality of sensor varies with the system conditions.

In this work, the system modeled is a gimbal camera that would be mounted to the underside of a helicopter. The complete system consists of the gimbal and actuating motor as well as models for the rate gyro sensors. Also, the motor rate control loop and the angular positioning control loop are designed to obtain performance necessary for LOS control applications. Once the control design is concluded, the closed-loop system is simulated, incrementally adding friction, structural resonance, environmental vibration, and dynamic command input, to simulate base motion cancellation, in order to compare the system performance based on the rate gyro selection for the given conditions.

The unique contribution of the research lies in the design of the ISP system and PI^2 controller using the PID-I approach and its application to generate system stability. The vibration, friction, resonance, and other disturbance effects are included to generate performance criteria for platform stabilization purposes. The results of the simulations of the four different types of commercially used gyro sensors are evaluated and presented. The detailed performances of the sensors including the noise and jitter levels are described in the conclusion section. This research verifies the necessity of modeling and simulating the sensor performance to ensure correct sensor selection for a desired application.

2. ISP System Model

An overview of the system design is described in this section corresponding to the system block diagram shown in Figure 1.

The system represented is a gimbal camera used to point toward an image and maintain the desired LOS. An angle command is generated as the system input. This angle command may be the result of an image tracking algorithm, base motion cancellation, or from other sources [1–6]. This work does not focus on the generation of the angle command but analyzes the system performance compared with the desired angle. To aim the gimbal system to the desired angle, an angle controller is designed. It compares the angle command input with the measured angle and this LOS error is used to generate motor rate commands for the motor to turn the camera in the desired direction and eliminate the LOS error.

A motor controller is designed to compare the commanded motor rate with the measured rate and output a motor voltage to achieve desired motor performance. This motor rate controller also helps to eliminate error due to disturbances such as friction and vibration which is explained later in Section 7.

The output rotation rates of the camera gimbal and direct drive motor are measured by the rate gyro to provide the angular rate for comparison by the rate controller. This measured rate is also integrated to calculate the measured angular position of the gimbal system to compare with the input angle command. Evaluating the effects of error in the rate measurement due to sensor quality on the LOS error performance is the main goal, which is significant in so many real design applications.

3. Gyro Modeling

The main objective is to evaluate the system performance based on the sensor selection; therefore the sensor is modeled first. While IMUs consist of accelerometers and rate gyros, the LOS simulation performed is only dependent on the angular positioning of the gimbal system and is therefore the only part of the inertial sensor that is modeled.

3.1. Basic Gyro Modeling. Rate gyros are instruments to measure the angular velocity with which they are rotating. While there are many different types of rate gyros [1, 2], their performance can all be modeled in a similar manner.

The first performance measure to model is the gyro frequency response or bandwidth. The frequency response is modeled as a second-order system. The general form of the gyro transfer function used, $G_{Gyro}(s)$, is shown in (1), where the bandwidth of the gyro, f_{BW} , is used to determine the

FIGURE 2: Simulink model of gyro II with ADC.

TABLE 1: Gyro specifications.

Gyro type	I	II	III	IV
Bandwidth	100 Hz	300 Hz	250 Hz	50 Hz
ARW	0.3 deg/ $\sqrt{\text{hr}}$	0.07 deg/ $\sqrt{\text{hr}}$	0.0009 deg/ $\sqrt{\text{hr}}$	50 $\mu\text{deg}/\sqrt{\text{hr}}$
Range	200 deg/sec	1000 deg/sec	1000 deg/sec	10 deg/sec
Sampling rate	200 Hz	400 Hz	500 Hz	100 Hz

TABLE 2: Analog to digital conversion specifications.

Gyro type	I	II	III	IV
ADC	Signed 16 bit	Signed 16 bit	Signed 32 bit	Signed 32 bit
Maximum rate	± 200 deg/s	± 1000 deg/s	± 1000 deg/s	± 10 deg/s
sensitivity	7.00 mdeg/s	35.0 mdeg/s	525 ndeg/s	5.25 ndeg/s

natural frequency of the second-order model, $\omega_n = 2\pi f_{\text{BW}}$. The damping ratio used is $\zeta = 0.707$.

$$G_{\text{Gyro}}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad (1)$$

Beside the gyro affecting the measured rate depending on the frequency of the angular rate change, internal noise can affect the measurement as well. Noise levels in gyro measurements are specified in terms of Angle Random Walk (ARW) values in units of deg/ $\sqrt{\text{hr}}$. If a stationary system is measured by a gyro, the internal noise will generate an error to the measurement. The ARW value represents the standard deviation in the resulting angle if the measurement on a stationary system with noise is integrated for one hour to obtain the angular position. The Simulink white noise block is used to model this noise and it requires a power spectral density (PSD) value in (deg/sec) 2 /Hz, which can be obtained from the ARW value as shown in (2):

$$\text{PSD} \left[\frac{(\text{deg/s})^2}{\text{Hz}} \right] = \left(\frac{1}{3600} \right) \cdot \left(\text{ARW} \left[\frac{\text{deg}}{\sqrt{\text{hr}}} \right] \right)^2. \quad (2)$$

The third specification used in modeling the gyro is the maximum measurement range. If the angular rate of the system being measured is greater than this limit, the rate is only measured as a value equal to the maximum range. This is modeled as a saturation block in Simulink. This rate saturation will have an effect on the control system if not accounted for and will be addressed in the control system design in Section 5.

A list of the gyro specifications for the four different qualities of gyros to be used in the simulations is shown in Table 1.

3.2. Analog to Digital Conversion. While gyro measurements can be used with analog circuitry, most modern applications use microprocessors to perform measurements and control calculations. Therefore analog to digital conversion (ADC) needs to be performed. This process is included in the modeling of the gyros.

The first element to account for is the digital data type used for storing the gyro measurement value for the ADC conversion. As shown in Table 2, the gyros use either a signed 16-bit integer, for types I and II, or a signed 32-bit integer, for types III and IV, to convert the analog measurement to digital counts. These measured integer numbers of counts are then converted back to a rate measurement using the sensitivity value for degrees/sec/bit. The sensitivity value is calculated by dividing the maximum range of the sensor by either 2^{15} or 2^{31} . For signed integer types, the first bit is used for the sign and the remaining 15 or 31 bits are used to represent a value from zero to the maximum range. To be able to account for bias on the sensor and still enable readings to the maximum range, a buffer of approximately 15% is used for the maximum scale prior to the ADC. This is done to calculate the single bit sensitivity values for the four different gyro types. The results of this calculation are listed in Table 2 for each corresponding gyro.

The method used to implement the ADC in the gyro model is shown in Figure 2. The measured value is first divided by the single bit sensitivity value. Then this value is rounded to the nearest integer to represent the quantized

FIGURE 3: Zero-rate gyro simulation.

digital count value. The quantized value is then multiplied by the single bit sensitivity value to obtain a quantized angular rate value.

3.3. Zero-Rate Gyro Simulations. In order to obtain a time history characterization of the gyros based on the performance specifications of Tables 1 and 2, the gyros will be simulated taking rate measurements of a stationary system. Therefore, any rate measured is due to the internal noise of the gyro. The rate input is set to a value of zero. Measurements are taken for 10 seconds. All simulations use the same random number generation seed. Because each gyro has a different sampling time, as per Table 1, the noise signal generated for each gyro is different as well. A similarity in the enveloping shape of the simulated measurement can be seen as a time compression or dilatation in each of the simulation data sets. The magnitudes of the measurement noise vary greatly depending on the gyro quality. The zero-rate measurement results for all four gyros are shown in Figure 3.

With the ADC implemented, only gyro II shows the gyro noise to have a magnitude comparable to the single bit sensitivity of the ADC process. To quantify the zero-rate gyro

TABLE 3: Zero-rate gyro simulation statistics.

Gyro type	I	II	III	IV
$1 - \sigma$	0.0599 deg/s	0.0241 deg/s	$2.90E - 4$ deg/s	$7.15E - 6$ deg/s

data above, the measured rate standard deviation ($1 - \sigma$) values are shown in Table 3.

4. Gimbal and Motor Control Modeling

In order to model the camera gimbal system as Figure 4 to effectively evaluate the system performance based on sensor selection, the model is based on a Cineflex Media HD ENG camera gimbal system for reference. Unit properties and performance specifications are given in Table 4 [5].

4.1. Gimbal and Motor Model. Assuming a spherical shape for the mass, the moment of inertia, J , corresponding to the gimbal mass and dimensions is given as $0.157 \text{ kg}\cdot\text{m}^2$. Also, assuming an ideal rotational system with no friction, the

FIGURE 4: Cineflex camera gimbal system used for [5].

TABLE 4: Gimbal performance specifications.

Size (diameter)	14 inch (35.6 cm)
Weight	27 lbs. (12.3 Kg)
Slew rate	55 deg/s
Max acceleration slew rate	100 deg/s ²

transfer function from motor torque, T , to angular rate, ω , is shown as follows:

$$\frac{T(s)}{\omega(s)} = \frac{1}{Js} = \frac{1}{0.157s}. \quad (3)$$

The motor specifications used to simulate the direct drive DC motor for the gimbal system are [1]

$$\begin{aligned} \text{Max. Voltage} &= 27 \text{ V}, \\ R &= 4.5 \, \Omega, \\ L &= 0.003 \text{ H}, \\ J_m &= 0.0017 \text{ kg} \cdot \text{m}^2, \\ K_t &= 0.85 \text{ Nm/A}, \\ K_e &= 0.85 \text{ V/(rad/s)}. \end{aligned} \quad (4)$$

The direct drive DC motor is modeled in Simulink as shown in Figure 5. The output motor voltage is less than the input voltage by the combination of the back emf and internal resistance voltage drop. The resulting output voltage across the motor inductance produces the time rate of change of current. The output current produces a torque scaled by the motor torque constant. The direct voltage to torque transfer function of motor input voltage, $V(s)$ to motor torque, $T(s)$, is shown as follows:

$$\frac{V(s)}{T(s)} = \frac{K_t}{Ls + R}. \quad (5)$$

For the direct drive motor-gimbal system, the back emf is generated from the motor rotational rate, which is the

FIGURE 5: Simulink model of DC motor.

FIGURE 6: Motor-gimbal block diagram.

same as the gimbal rotational rate. Therefore the motor and gimbal must be modeled together to obtain the angular rate to calculate the motor-gimbal system transfer function, $G_{mg}(s)$. When the feedback loop is closed with the system specifications, as shown in the block diagram in Figure 6, the motor-gimbal transfer function is shown as follows:

$$G_{mg}(s) = \frac{1805}{s^2 + 1500s + 1534}. \quad (6)$$

4.2. Motor Controller Design. With the motor-gimbal system modeled in the previous section, a motor controller must be designed to control the motor-gimbal system to the desired rate without error and to the desired performance specifications of Table 4.

Figure 7 shows the block diagram for the gimbal motor rate control loop that implements a PID controller on the rate error signal from the angular rate measured by the gyro. The gyro select block contains the four models for the different gyro specifications to allow for quickly changing sensor values for simulations. The control signal is constrained to the 27 V motor specification limit. With the output constrained, the PID block also implements an anti-integral windup algorithm to compensate for when the control signal is saturated [9, 10].

The eventual motor controller is implemented as a PID compensator; however the initial controller design will begin with a PI compensator. The system is designed using root locus and pole-zero placement methods. Using step response performance to design the controller, the step response rise time must correspond to a maximum acceleration of 100 deg/s², taken from the gimbal performance specifications

FIGURE 7: Gimbal motor rate control loop.

FIGURE 8: Root locus of uncompensated system.

FIGURE 9: Comparison of open loop and uncompensated closed-loop motor rate performance.

of Table 4. Since the motor rate is limited to a maximum of 55 deg/s, to match an average acceleration of 100 deg/sec² the 10–90% rise time levels of the step response correspond to a range of 44 deg/s. Accelerating through a 44 deg/s range in 0.44 seconds corresponds to an average acceleration of 100 deg/sec².

The root locus is plotted for the motor-gimbal system closing the loop with only proportional control shown in Figure 8. Figure 9 shows the open loop and uncompensated closed-loop step response plots.

In order to remove the steady state error, integral control is required. To obtain the desired rise time of 0.44 s, the dominate system pole must be slower than $s = -6.3$ in order to not go over the maximum acceleration specification. To compensate the system pole near $s = -1$, a compensator zero is added at $s = -0.8$ near this pole. Plotting the root locus for this PI compensated system, shown in Figure 9, a proportional gain of 4.5 moves the pole at $s = -1$ to the left to a value of $s = -5.7$, near but not past the fastest value allowed of $s = -6.3$. The integral pole at $s = 0$ moves toward the zero at $s = -0.8$ and is nearly cancelled with this compensation. This proportional gain results in similar performance if the pole of the plant is shifted at all due to design tolerance or inertia change due to angular position. The compensator design is

given in (7). The root locus and gain selection for PI controller design is shown in Figure 10.

$$G_{c-PI}(s) = \frac{4.5 * (s + 0.8)}{s} = \frac{4.5s + 3.6}{s}. \quad (7)$$

An additional derivative control term is needed to aid in disturbance rejection. A derivative gain of 0.05 is added, which adds a zero to the controller at $s = -89.19$ and shifts the zero from the original PI controller from -0.8 to -0.807 . The step response plot in Figure 11 shows the similar performance with the derivative control added. The resulting PID controller is given as follows:

$$G_{c-PID}(s) = \frac{0.05s^2 + 4.5s + 3.6}{s}. \quad (8)$$

The rise time achieved by PI design is 0.456 s with gains of $K_p = 4.5$, $K_i = 3.6$. With the additional derivative gain, $K_d = 0.05$, the PID controller's 10–90% rise time is 0.485 seconds, with an average acceleration of 90.7 deg/s² (Figure 11). This rate is a reasonable model for the maximum acceleration of 100 deg/s², since a rate higher than the average occurs during the 10–90% rise time. The transfer function for the

FIGURE 10: Root locus and gain selection for PI controller design.

FIGURE 12: Uncompensated root locus near dominant pole region.

FIGURE 11: Motor rate step response.

PID controlled motor-gimbal system, $G_{\text{mg-PID}}(s)$, is given as follows:

$$G_{\text{mg-PID}}(s) = \frac{90.23s^2 + 8121s + 6497}{s^3 + 1590s^2 + 9655s + 6497}. \quad (9)$$

5. Angle Control Loop Design

In order to control the line of sight (LOS) of the PID compensated motor-gimbal system to a desired angle, an outer angle control loop is required. Initial designs are for a PID controller. After initial simulations are conducted, it becomes apparent that a higher type controller is needed to reduce LOS error in dynamic environments. Therefore an integrator is cascaded with the initial PID controller. This controller is designed and implemented as a PID-I, which,

as a single block simplification, becomes a PI^2 controller. This section describes the PID and PI^2 angle controller design process, simulation, and performance comparison of the two controllers.

5.1. Angle Controller Design. The angle controller output drives the PID compensated motor-gimbal system in order to achieve desired LOS tracking. This is done by comparing the integrated angular rate measured by the gyro, which results in the measured gimbal angle, to the desired angle command as shown in block diagram of Figure 1. The angle error value from this comparison is what must be compensated by the angle controller to achieve the desired LOS tracking performance. Therefore the compensated motor-gimbal transfer function of (9) is the system plant for the angle controller design. Although there are many methods of designing LOS angle controllers [10–14], the method performed in this design is explained as follows.

The design process begins by plotting the root locus and step response for the uncompensated, closed-loop angle system shown in Figures 12 and 13.

Since the uncompensated system step response shows no steady state error, only a PD controller is necessary for satisfactory angle step response. However to track higher order inputs without steady state error a PID controller would be preferred. It is desired to design a PID controller to obtain step response performance that operates with a short rise time and settling time. This performance specification is balanced by also attempting to minimize the time that the control signal is saturated as well. To achieve this, the PID controller can be designed in two steps: a PD controller cascaded with a PI controller. MATLAB is used to plot the root locus and calculate preliminary gains. Then the Simulink model is used to analyze the actual system performance and monitor the motor command signal to verify that the time the signal is in saturation is minimized. All system performance data is generated from simulations using the Simulink model.

FIGURE 13: The uncompensated closed-loop angle step response.

FIGURE 14: Root locus for PID angle controller design.

The PD portion of the controller is designed to place a zero at $s = -5.5$. Then a PI controller is cascaded to add a zero at $s = -0.75$ as well as a pole at the origin. The corresponding root locus is shown in Figure 14. The proportional gain chosen for the cascaded PD-PI controller to place the closed-loop poles at the desired locations is 1.75 resulting in the PID gains of $K_p = 11$, $K_i = 7.2$, and $K_d = 1.75$. The performance achieved by these gains is a rise time of 0.5 seconds and settling time of 3.5 seconds, as shown in Figure 15. The transfer function of the resulting PID controller is shown as follows:

$$G_{c-PID_{angle}}(s) = \frac{1.75s^2 + 11s + 7.2}{s}. \quad (10)$$

5.2. Higher Order Input Consideration, Design, and Comparison. Since the main purpose of a camera gimbal system is tracking, as well as tracking in dynamic environments,

FIGURE 15: 10-degree step response, PID versus PI^2 .

the PID controller performance is simulated for ramp and sinusoidal inputs as well. The ramp input is a 10 degrees/s' ramp that lasts for 3 seconds. The sinusoidal input is of amplitude of 10 degrees and a period of 12.57 seconds. For the PID angle controller, the tracking error overshoot for the ramp input is nearly one degree. The maximum LOS error for the sinusoidal input is approximately 0.3 degrees. These LOS errors appear to be large for the system and therefore a higher order controller is considered.

To increase the order of the system an integral gain is cascaded with the PID controller, thus creating a PI^2 controller [9–11]. The cascaded integral gain is tuned using the sinusoidal input and simulating with the Simulink model. It is also verified by root locus methods on the linear model in MATLAB.

For tuning in Simulink, first the gain is increased until the system is stable. Once the minimum gain achieves system stabilization, the gain is further increased to remove oscillations about the commanded sinusoidal response. The motor command voltage signal is monitored during this simulation as well. The gain that achieves no noticeable steady state tracking error while removing oscillations about the sinusoidal command signal without saturating the motor command voltage for extended periods is a gain of 500. The same process is repeated and verified for the system input ramp command. The gain of 500 removes oscillations and overshoots in the ramp response as well. With the integral controller cascaded with the previously designed PID controller, the resulting cascaded form of the controller transfer function is

$$G_c(s) = \frac{1.75s^2 + 11s + 7.2}{s} \cdot \frac{500}{s} \quad (11)$$

FIGURE 16: PI2 controller root locus.

or the single block implementation of the controller in PI² form is

$$G_c(s) = \frac{875s^2 + 5500s + 3600}{s^2}. \quad (12)$$

The units for $G_c(s)$ are (deg/s)/deg.

The PI² design is also confirmed by the root locus method plotted in Figure 16. This plot shows the poles initial split into the right hand plane and therefore requires a minimum gain to move them to the left of the imaginary axis for stabilization. If the plot is expanded, it is seen that the break in point of the two complex poles occurs at a gain of 4000. When this gain is simulated in Simulink the motor control signal is saturated nearly continuously, driving the motor too hard. For this reason the gain value of 500 determined by simulation tuning is chosen for the controller design. Even though the high frequency poles have a damping ratio less than one, the overshoot and oscillation are low or not noticeable, depending on the input command type, due to the high frequency of these poles compared with the dominant poles of the system.

The system responses of both the PID and PI² controllers are compared using gyro III to produce feedback angle measurements. Figure 15 shows the 10-degree step response comparison for the two controllers with similar performance from both, with the PI² controller having marginally better performance.

Figure 17 shows the system's response to an angle ramp input of 10 degrees/s for three seconds. Comparing the system response of the two control methods, the LOS error is a magnitude smaller for the PI² controller. Figure 18 compares the system performance with a sinusoidal angle input command for the two control strategies. The LOS error for the PI² controller is at least one magnitude smaller than the PID controller. For these reasons, the PI² controller is chosen as the angle controller for the system and is used for all further simulations.

Figure 19 shows the complete block diagram of the system with both the PID motor-gimbal controller and the PI² angle controller implemented. The rate saturation term is put into the Simulink model. For instance, for gyro IV, the saturation level is 10 degrees/s. This is lower than the others by at least a factor of 10. So when the performance of this gyro was analyzed, the rate command level limit was changed accordingly so the platform would not be commanded at a rate the gyro could not measure. It is set 7 degrees/s, so any noise, overshoot, and so forth would not take the rate over 10 degrees/s. Restated, the command signals to the system had different limits when using gyro IV. Both controller outputs are limited with saturation blocks to represent the maximum motor rate command of the angle controller of 55 degrees/s and the maximum motor voltage command signal of the motor-gimbal controller of 27 V. Therefore both controllers use blocks containing anti-integral windup algorithms to eliminate integral windup effects.

6. Ideal System Performance Comparison versus Gyro

Now, the modeling of the gyros, the camera gimbal system, the drive motor selection and modeling, and both the motor controller and angle controller have been completed. The major contribution of this research is to compare the system performance of an inertially stabilized platform based on the sensor selection. Simulations are conducted with the same input commands executed with the system using each of the four different gyros to provide feedback measurements. The four different gyros have been modeled to the different performance specifications provided in Table 1.

The first simulation is performed on the ideal system as a zero-angle hold. The command angle is set to zero and the closed-loop system is simulated for 60 seconds. The simulation results are shown in Figure 20. It can be seen that the higher performance gyro, with lower noise specifications, provides better performance for the ideal system. The statistics shown in Table 5 show the standard deviation of the angle and the mean angle for the 60-second simulation. The last three columns provide a description of the image size of the angle standard deviation at three different distances. This is done to provide some understanding of such small angle deviation values.

7. Further System Modeling and Simulation-Friction, Vibration, and Structural Resonance

The ideal system has been simulated; the next goal is to determine if environmental disturbances, such as vibration, or physical system parameters, for instance friction or structural resonance, affect the system in such a way that a higher quality sensor will no longer provide increased performance over a lower quality, and therefore lower priced, sensor. Therefore the system is now modeled with friction, structural resonance, and disturbance torque.

FIGURE 17: Controller performance comparison for ramp input.

FIGURE 18: Controller performance comparison for sinusoidal input.

FIGURE 19: Ideal system closed-loop block diagram.

7.1. Friction Modeling. Friction is the first additional system parameter to be added to the model. It will be considered as frictional force acting on the rotational elements of the gimbal.

Many engineering applications [15–22] generally model friction with three terms: static friction when the velocity is zero, coulomb friction, and viscous friction. This frictional torque, T_f , model is shown in (13) when the velocity is nonzero;

$$T_f = T_c * \sin(\omega) + T_v * \omega, \quad (13)$$

where T_c is the coulomb friction in Nm, T_v is the viscous friction coefficient in Nm/rad/s, and ω is the angular velocity in rad/s.

However, to model the system more closely to a true physical system, the friction is modeled with a transition from

static friction to the commonly used kinetic friction model. Modeling friction in this manner is referred to as stick-slip friction modeling and has been modeled in more than one manner. The method chosen is the Tustin model [15] shown as follows:

$$T_f = \left(0.7 * T_s + 0.3 * T_s * \exp\left(\frac{-|\omega|}{\omega_s}\right) \right) * \text{sign}(\omega) + T_v * \omega, \quad (14)$$

where ω_s is the stick friction transition rate and T_s is the static friction torque.

Values for static friction and viscous friction coefficients are taken from [4] shown in Table 6, which is using a similar camera gimbal system.

TABLE 5: Zero-angle hold statistics.

Gyro type	$1 - \sigma$ (deg)	Mean (deg)	Two-sided standard deviation image size (m) at the following distances		
			1 km	6 km (20,000 ft)	24 km (80,000 ft)
I	0.0164	-0.0197	0.572	3.434	13.74
II	0.00285	$-7.97E-04$	0.0995	0.597	2.39
III	$2.57E-05$	$-3.78E-05$	$8.99E-04$	0.00539	0.0216
IV	$3.94E-06$	$-4.22E-06$	$1.37E-04$	$8.25E-04$	0.0033

FIGURE 20: Zero-angle hold performance comparison versus gyro type.

TABLE 6: Friction ranges.

Static friction	Viscous friction
0.05–0.45 (Nm)	0.015–0.045 (Nm/rad/s)

The static friction values range from 0.05 to 0.45 Nm and the viscous friction coefficients range within $1.5E-2$ – $4.5E-2$ Nm/rad/s. Values used are $T_s = 0.2$ Nm and $T_v = 0.025$ Nm/rad/s. As the system goes from static to kinetic friction, the kinetic coulomb friction is modeled to be 0.7 of the static friction, T_s , and the stick friction transition rate, $\omega_s = 0.01$ rad/s.

The frictional torque model is implemented in Simulink with an s function, calculating the torque and direction based on the angular rate of the gimbal. Also if the system is stationary, the motor torque must overcome the static friction before the system would move.

Running the zero-angle hold simulation on the system with friction, the system is first disturbed from zero degrees with one sample impulse command of one degree. This

is done because otherwise the system friction holds the gimbal at zero degrees without motion in most cases. When simulated with this initial system disturbance, as the system returns toward a zero-degree angle, there is some oscillation in the system near 5 to 10 seconds when either gyro III or gyro IV is used, as shown in Figure 21. Although in the order of 10^{-3} degrees, this happens in both the gyro III and gyro IV simulations. The ODE solver in Simulink is changed and the simulation is run with multiple different solver choices. This phenomenon occurs in all cases. This points to this effect being a control system induced oscillation as the systems transitions between friction regions and not a numerically induced effect. Due to this result, all zero-angle hold simulations with friction are shown and analyzed for steady state from 15 to 60 seconds in Figure 22.

The complete zero-angle hold statistics for model with friction are listed in Table 7. The oscillatory response of gyro IV results in degraded performance for this gyro with this system model. It seems the low noise level of the high performance gyro and the low velocities it reaches give rise to sticking effects in the static/kinetic friction transition region.

FIGURE 21: Zero-angle hold with friction for gyro III.

FIGURE 22: Zero-angle hold with friction model included.

This gives rise to a friction induced system oscillation as the controller attempts to overcome the sticking friction.

7.2. Structural Resonance. Structural resonance is the next physical parameter to be added to the system model. The structural resonance considered is that of the motor drive shaft and structural connections. Second-order resonant systems are used. A damping factor, ζ , of 0.05 is used to simulate a metallic resonance. The transfer function for any single given resonant frequency of f_{res} is

$$G_{\text{struct}}(s) = \frac{(2\pi f_{\text{res}})^2}{s^2 + 2\zeta(2\pi f_{\text{res}})s + (2\pi f_{\text{res}})^2}, \quad (15)$$

where multimodal structural resonances can be modeled by cascading second-order models for each resonant frequency peak. Without a structure to model and analyze, structural resonant frequencies are used from [4], which uses a similar camera gimbal system. The multimodal modeling of the system is accomplished by cascading four transfer functions with the frequencies of 140 Hz, 220 Hz, 260 Hz, and 295 Hz to represent the system structure. The resonance response for these frequencies is shown in Figure 23.

The zero-angle hold simulation is again run on the system, now including friction and structural resonance. The results are shown in Figure 24 and the simulation summary statistics are shown in Table 8.

FIGURE 23: Structural resonance frequency response.

FIGURE 24: Zero-angle hold with friction and resonance modeled.

TABLE 7: Zero-angle hold statistics for model with friction.

Gyro type	1 – σ (deg)	Two-sided standard deviation image size (m) at the following distances		
		1 km	6 km (20,000 ft)	24 km (80,000 ft)
I	0.0142	0.495	2.968	11.87
II	0.00257	0.0897	0.538	2.15
III	2.538E – 05	8.86E – 04	0.0053	0.0213
IV	0.00112	0.0391	0.235	0.938

TABLE 8: Simulation statistics including resonance.

Gyro type	$1 - \sigma$ (deg)	Two-sided standard deviation image size (m) at the following distances		
		1 km	6 km (20,000 ft)	24 km (80,000 ft)
I	0.01404	0.49	2.941	11.77
II	0.00238	0.083	0.498	1.99
III	$5.17E - 04$	0.018	0.108	0.433
IV	$5.01E - 04$	0.017	0.105	0.419

TABLE 9: Simulation statistics for 1 mm CG offset, vibration only.

Gyro type	$1 - \sigma$ (deg)	Two-sided standard deviation image size (m) at the following distances		
		1 km	6 km (20,000 ft)	24 km (80,000 ft)
I	0.01713	0.598	3.588	14.35
II	0.01699	0.593	3.558	14.23
III	$3.84E - 03$	0.134	0.805	3.221
IV	$6.45E - 03$	0.225	1.351	5.403

TABLE 10: Simulation statistics for 5 mm CG offset, vibration only.

Gyro type	$1 - \sigma$ (deg)	Two-sided standard deviation image size (m) at the following distances		
		1 km	6 km (20,000 ft)	24 km (80,000 ft)
I	0.0281	0.981	5.888	23.55
II	0.0192	0.671	4.024	16.1
III	0.0192	0.672	4.029	16.12
IV	0.2102	7.336	44.02	176.1

The results show that gyro IV performed with a smaller standard deviation with structural resonance included compared with friction alone. It seems that the small vibrations induced by the structural resonance help to keep the system from being stuck in the static friction region. This is similar to the use of dithering to overcome sticking friction. Although the performance of gyro IV improved from the previous simulation with friction only, its performance here is very similar to gyro III.

7.3. Vibration Induced Disturbance Torque. Disturbance torques due to base vibration can be caused when there is a mass imbalance of the system. If the center of mass of the camera gimbal system is offset from the rotational axis, a disturbance torque will be caused by external vibration [23]. This disturbance torque can be modeled as a band limited white noise torque added to the motor torque. Vibration levels in helicopters from [24, 25] are used to approximate the vibration level applied to the imbalanced gimbal. The vibration levels in [24] have frequency spectrum peaks from 0.84 m/s^2 to 1.68 m/s^2 . To take an average value to consider for white noise levels, 0.5 m/s^2 is used. This vibration induces the disturbance torque by modeling a center of mass offset ranging between 1 mm and 5 mm. The disturbance torque magnitude level is calculated as the product of the vibration

acceleration, a , the gimbal mass, m , and the offset radius, r_{offset} :

$$T_d = a * m * r_{\text{offset}}. \quad (16)$$

To calculate power spectral density (PSD) torque levels to apply for the Simulink noise generation, the torque level must be squared. For the range of offset radii, the PSD levels range within $3.76E - 5$ – $9.41E - 4 \text{ (N}\cdot\text{m)}^2/\text{Hz}$.

Simulations including disturbance torque are run in three different configurations: the disturbance torque acting on the ideal system without friction, acting on the system with only friction added to the model, and finally acting with friction and structural resonance is included. These cases are performed for center of mass offsets of both 1 mm and 5 mm. Simulation plots are shown in Figures 25–30 with simulation statistics shown in Tables 9 and 10 for the ideal system. For the cases including friction and resonance, simulation performance statistics are summarized in the conclusion in Table 11.

7.4. Dynamic Input. Lastly, the system is simulated with a dynamic sinusoidal command input. This is to represent the command signal generated to stabilize the camera gimbal while mounted under a helicopter making dynamic flight maneuvers. This sinusoidal input signal has amplitude of

FIGURE 25: Zero-angle hold for CG offset of 1 mm, vibration only.

FIGURE 26: Zero-angle hold for CG offset of 5 mm, vibration only.

FIGURE 27: Vibration for CG offset of 1 mm, friction only.

FIGURE 28: Vibration for CG offset of 1 mm, friction, and structural resonance.

FIGURE 29: Vibration for CG offset of 5 mm, friction only.

FIGURE 30: Vibration for CG offset of 5 mm, friction, and structural resonance.

10 degrees and a period of 12.57 seconds. Simulations are conducted for two different model configurations: the system with friction included and the system with friction, low level vibration, and structural resonance included.

Simulation plots are shown for 30 seconds of tracking the input command. Figure 31 shows the response when gyro III is used for rate and angle measurements. The tracking error

can be seen when the rotation changes directions and static friction must be overcome.

Figure 32 plots the LOS error for the simulation for the four different sensors. Gyro III results in the best performance with the lowest LOS error magnitude and no noticeable jitter. Gyro II also has low LOS error magnitude, but jitter is present in its response. Gyro IV has no apparent jitter, but the LOS

TABLE 11: Summary of zero-angle hold performance.

1 – sigma angle (degrees) for gyro type	I	II	III	IV
Zero-angle hold, ideal	0.0164	0.00285	$2.57E-05$	$3.94E-06$
(i) Friction	0.01417	0.00257	$2.54E-05$	0.00112
(ii) Friction and resonance	0.01404	0.00238	$5.17E-04$	$5.01E-04$
Vibration				
(i) Low, $(3.8E-5 \text{ (Nm)}^2/\text{Hz})$				
(ii) Ideal	0.0171	0.017	0.00384	0.00645
(iii) Friction	0.0166	0.00329	0.0023	0.00304
(iv) Friction and resonance	0.0168	0.00429	0.00343	0.00487
Vibration				
(i) High, $(9.5E-4 \text{ (Nm)}^2/\text{Hz})$				
(ii) Ideal	0.0281	0.0192	0.0192	0.21
(iii) Friction	0.0254	0.0172	0.0171	0.129
(iv) Friction and resonance	0.0273	0.0194	0.0194	0.152

FIGURE 31: Sinusoidal input signal and response for gyro III.

error is approximately five times larger than that of gyro III. Gyro I displays the worst performance of all.

When structural resonance and vibration disturbance torque are included, the performance of gyro III is shown in Figure 33. In the zoomed in plot, the jitter about the command reference can be seen. The LOS error for all four gyros is plotted in Figure 34. Under these conditions gyro II and gyro III have similar system performance and outperform both gyro I and IV.

8. Conclusion

This work presented a systematic procedure for modeling and simulating a camera gimbal system, to perform LOS tracking

performance based on the quality of the sensors selected to provide feedback measurements to stabilize the system.

One significant result of this work is in the design of the controller. An effective method of increasing the controller type of a system to reduce LOS error for tracking dynamic input commands, in this case designing a PI^2 controller, is performed. This method is to design a PID controller using well established methods to achieve the desired system response for lower type inputs, such as a step input. Then an additional integral stage can be cascaded with the original controller. This integral gain can then be increased during simulations on higher type input commands, such as ramp or sinusoidal inputs, until the desired performance for the higher type inputs is achieved. During this process the

TABLE 12: Summary of dynamic input system performance.

Steady state max LOS error (deg.) for gyro type	I	II	III	IV
(i) Friction only	0.054	0.015	0.0059 (0.00475)	0.0228
(ii) Qualitative	Jitter/noise mean drift	Jitter/noise mean drift	Smooth	Smooth
(iii) Friction, resonance, and vibration	0.0612	0.0215	0.0153	0.109

FIGURE 32: Sinusoidal input LOS error for all four gyros.

FIGURE 33: Sinusoidal input signal, friction, vibration, and resonance for gyro III.

FIGURE 34: Sinusoidal input LOS error for friction, vibration, and resonance.

performance for the lower type input remains nearly identical to the original controller design.

The other result is of main interest. Tables 11 and 12 summarize the simulation performance results for the various cases performed in this work. When the ideal system is simulated, the system performance increases with each increase in the quality of the sensor used for measurement. However, when friction, structural resonance, and vibration are introduced into the model, there are instances when higher quality gyros no longer provide improved system performance.

When friction is included, the performance of gyro IV suffers from the sticking effects of friction and is slightly outperformed by gyro II. However, when structural resonance is included, the added motion helps to overcome the sticking effects of friction and increases the performance of gyro IV, but it is still matched by gyro III.

Of significance is the fact that the sticking effects of friction with gyro IV in a vibration-free environment were observed by NGC in applications in similar conditions. This confirms that the modeling performed not only is for an ideal system, but truly represents a system with more complex realistic effects.

When low level vibration is added, the performance of gyro IV degrades and is outperformed by gyro III. In this

case gyro I and II have matching performance. However when the vibration level is increased, gyro IV has the worst performance. Gyro III and gyro II have matching performance, only slightly outperforming gyro I.

For the simulations with a dynamic input command signal, gyro III has the best performance. The LOS error reaches its peak value when overcoming friction. For gyro III this is the maximum LOS error. The value in parentheses is the maximum LOS error apart from the region of overcoming friction. When all the system parameters are included with low level vibration, gyro III still performs the best, nearly matched by gyro II and outperforming gyro I by a small margin.

This verifies the value of modeling and simulation in system engineering applications to understand the component trade studies to ensure correct sensor selection for the desired application. The benefits of modeling system parameters accurately and adding environmental disturbances can lead to valuable information for choosing sensors that will provide desired system performance at a reasonable cost.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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